



**Yao.jl:** Extensible, Efficient Framework  
for Quantum Algorithm Design

<http://yaoquantum.org/>

# Creators of Yao



Xiu-Zhe Luo, U Waterloo & PI



Jin-Guo Liu, IOP CAS

# *What is the killer app of a near-term quantum computer?*



In about next 3 years

Small:  $O(10)$ - $O(10^3)$  qubits

Shallow:  $O(10^2)$ - $O(10^4)$  gates

Noisy: no error correction



# Quantum Algorithms



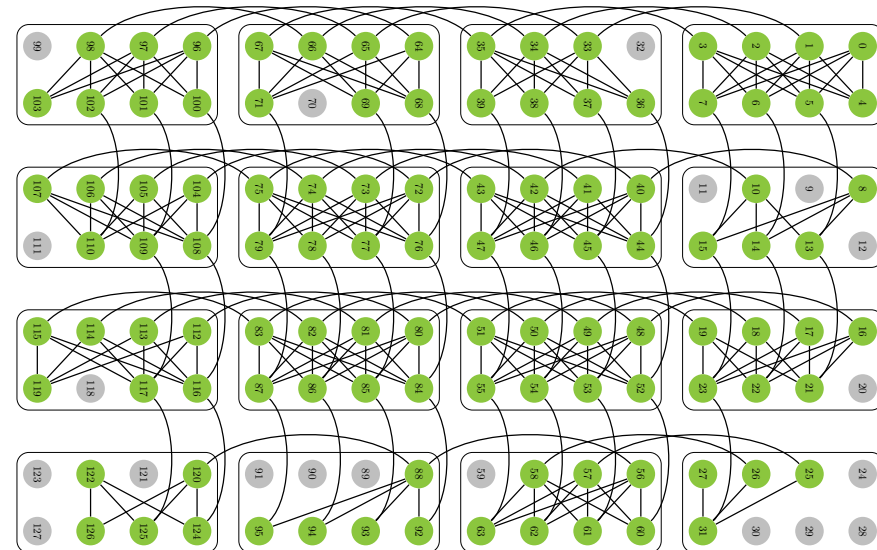
Cryptography



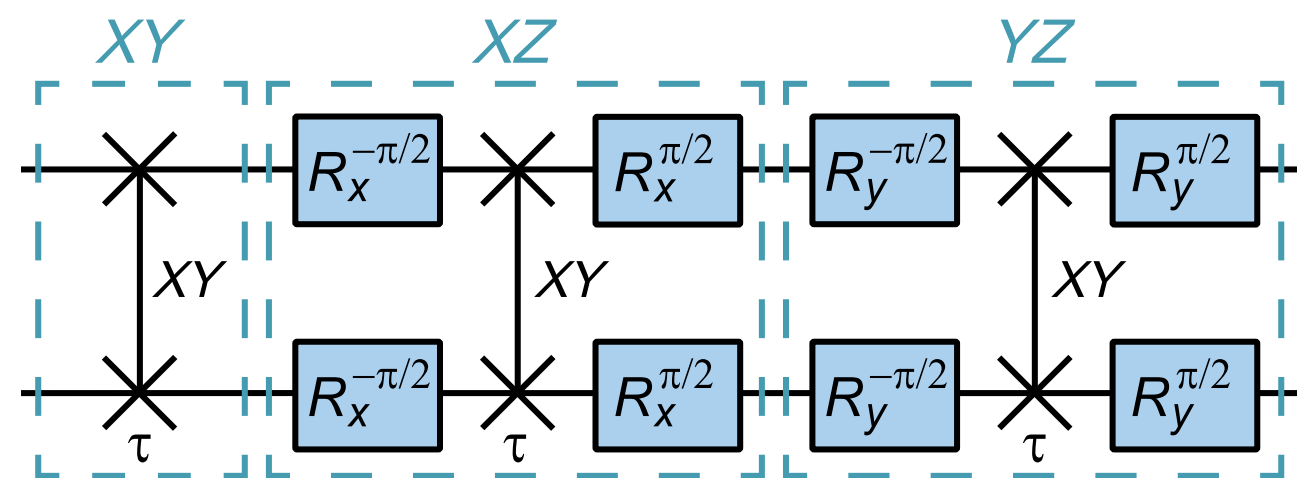
Search

$$A|x\rangle = |b\rangle$$

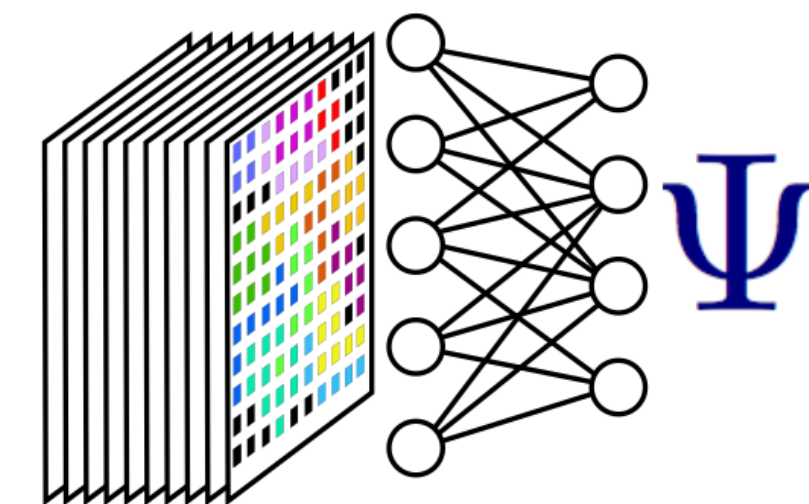
Linear Algebra



