# ifferentiable programming tensor networks and quantum circuits

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## Collaborators





#### Hai-Jun Liao





### Jin-Guo Liu (on the market)

Tao Xiang

1903.09650 <u>https://github.com/wangleiphy/tensorgrad</u>



## Differentiable Programming





#### Andrej Karpathy

Director of AI at Tesla. Previously Research Scientist at OpenAI and PhD student

https://medium.com/@karpathy/software-2-0-a64152b37c35

#### Writing software 2.0 by gradient search in the program space

## Differentiable Programming

#### **Benefits of Software 2.0**

- Computationally homogeneous
- Simple to bake into silicon
- Constant running time
- Constant memory usage
- Highly portable & agile
- Modules can meld into an optimal whole
- Better than humans

### Writing software 2.0 by gradient search in the program space



#### Andrej Karpathy

Director of AI at Tesla. Previously Research Scientist at OpenAI and PhD student at Stanford. I like to train deep neural nets on large datasets.

https://medium.com/@karpathy/software-2-0-a64152b37c35





### The engine of deep learning



## Compose differentiable components to a program e.g. a neural network, then optimize with gradient

## Computation Graph



#### "comb" graph

data

### Pullback the adjoint through the graph







#### "comb" graph



#### Define "ad

### Pullback the adjoint through the graph

djoint" 
$$\overline{x} = \frac{\partial \mathscr{L}}{\partial x}$$

## Computation Graph





#### "comb" graph



### Pullback the adjoint through the graph



## Computation Graph





#### "comb" graph



#### Define "ac

### Pullback the adjoint through the graph

djoint" 
$$\overline{x} = \frac{\partial \mathscr{L}}{\partial x}$$

## Computation Graph



### Define "adjoint" $\overline{x} = \frac{\partial \mathscr{L}}{\partial x}$

### Pullback the adjoint through the graph

## Computation Graph



#### "comb" graph

### Pullback the adjoint through the graph



## Computation Graph



#### directed acyclic graph

Message passing for the adjoint at each node



### Advantages of automatic differentiation

Accurate to the machine precision

 Same computational complexity as the function evaluation: Baur-Strassen theorem '83

Supports higher order gradients





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## Applications of AD

#### **Computing force Quantum optimal control**





Sorella and Capriotti J. Chem. Phys. '10



Leung et al PRA '17

Tamayo-Mendoza et al ACS Cent. Sci. '18

## Understandings of AD





Black magic box Chain rule



with Will Farr

#### Functional differential geometry

https://colab.research.google.com/ github/google/jax/blob/master/ notebooks/autodiff\_cookbook.ipynb



## Reverse versus forward mode

- Backtrace the computation graph
- Needs to store intermediate results
- Efficient for graphs with large fan-in



Reverse mode AD: Vector-Jacobian Product of primitives

$$v_o(J)_{o \times i}$$

**Backpropagation = Reverse mode AD applied to neural networks** 

# Reverse versus forward mode

- Same order with the function evaluation
- No storage overhead
- Efficient for graph with large fan-out

 $\frac{\partial \mathscr{L}}{\partial \theta} = \frac{\partial \mathscr{L}}{\partial x_n} \frac{\partial x_n}{\partial x_{n-1}} \frac{\partial x_2}{\partial x_1} \frac{\partial x_1}{\partial \theta}$ 

Forward mode AD: Jacobian-Vector Product of primitives

 $(J)_{o \times i} v_i$ 

Less efficient for scalar output, but useful for higher-order derivatives



## How to think about AD ?

- AD is modular, and one can control its granularity
- Benefits of writing customized primitives
  - Reducing memory usage
  - Increasing numerical stability
  - Call to external libraries written agnostically to AD (or, even a quantum processor)



(X) P E N N Y L A N E

## Example of the primitives

### ~200 functions to cover most of numpy in HIPS/autograd https://github.com/HIPS/autograd/blob/master/autograd/numpy/numpy\_vjps.py