Quantum Annealing  
and Optimization



Quantum  
Simulation



Quantum  
Machine Learning

# Quantum Algorithms

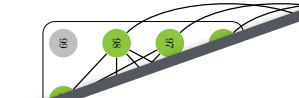


Cryptography

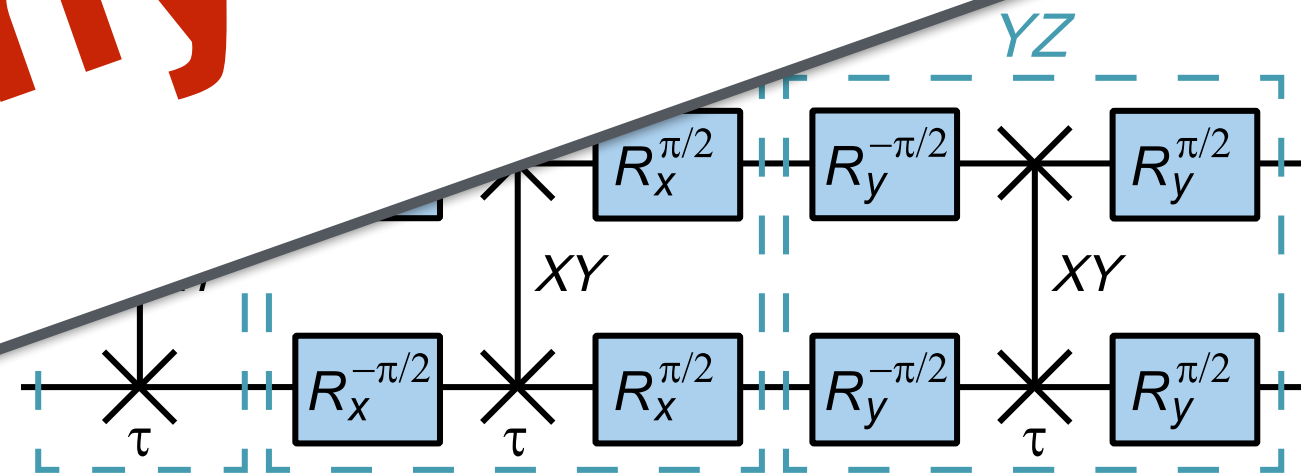


**Many caveats ...**

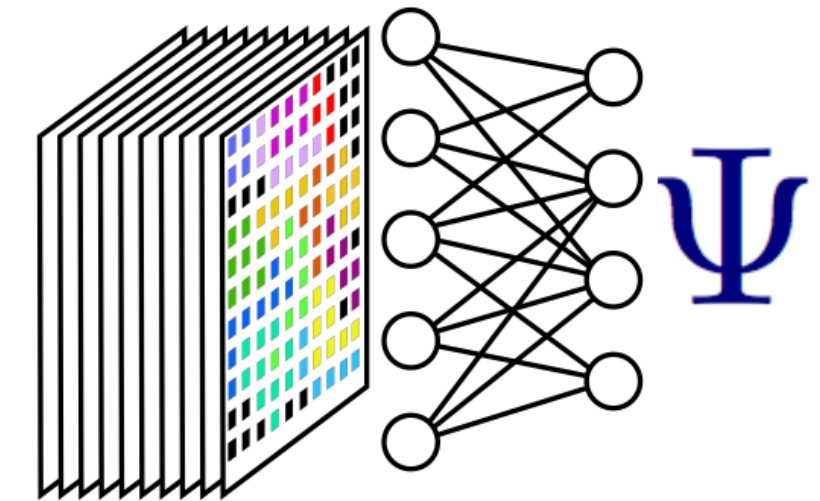
Algebra



Quantum Annealing  
and Optimization

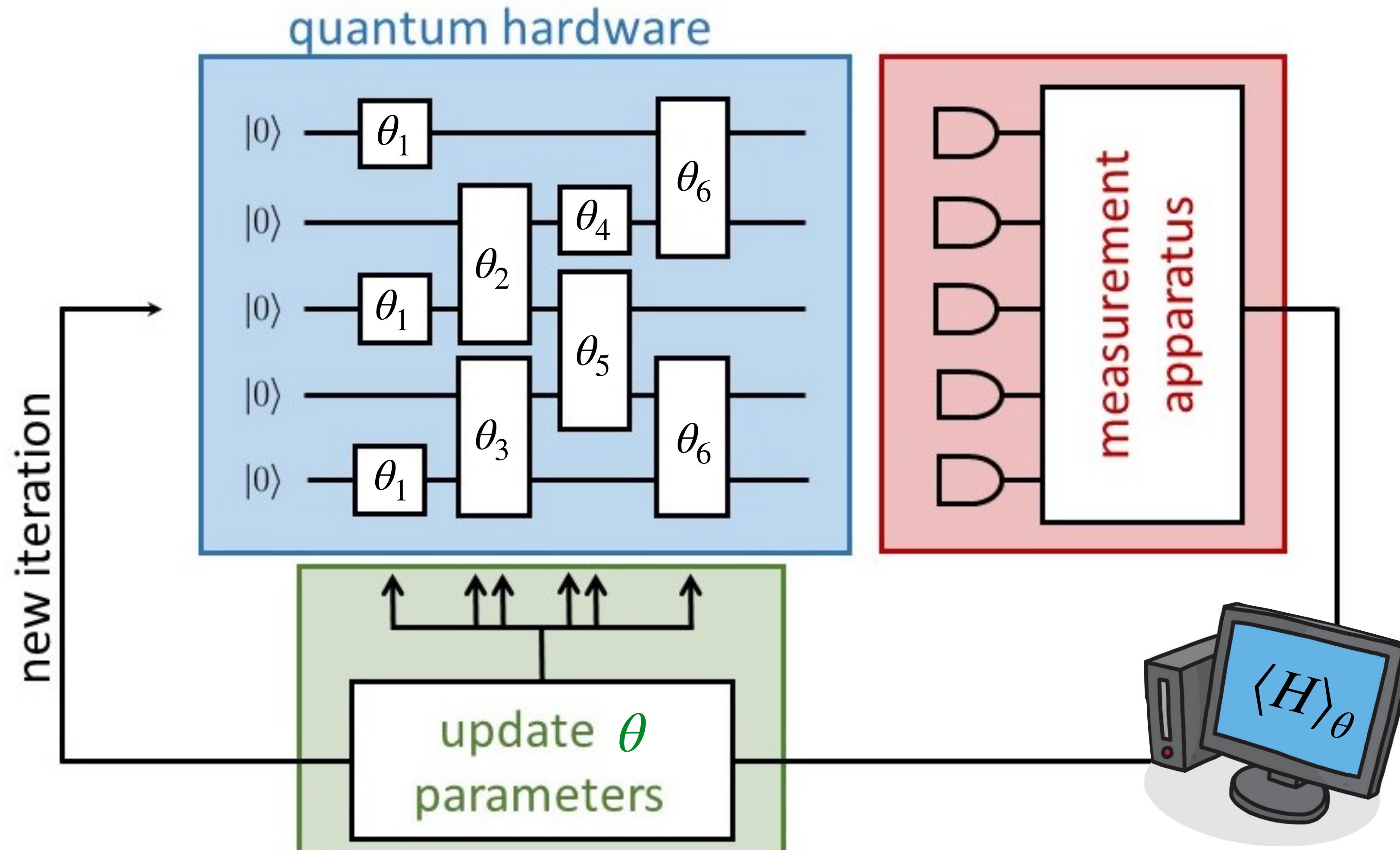


Quantum  
Simulation



Quantum  
Machine Learning

# Variational quantum eigensolver

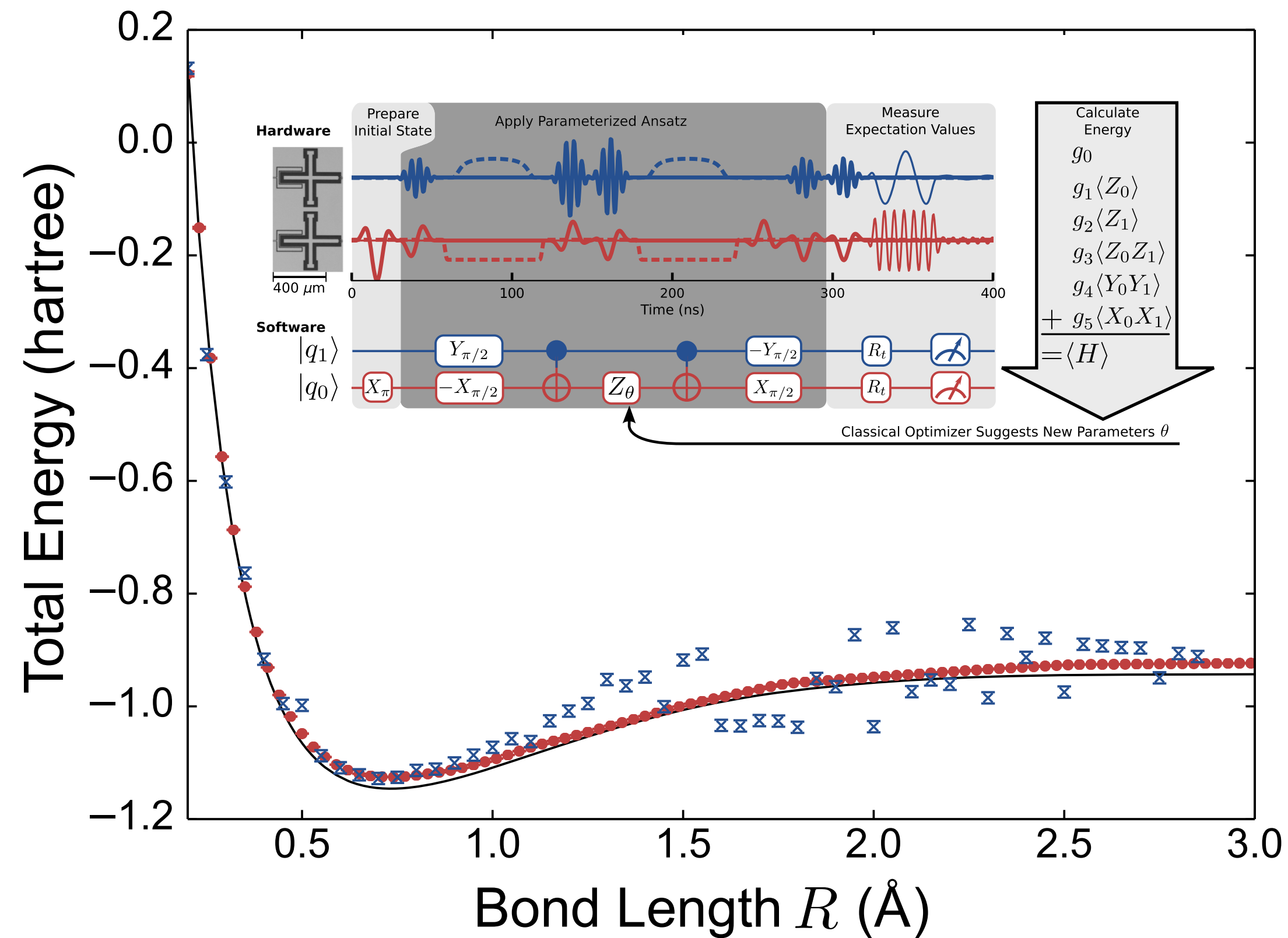


**Quantum circuit as a variational ansatz**

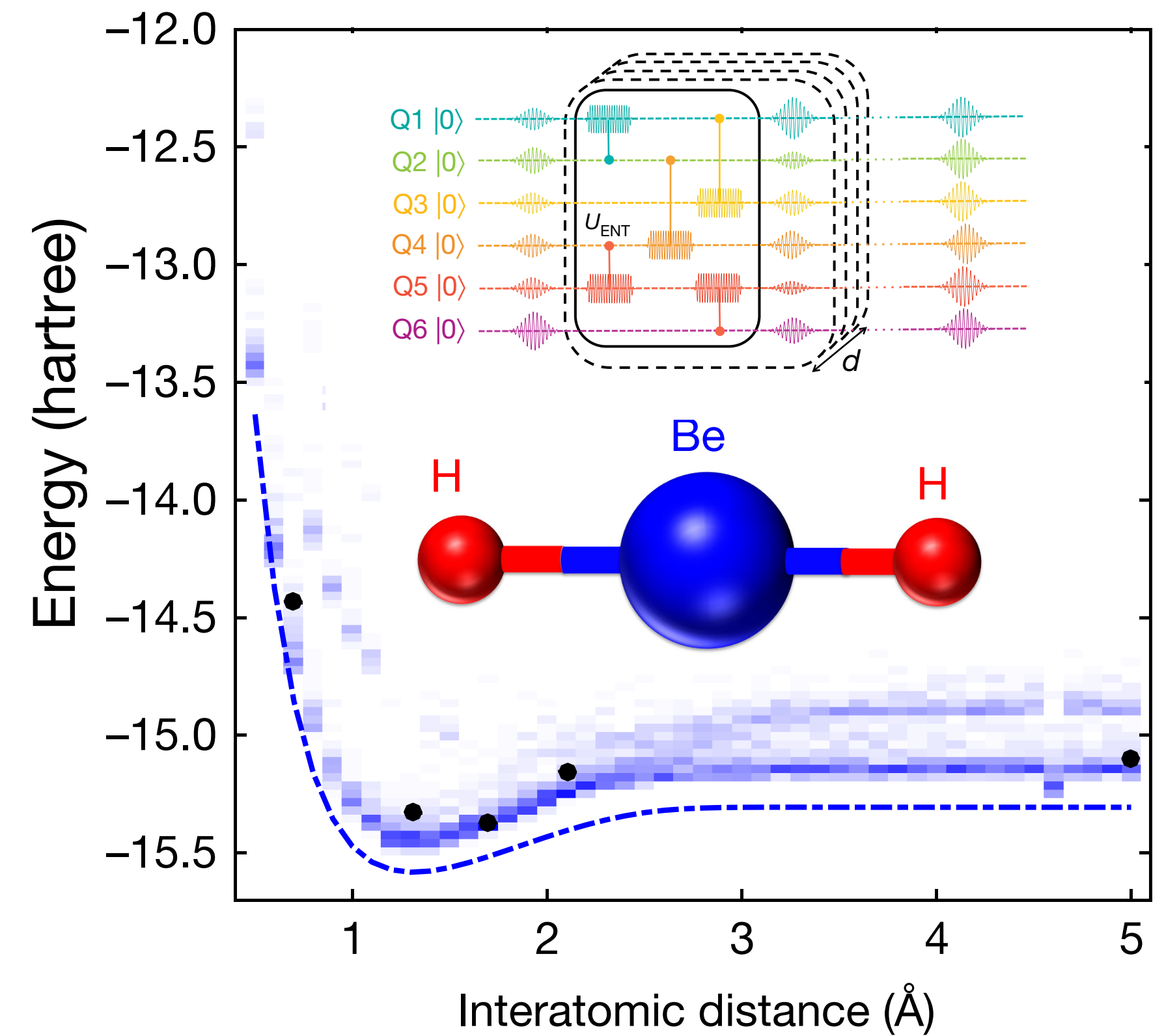
Peruzzo et al,  
Nat. Comm. '13

# VQE on actual quantum devices

## H<sub>2</sub> molecule with 2 qubits

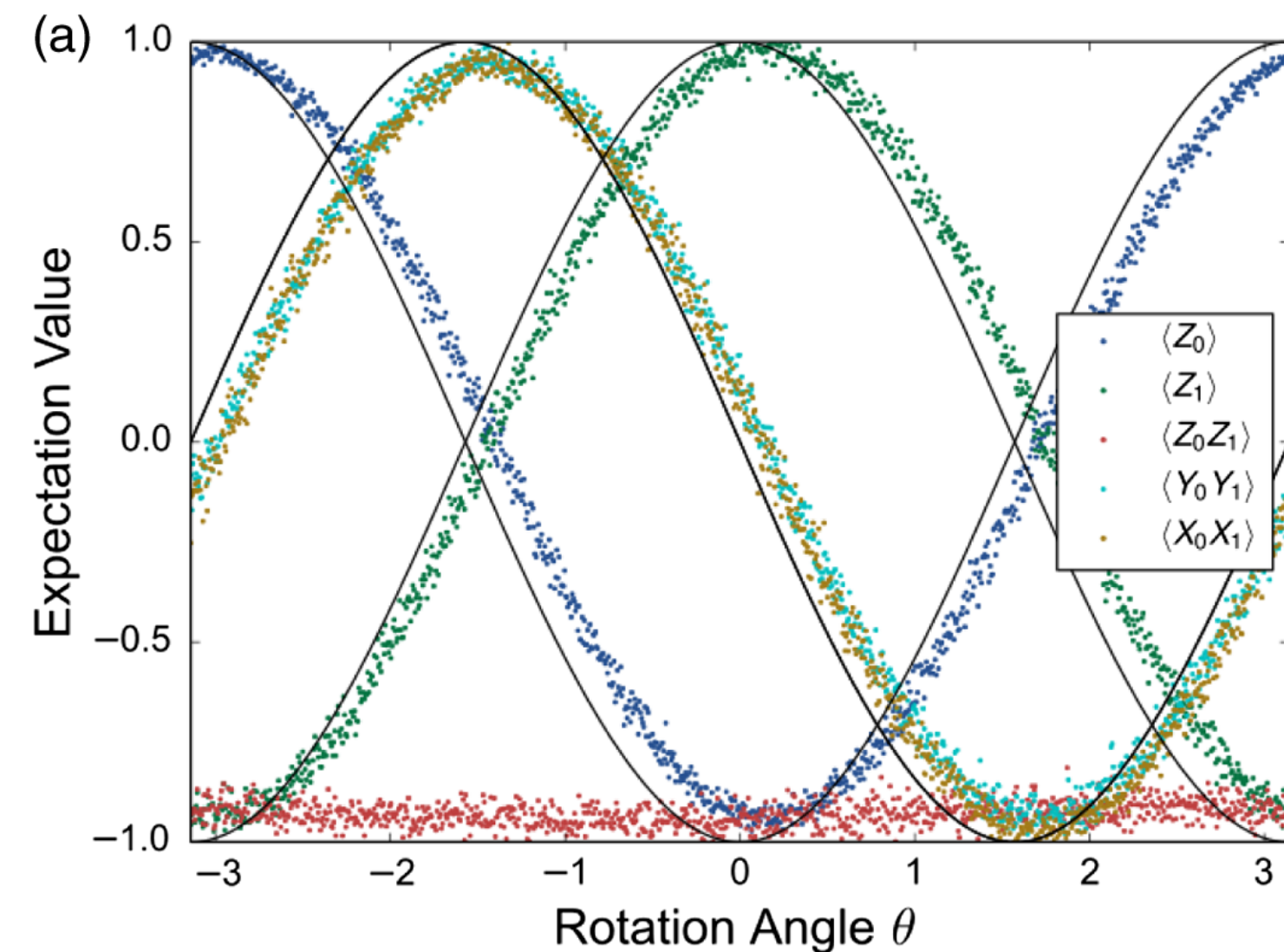


## BeH<sub>2</sub> molecule with 6 qubits



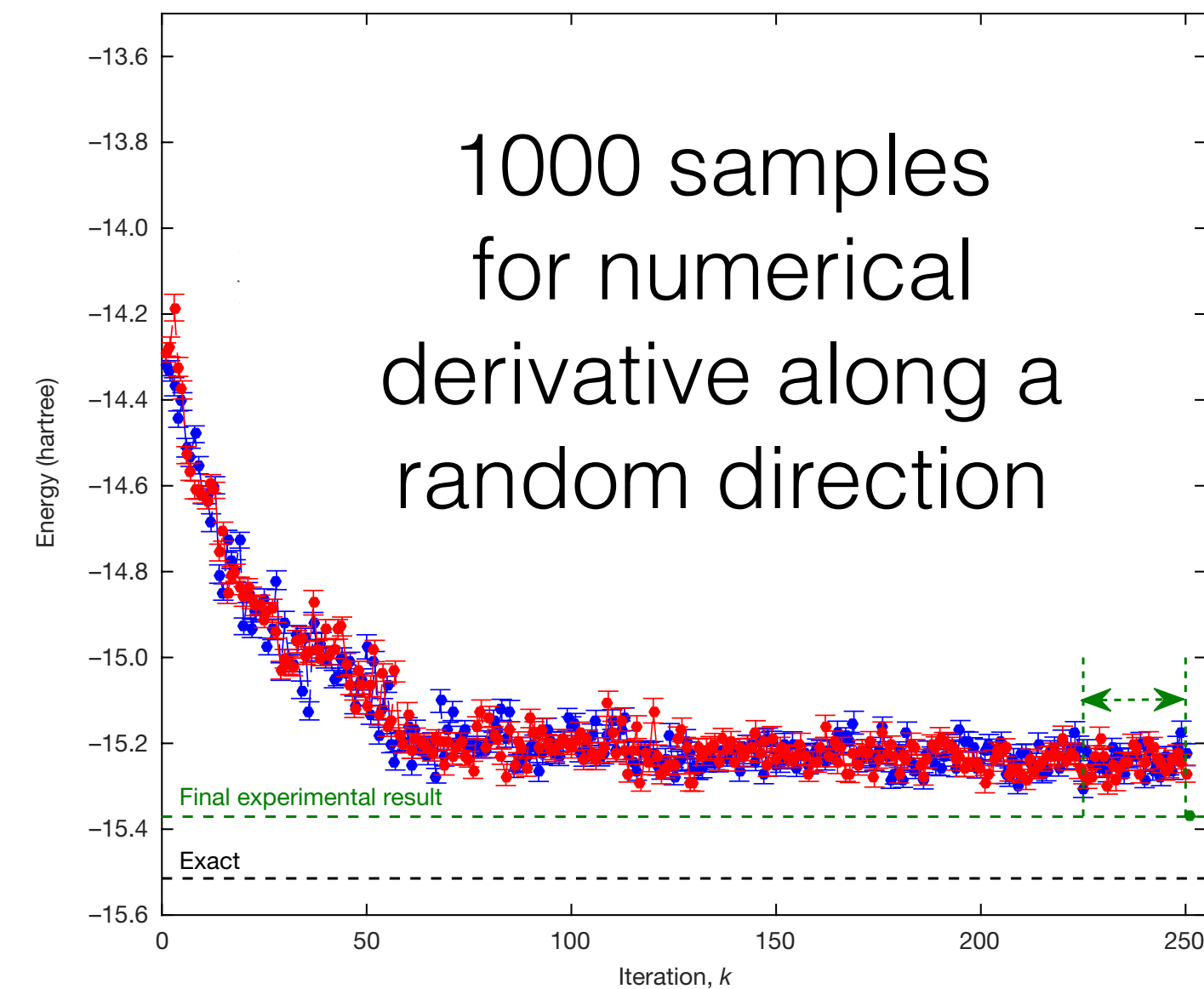
# Optimize the quantum circuit

Scan 1000 values of the single variational parameter



Google PRX '16

Stochastic gradient descend with numerical derivative

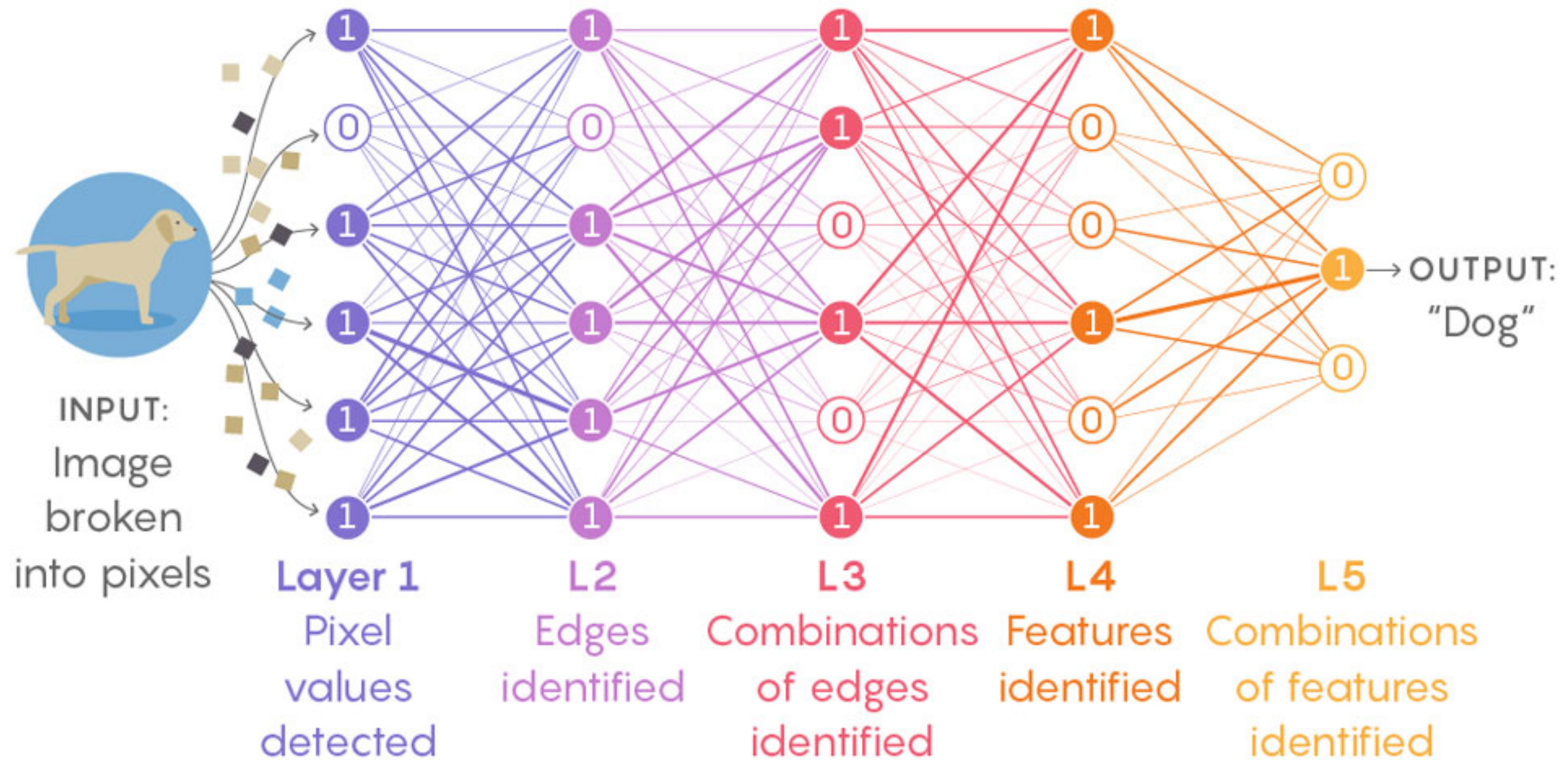


IBM Nature '17

**These optimization schemes do not scale to higher dimensions**

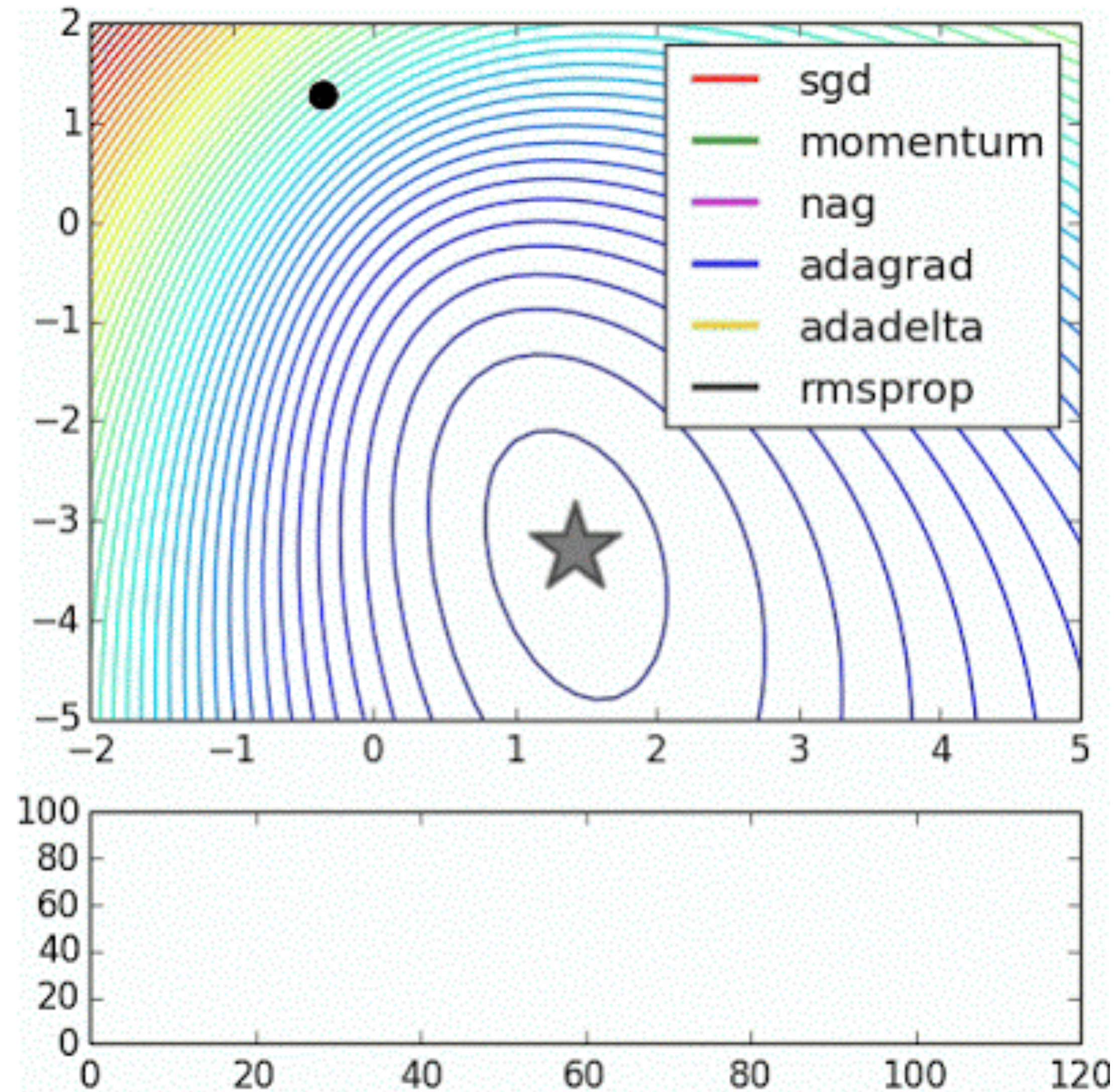