Operators	+, -, *, /, (-), **, %, <, <=, ==, !=, >=, >				
Basic math functions	exp, log, square, sqrt, sin, cos, tan, sinh,				
	cosh, tanh, sinc, abs, fabs, logaddexp,				
	logaddexp2, absolute, reciprocal, exp2,				
	expm1, log2, log10, log1p, arcsin, arccos,				
	arctan, arcsinh, arccosh, arctanh, rad2deg,				
	degrees, deg2rad, radians				
Complex numbers	real, imag, conj, angle, fft, fftshift,				
•	ifftshift, real_if_close				
Array reductions	sum, mean, prod, var, std, max, min, amax, amin				
Array reshaping	reshape, ravel, squeeze, diag, roll,				
	array_split, split, vsplit, hsplit, dsplit,				
	expand_dims, flipud, fliplr, rot90, swapaxes,				
	rollaxis, transpose, atleast_1d, atleast_2d,				
	atleast_3d				
Linear algebra	dot, tensordot, einsum, cross, trace, outer,				
	det, slogdet, inv, norm, eigh, cholesky, sqrtm,				
	solve_triangular				
Other array operations	cumsum, clip, maximum, minimum, sort,				
	msort, partition, concatenate, diagonal,				
	truncate pad, tile, full, triu, tril, where,				
	diff, nan_to_num, vstack, hstack				
Probability functions	t.pdf, t.cdf, t.logpdf, t.logcdf,				
	multivariate_normal.logpdf,				
	multivariate_normal.pdf,				
	multivariate_normal.entropy, norm.pdf,				
	norm.cdf, norm.logpdf, norm.logcdf,				

### Loop/Condition/Sort/Permutations are also differentiable

•••	Q autograd/numpy_vjps.py at mas × +
←) →	(i) 🔒 GitHub, Inc. (US) https://github.com/HIPS/autograd/b 133% ···· ♡ ☆ 🖳 🖞 💷 😨 🔶 💌 T ≫ 🖆
67	# Simple grads
68	
69	defvjp(anp.negative, lambda ans, x: lambda g: -g)
70	defvjp(anp.abs,
71	<code>lambda</code> ans, x : <code>lambda</code> g: g * replace_zero(anp.conj(x), 0.) / replace_zero(ans, 1.))
72	defvjp(anp.fabs, lambda ans, x : lambda g: anp.sign(x) $*$ g) # fabs doesn't take complex numbers.
73	defvjp(anp.absolute, lambda ans, x : lambda g: g * anp.conj(x) / ans)
74	defvjp(anp.reciprocal, <mark>lambda</mark> ans, x : <mark>lambda</mark> g: − g / x <b>*</b> *2)
75	defvjp(anp.exp, lambda ans, x : lambda g: ans * g)
76	defvjp(anp.exp2, lambda ans, x : lambda g: ans * anp.log(2) * g)
77	defvjp(anp.expm1, lambda ans, x : lambda g: (ans + 1) * g)
78	defvjp(anp.log, lambda ans, x : lambda g: g / x)
79	defvjp(anp.log2, lambda ans, x : lambda g: g / x / anp.log(2))
80	defvjp(anp.log10, lambda ans, x : lambda g: g / x / anp.log(10))
81	defvjp(anp.log1p, lambda ans, x : lambda g: g / (x + 1))
82	defvjp(anp.sin, lambda ans, x : lambda g: g * anp.cos(x))
83	defvjp(anp.cos, lambda ans, x : lambda g: - g * anp.sin(x))
84	defvjp(anp.tan, lambda ans, x : lambda g: g / anp.cos(x) **2)
85	defvjp(anp.arcsin, lambda ans, x : lambda g: g / anp.sqrt(1 – x**2))
86	defvjp(anp.arccos, lambda ans, x : lambda g:-g / anp.sqrt(1 - x**2))
87	defvjp(anp.arctan, lambda ans, x : lambda g: g / (1 + x**2))
88	detvjp(anp.sinh, lambda ans, x : lambda g: g * anp.cosh(x))
89	defvjp(anp.cosh, lambda ans, x : lambda g: g * anp.sinh(x))
90	defvjp(anp.tanh, lambda ans, x : lambda g: g / anp.cosh(x) **2)
91	defvjp(anp.arcsinh, lambda ans, x : lambda g: g / anp.sqrt(x**2 + 1))
92	defvjp(anp.arccosn, lambda ans, x : lambda g: g / anp.sqrt(x**2 - 1))
93	detvjp(anp.arctann, lambda ans, x : lambda g: g / (1 – x**2))
94	defvjp(anp.rad2deg, lambda ans, x : lambda g: g / anp.p1 * 180.0)
95	defvip(anp.degrees, lambda ans, x : lambda g: g / anp.pl * 180.0)
90	definition redience lembda and x + lembda q; q + anp.pl / 180.0)
97	defvip(anp.raulans, lambda ans, x : lambda g: g * anp.pl / 180.0)
90	definition can t = lombdo construction (construction) = lombdo construction (constr
99	uervjp(anp.syrt, lambda ans, x : lambda g: g * 0.5 * X**-0.5)