# The engine of deep learning



**Compose differentiable components to form a program  
e.g. a neural network, then optimize it with gradients**

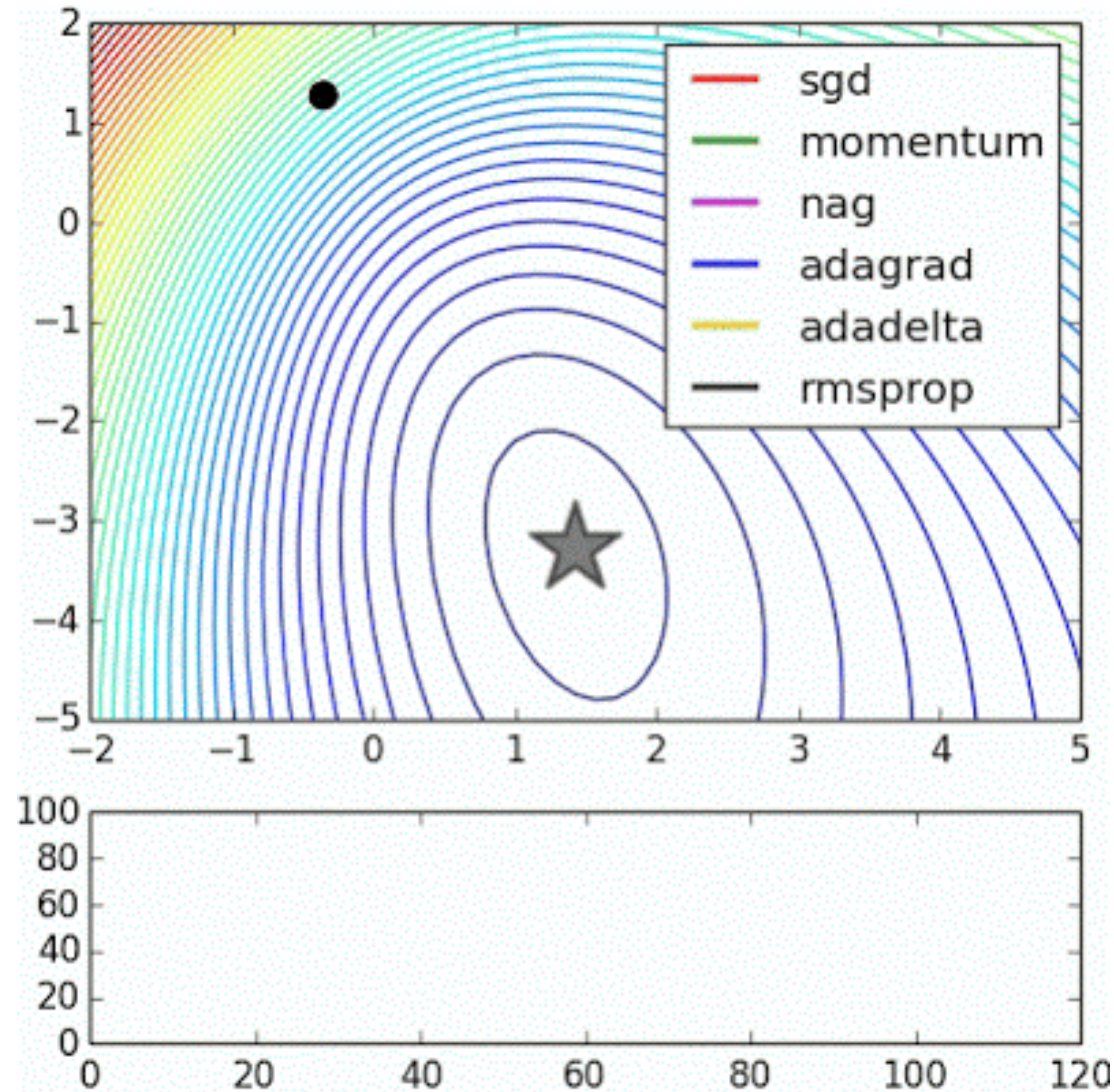
# Optimization with noisy gradients



Ruder, 1609.04747

**VQE encounters the “same type” of stochastic optimization in deep learning**

# Optimization with noisy gradients



Ruder, 1609.04747

**VQE encounters the “same type” of stochastic optimization in deep learning**

# *Differentiable Programming*

## *Quantum Circuits*

Neural Nets  $\leftrightarrow$  Probabilistic Graphical Models  $\leftrightarrow$  Tensor Nets  $\leftrightarrow$  Quantum Circuits

# Differentiable Programming

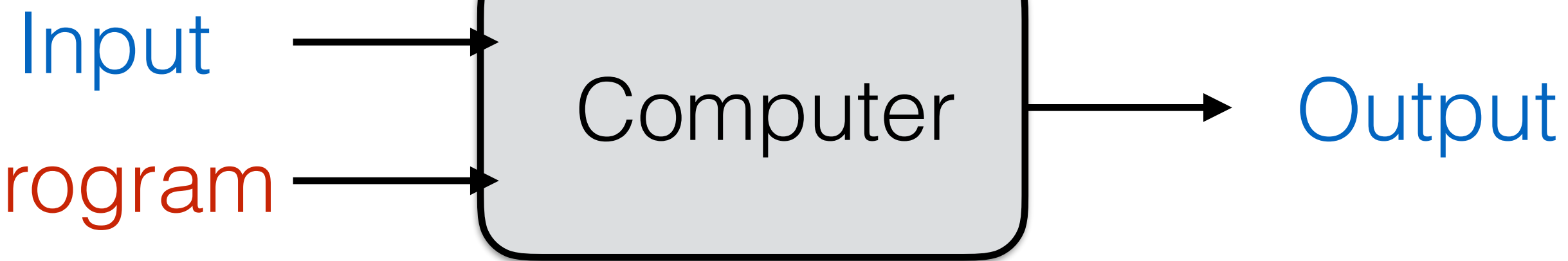


**Andrej Karpathy**

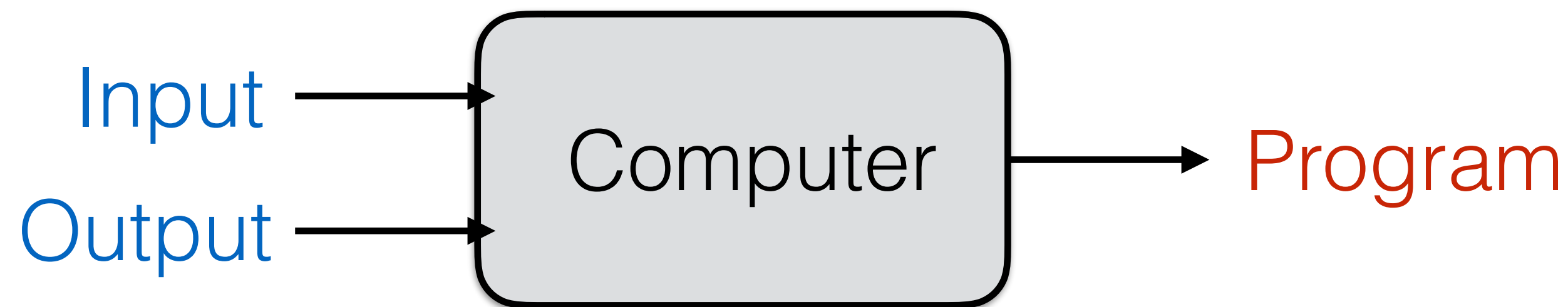
Director of AI at Tesla. Previously Research Scientist at OpenAI and PhD student at Stanford. I like to train deep neural nets on large datasets.

<https://medium.com/@karpathy/software-2-0-a64152b37c35>

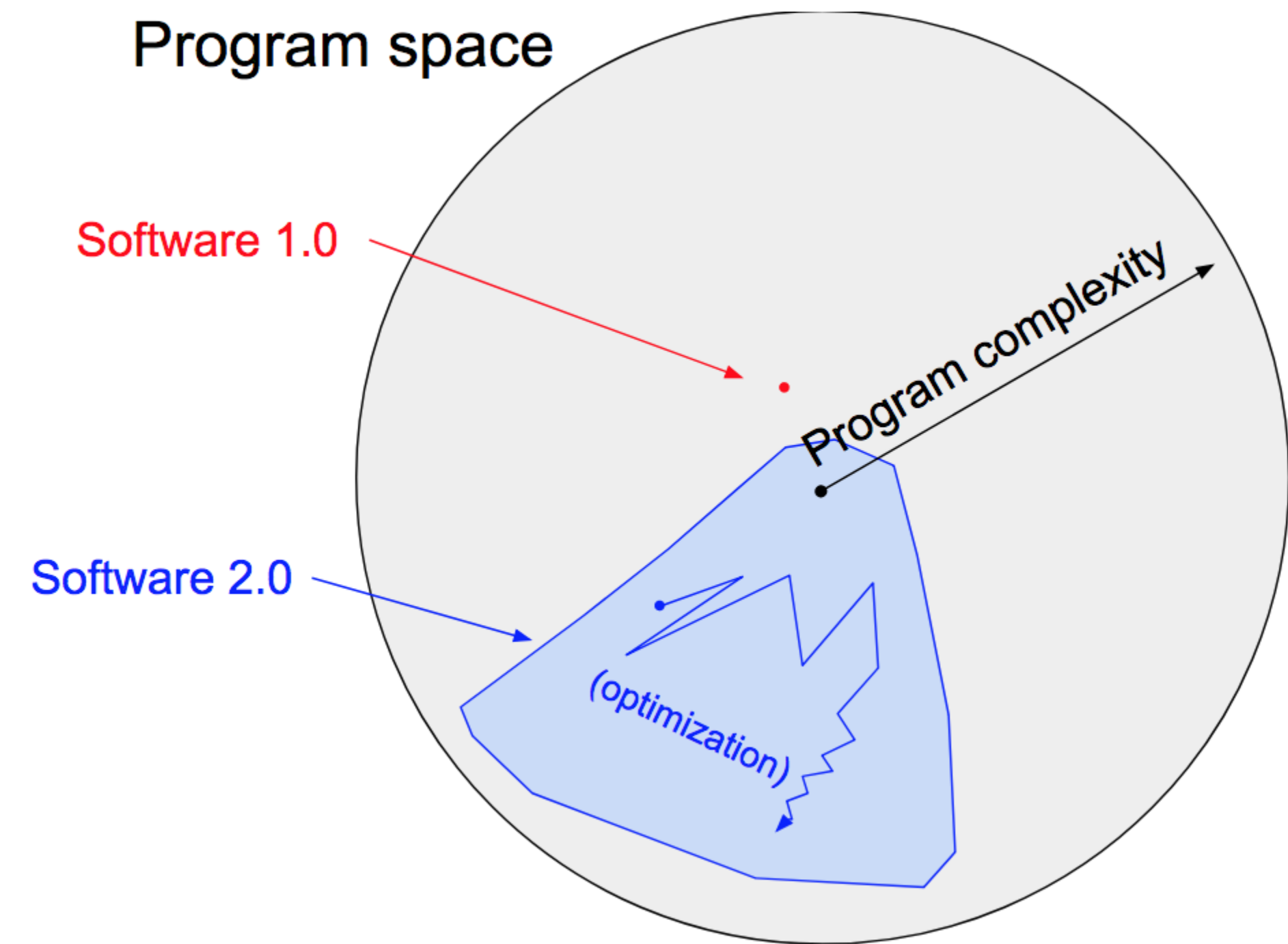
Traditional



Machine Learning



Program space



**Writing software 2.0 by gradient search in the program space**

# Differentiable Programming

## Benefits of Software 2.0

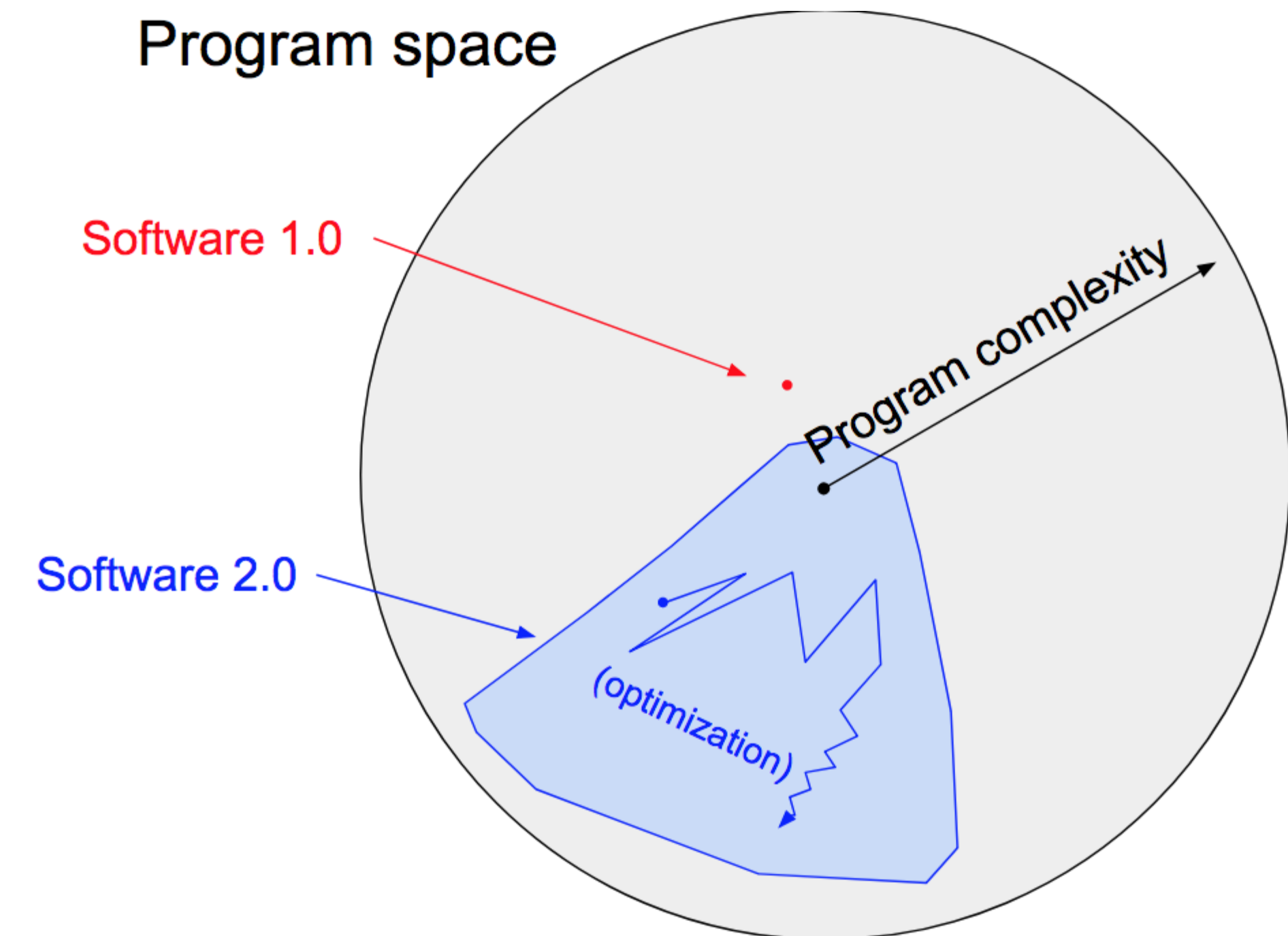
- Computationally homogeneous
- Simple to bake into silicon
- Constant running time
- Constant memory usage
- Highly portable & agile
- Modules can meld into an optimal whole
- **Better than humans**