### Differentiable programming tools

### **HIPS/autograd**

### **O** PyTorch





theano







## Current support for AD\*

	linalg	comp
PyTorch	$\checkmark$	X
TensorFlow	$\checkmark$	Ť
Autograd	$\checkmark$	<b>M</b>
Jax	$\checkmark$	Ť
Flux.jl/Zygote.jl	X	$\checkmark$

Jin-Guo's blog post https://giggleliu.github.io/2019/04/02/einsumbp.html



\*as of July 2019



## Differentiable Scientific Programming

- Most linear algebra operations (Eigen, SVD!) are <u>differentiable</u>
- ODE integrators are differentiable with O(1) memory
- <u>Differentiable ray tracer and Differentiable fluid simulations</u>
- Differentiable Monte Carlo/Tensor Network/Functional RG/ Dynamical Mean Field Theory/Density Functional Theory/ Hartree-Fock/Coupled Cluster/Gutzwiller/ Molecular Dynamics...

Differentiable programming is more than training neural networks





## Differentiable Eigensolver $H\Psi = \Psi \Lambda$

#### What happen if $H \rightarrow H + dH$ ? **Forward mode:**

### **Reverse mode:** How should I change H given

### Hamiltonian engineering via differentiable programming



Perturbation theory

Inverse perturbation theory!  $\partial \mathscr{L}/\partial \Psi$  and  $\partial \mathscr{L}/\partial \Lambda$ ?

https://github.com/wangleiphy/DL4CSRC/tree/master/2-ising See also Fujita et al, PRB '18













### **AD** computes physical observables as high-order gradients

## Tensor network quantum states

### Optimization



- Trotterized imaginary-time projection
- Update schemes: "simple", "full" "cluster", "faster full"...

### Contraction





- #P hard in general
- Approximated schemes: TRG, Boundary MPS, Corner transfer matrix RG

### Expressibility v.s. Optimization: an eternal problem



Osorio, Corboz, Troyer, PRB '14



Typically, one finds ordered states at small D and tries hard to push up the bond dimensions

## Variational optimization infinite tensor networks $\mathcal{L}_{\theta} = \left\langle \Psi_{\theta} | \hat{H} | \Psi_{\theta} \right\rangle / \left\langle \Psi_{\theta} | \Psi_{\theta} \right\rangle$



Corboz, PRB '16 Vanderstraeten et al, PRB '16

Variational optimization with gradient indeed help! However, manually deriving gradients is cumbersome

Corboz et al, PRX '18 Rader et al, PRX '18



## Variational optimization infinite tensor networks $\mathscr{L}_{\theta} = \langle \Psi_{\theta} | \hat{H} | \Psi_{\theta} \rangle / \langle \Psi_{\theta} | \Psi_{\theta} \rangle$



Corboz, PRB '16 Vanderstraeten et al, PRB '16 Variational optimization with gradient indeed help! Corboz et al, PRX '18 However, manually deriving gradients is cumbersome Rader et al, PRX '18