**Andrej Karpathy**

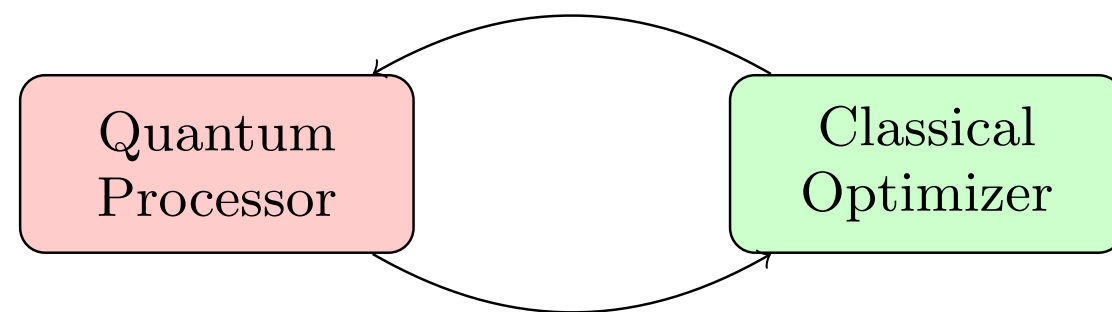
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<https://medium.com/@karpathy/software-2-0-a64152b37c35>



**Writing software 2.0 by gradient search in the program space**

# Differentiable Quantum Programming



**It is a paradigm beyond quantum-classical hybrid**

- Variational quantum eigensolver (VQE)
- Quantum circuit Born machine (QCBM)
- Quantum approximate optimization algorithm (QAOA)
- Quantum pattern recognition

...

Quantum circuit classifier

Farhi, Neven, 1802.06002 Havlicek et al, 1804.11326

TNS inspired circuit architecture

Huggins, Patel, Whaley, Stoudenmire, 1803.11537

VQE with fewer qubits

Liu, Zhang, Wan, LW, 1902.02663

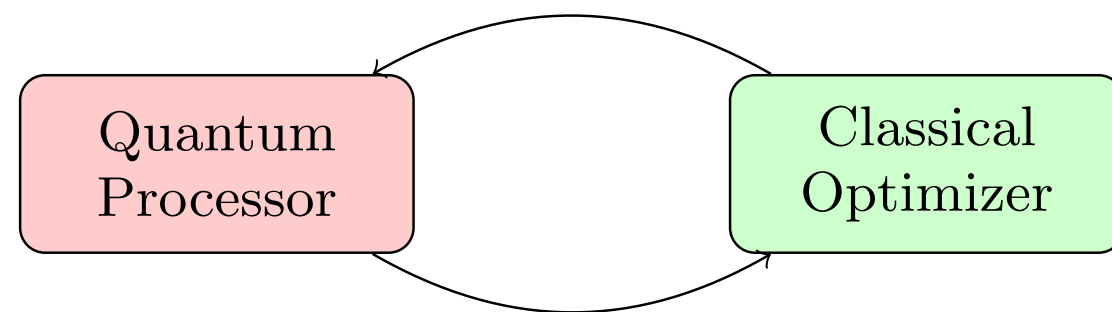
Quantum generative model

Gao, Zhang, Duan, 1711.02038

Quantum adversarial training

Dallaire-Demers, Lloyd, Benedetti 1804.08641, 1804.09139, 1806.00463

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**Near term:**

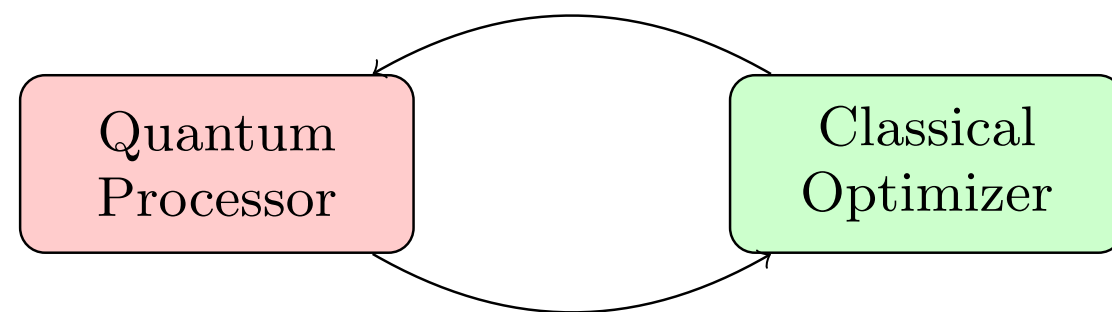
What can we do with noisy circuits of limited depth ?

**Long term:**

Are we really good at programming quantum computers ?



# Differentiable Quantum Programming



**It is a paradigm beyond quantum-classical hybrid**

**Quantum code**

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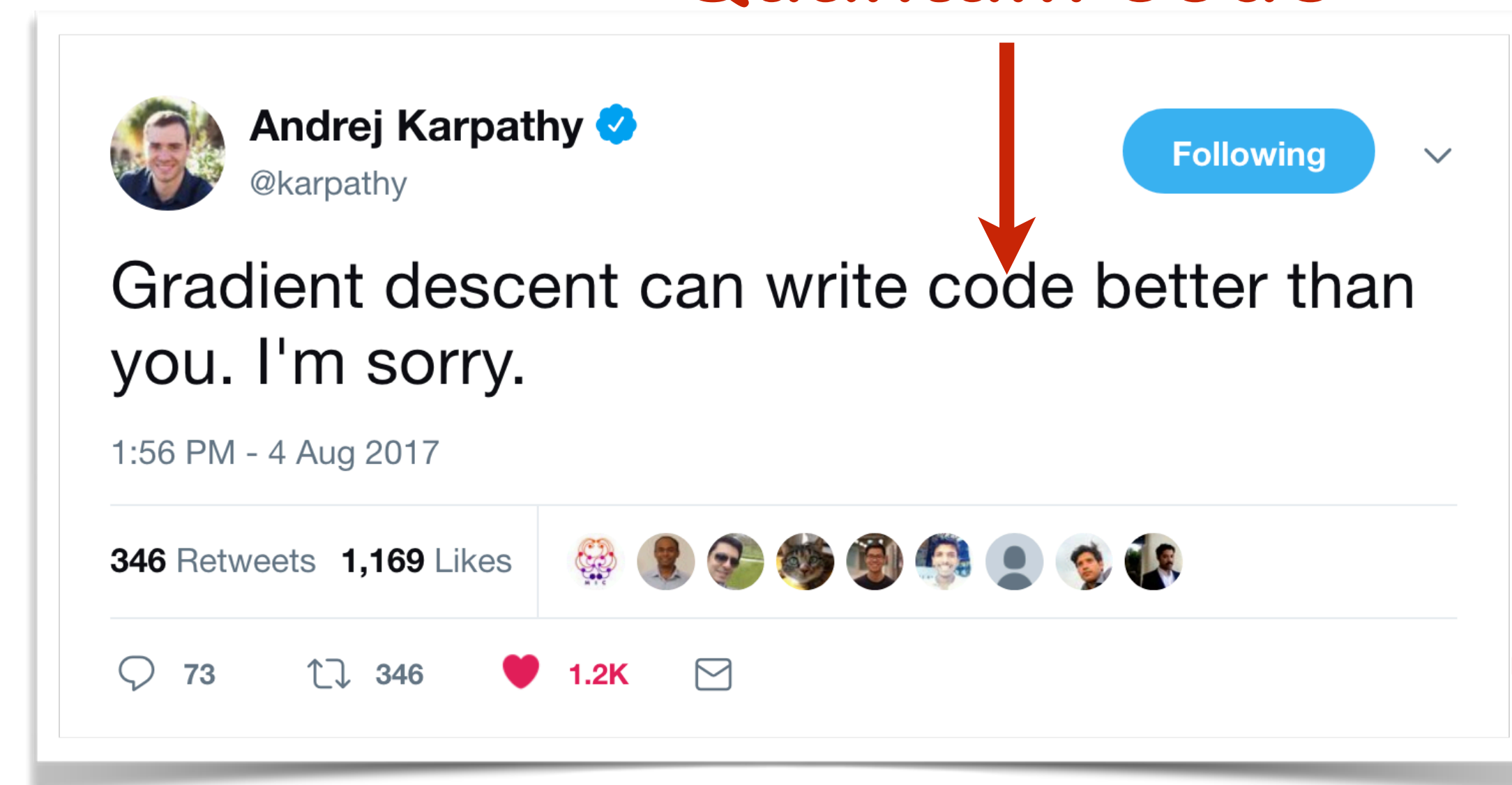
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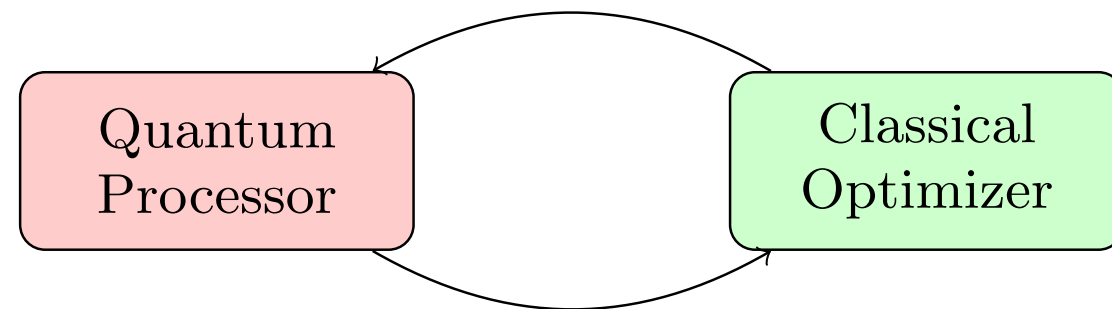
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# Differentiable Quantum Programming