### Automatic differentiation to the rescue

### Optimization



conjugate-gradient, quasi-Newton, etc

### Any lattice, any Hamiltonian, any contraction scheme

Human only cares about tensor contraction Differentiable programing takes care of the optimization

#### Contraction









conjugate-gradient, quasi-Newton, etc

### Any lattice, any Hamiltonian, any contraction scheme

Human only cares about tensor contraction Differentiable programing takes care of the optimization

CTMRG, Nishino, Okunishi, JPSJ, '95





## Nuts and Bolts

Numerical stable backward through SVD

$$T_{i+1} = f(T_i, \theta) \xrightarrow{\text{Iterate}} T^* = f(T^*, \theta) \qquad \overline{\theta} = \overline{T^*} \left[ 1 - \frac{\partial f}{\partial T^*} \right]^{-1} \frac{\partial f}{\partial \theta}$$

 $A = UDV^T \qquad \overline{A} \xleftarrow{?} \overline{U}, \overline{D}, \overline{V}$ 

• Reduce memory via checkpointing or exploiting RG fixed point property

Liao, Liu, LW, Xiang, 1903.09650













# **Infinite size**

AD optimized iPEPS



Liao, Liu, LW, Xiang, 1903.09650

#### Square lattice Heisenberg model **Finite size** VMC of an RVB-type state -0.6692 -0.6694 1x10-4@L=64 E/N $-\bullet L = 64$ (variational) -0.6696 -0L = 32 (variational) = 16 (variational) = 64 (exact QMC) = 32 (exact QMC) -0.6698 -L = 16 (exact QMC) $4 \times 10^{-5} @L=16$ -0.6700 3 0 correlation level

Lin, Tang, Sandvik, PRB '12





### **Infinite size** AD optimized iPEPS



### Square lattice Heisenberg model

### **Finite size** VMC optimized RBM



Carleo & Troyer, Science '17

#### **Infinite size** AD optimized iPEPS



### Square lattice Heisenberg model

### **Finite size** VMC optimized RBM



Carleo & Troyer, Science '17

### Lowest variational energy for infinite system https://githtlb.com/wangleiphy/tensorgrad 1 GPU (Nvidia P100) week

### Kitaev honeycomb model



#### **Reaches lower energy even at smaller bond dimensions** with substantially reduced magnetic order

c.f. analytically constructed iPEPS, Lee et al, 1901.05786



### The morals

- iPEPS with small bond dimensions are more expressive than we thought. We just did not optimize them hard enough
- Differentiable programming tensor networks has a bright future: variational contraction, gauge fixing, fermions...





## The morals

- iPEPS with small bond dimensions are more expressive than we thought. We just did not optimize them hard enough
- Differentiable programming tensor networks has a bright future: variational contraction, gauge fixing, fermions...



BTW, the difficulty of optimizing neural network quantum states with VMC: stochastic optimization with correlated samples and poor gradient estimator (potentially can also be fixed by ML).

### Ground state phase diagram of the J1-J2 model



+ many many others





# Ground state phase diagram of the J1-J2 model





### Ground state phase diagram of the J1-J2 model





#### IOP, CAS Hai-Jun Liao





#### Neural Nets — Probabilistic Graphical Models — Tensor Nets — Quantum Circuits

Differentiable Programming Quantum Circuits



## Variational quantum eigensolver



**Quantum circuit as a variational ansatz** 

Peruzzo et al, Nat. Comm. '13





Google PRX '16

## VQE on actual guantum devices



#### Scan 1000 values of the single variational parameter



### These optimization schemes do not scale to higher dimensions

### Optimize the quantum circuit

Stochastic gradient descend with numerical derivative





Parametrized gate of the form



### Differentiable quantum circuits

Li et al, PRL '17, Mitarai et al, PRA '18 Schuld et al, PRA '19, Nakanishi et al '19

$$\sum \left( \nabla \langle H \rangle_{\theta} = \left( \langle H \rangle_{\theta + \pi/2} - \langle H \rangle_{\theta - \pi/2} \right) \right)$$