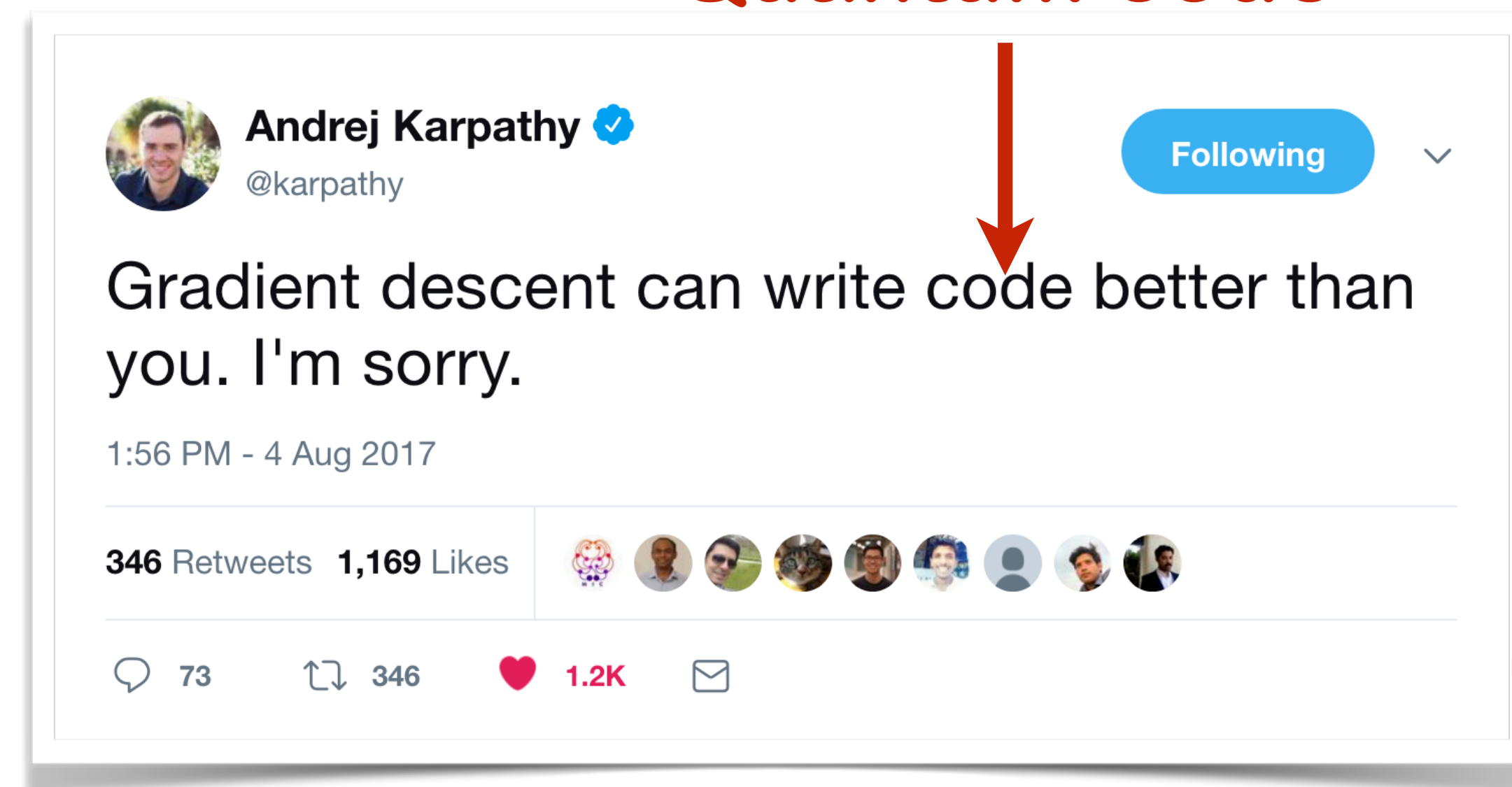
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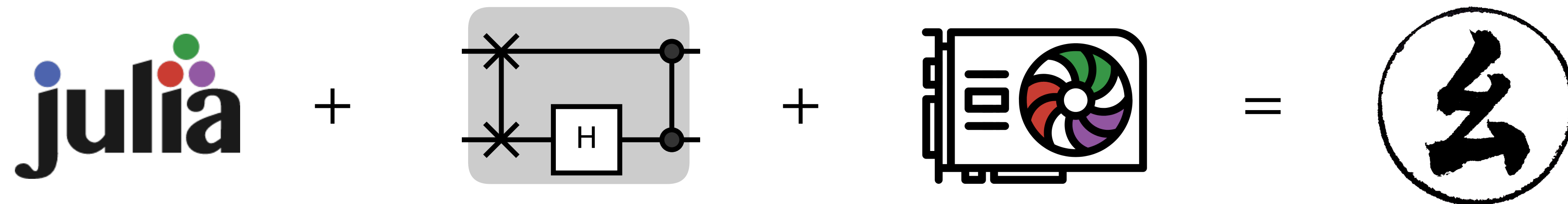
Quantum circuit  
TNS inspired circ  
VQE with fewer c  
Quantum genera  
Quantum advers



## Quantum Software 2.0

# Be prepared for Quantum Software 2.0

<https://yaoquantum.org/>

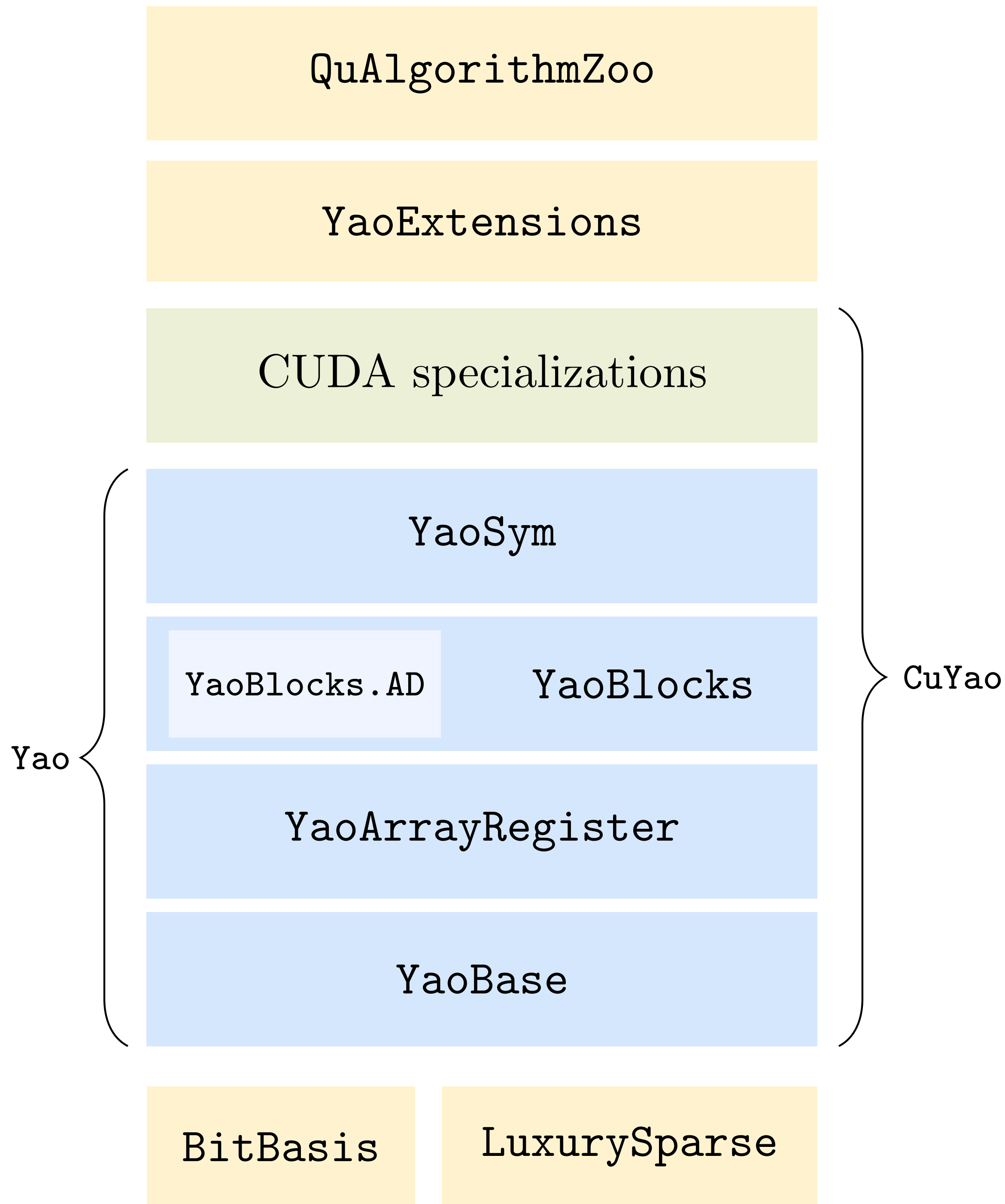


**Xiu-Zhe Luo (IOP, CAS → Waterloo & PI)**

**Jin-Guo Liu (IOP, CAS → Harvard)**

Features:

- Differentiable programming quantum circuits
- Batched quantum register with GPU acceleration
- Quantum block intermediate representation