#### Unbiased gradient estimator measured on quantum circuits



#### Monte Carlo Gradient Estimation in Machine Learning

Shakir Mohamed\* Mihaela Rosca\* Michael Figurnov<sup>\*</sup> Andriy Mnih<sup>\*</sup> \*Equal contributions; DeepMind, London

#### 59 pages survey, three types of gradient estimators 1906.10652

With so many competing approaches, we offer our rules of thumb in choosing an estimator, which follow the intuition we developed throughout the paper:

- have variance that is low enough so as not to interfere with the optimisation.

SHAKIR@GOOGLE.COM MIHAELACR@GOOGLE.COM MFIGURNOV@GOOGLE.COM AMNIH@GOOGLE.COM

10.1 Guidance in Choosing Gradient Estimators  $\nabla_{\theta} \mathbb{E}_{x \sim p_{\theta}} \left[ f(x) \right]$ 

• If our estimation problem involves continuous functions and measures that are continuous in the domain, then using the pathwise estimator is a good default. It is relatively easy to implement and a default implementation, one without other variance reduction, will typically

• If the cost function is not differentiable or a black-box function then the score-function or the measure-valued gradients are available. If the number of parameters is low, then the measurevalued gradient will typically have lower variance and would be preferred. But if we have a high-dimensional parameter set, then the score function estimator should be used.

• If we have no control over the number of times we can evaluate a black-box cost function, effectively only allowing a single evaluation of it, then the score function is the only estimator



## Optimization with noisy gradient



#### VQE encounters the "same type" of stochastic optimization in deep learning

Ruder, 1609.04747



## Optimization with noisy gradient



#### VQE encounters the "same type" of stochastic optimization in deep learning

Ruder, 1609.04747





### Train quantum circuits as probabilistic g



It is a paradigm beyond quantum-classical hybrid

- Variational quantum eigensovler (VQE)
- Quantum circuit Born machine (QCBM)
- Quantum approximate optimization algorithm (QAOA)
- Quantum pattern recognition

. . .

Quantum circuit classifier TNS inspired circuit architecture VQE with fewer qubits Quantum generative model Quantum adversarial training

Farhi, Neven, 1802.06002 Havlicek et al, 1804.11326 Huggins, Patel, Whaley, Stoudenmire, 1803.11537 Liu, Zhang, Wan, LW, 1902.02663 Gao, Zhang, Duan, 1711.02038 Dallaire-Demers, Lloyd, Benedetti 1804.08641,1804.09139, 1806.00463







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#### Near term:

What can we do with noisy circuits of limited depth?

#### Long term:

Are we really good at programing quantum computers?







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### It is a paradigm beyond quantum-classical hybrid

#### Quantum code







- Variational quantum eigensovler (VQE)
- Quantum circuit Born machine (QCBM)
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. . . Quantum circuit TNS inspired circ VQE with fewer ( Quantum genera Quantum advers

#### It is a paradigm beyond quantum-classical hybrid

#### Quantum code







### Be prepared for Quantum Software 2.0

https://github.com/QuantumBFS/Yao.jl



#### Jin-Guo Liu (IOP, CAS) Xiu-Zhe Luo (Waterloo & Pl)

Features:



• Differentiable programming quantum circuits Batch parallelization with GPU acceleration Quantum block intermediate representation





#### **Differentiable Programming Tensor Networks**

Hai-Jun Liao, Jin-Guo Liu, LW, Tao Xiang, 1903.09650, PRX in press

### Yao.jl: Extensible, Efficient Framework for Quantum Algorithm Design Xiu-Zhe Luo, Jin-Guo Liu, Pan Zhang, LW, up coming



### Thank You!



 $|0\rangle_{Q_0} |0\rangle_{Q_1} - |H|$  $|0\rangle_{Q_2} - |H|$  $|0\rangle_{Q_3} - |H|$ 

**Tensor Networks** 

**Quantum Circuits** 