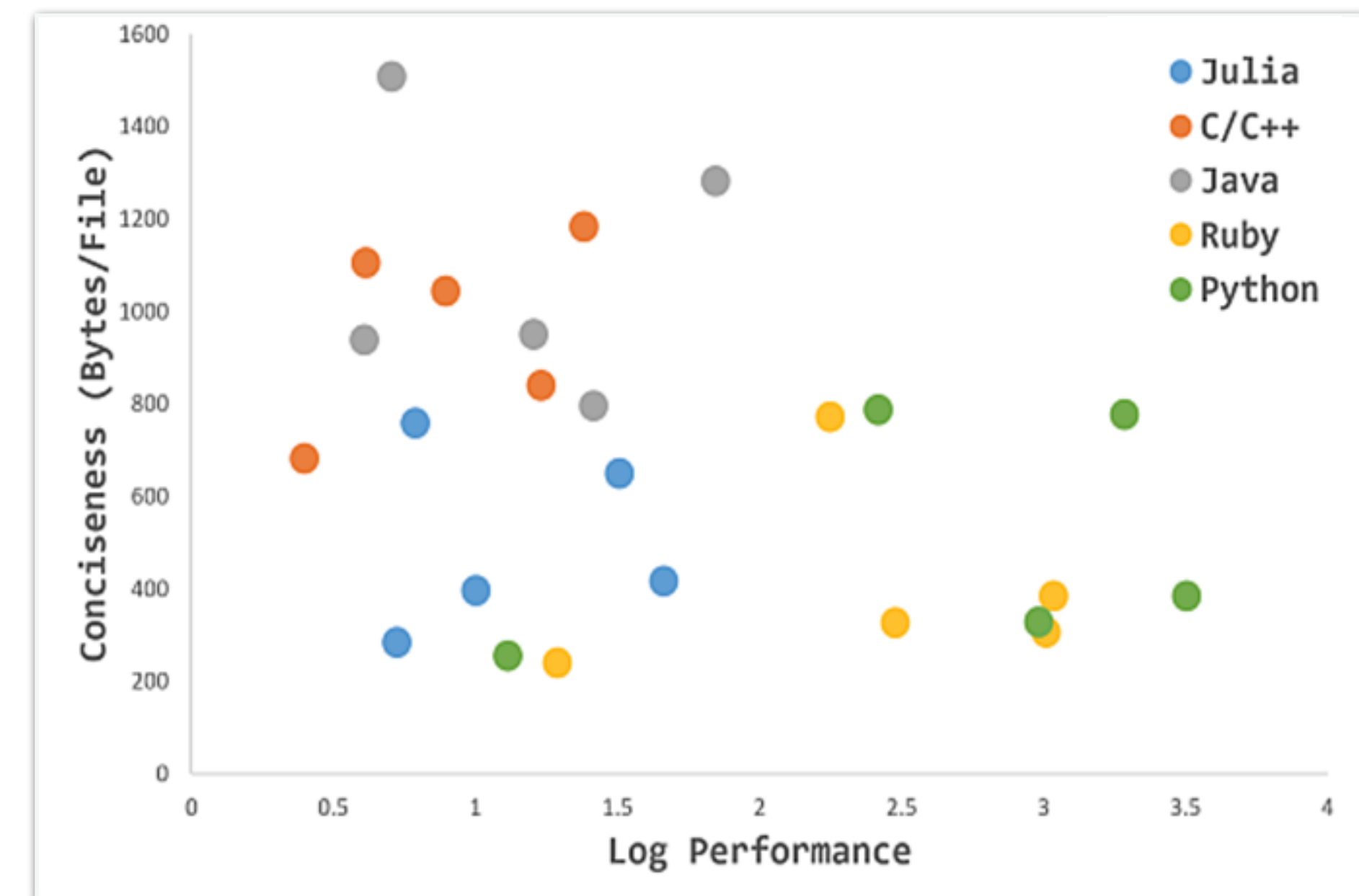


# Stacks of Yao

<https://github.com/QuantumBFS>

# Why Julia ?

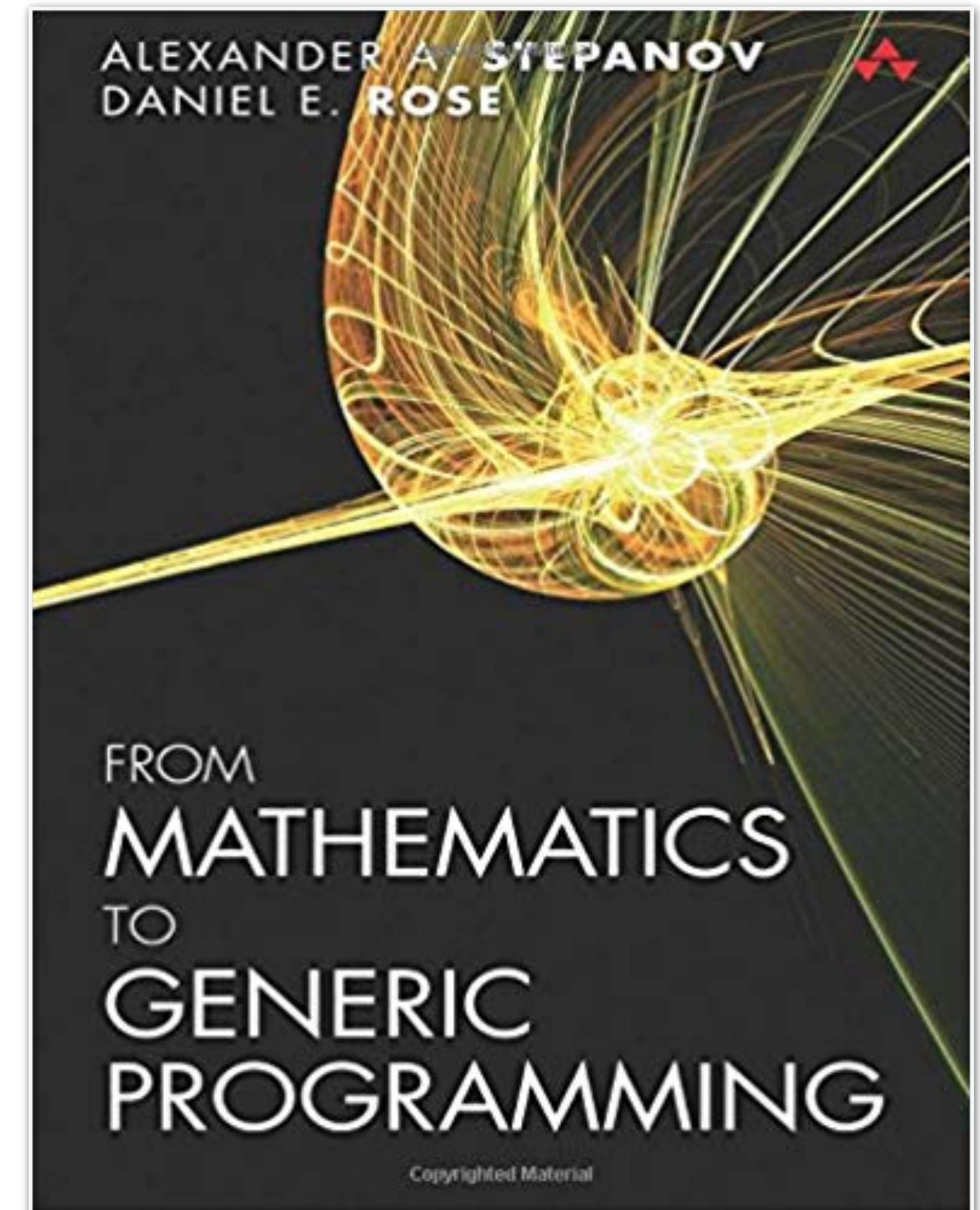
- [Julia is fast!](#)
- Generic programming (type system and multiple dispatch)
- The future of technical computing



<http://ljuug.org>

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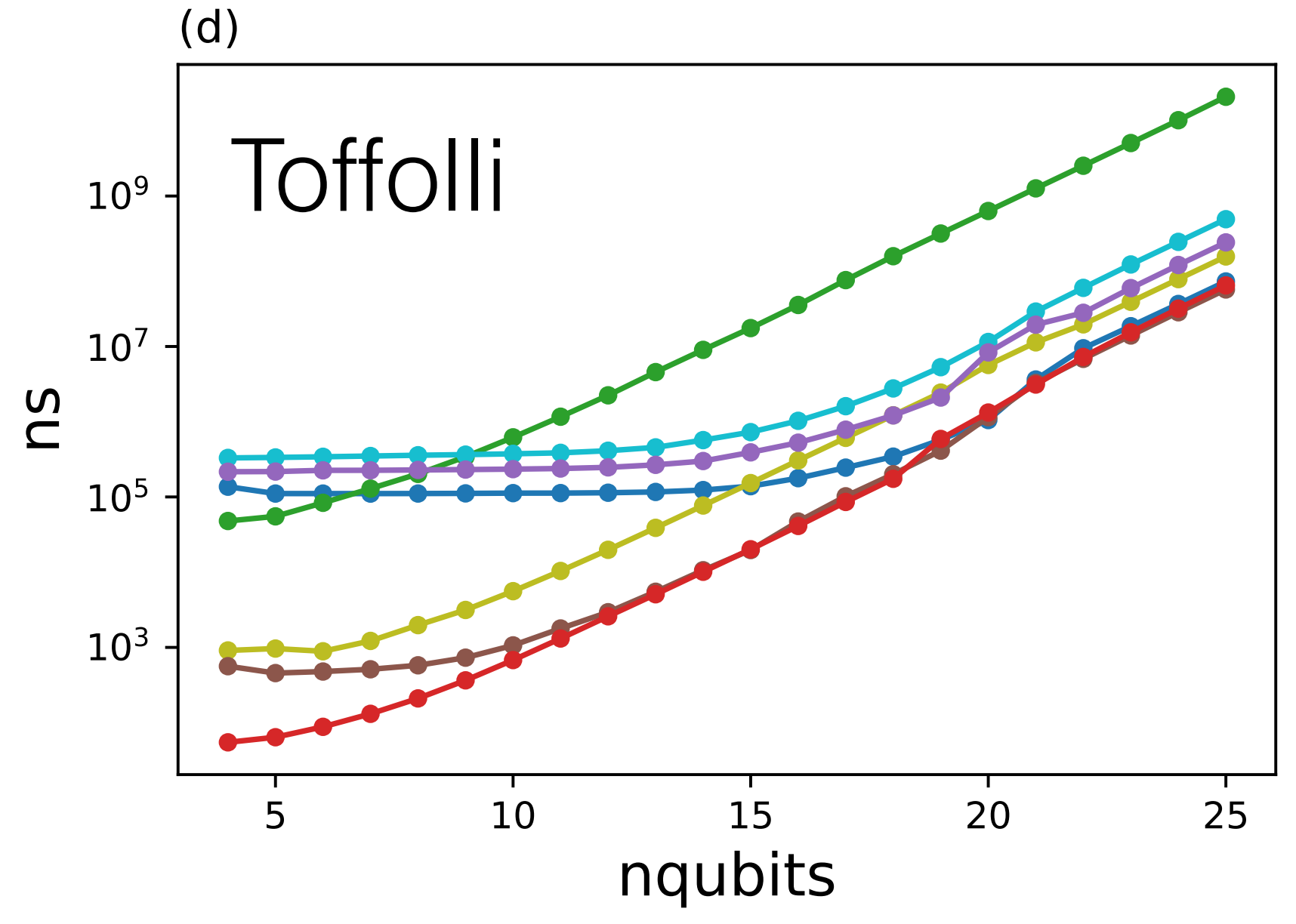
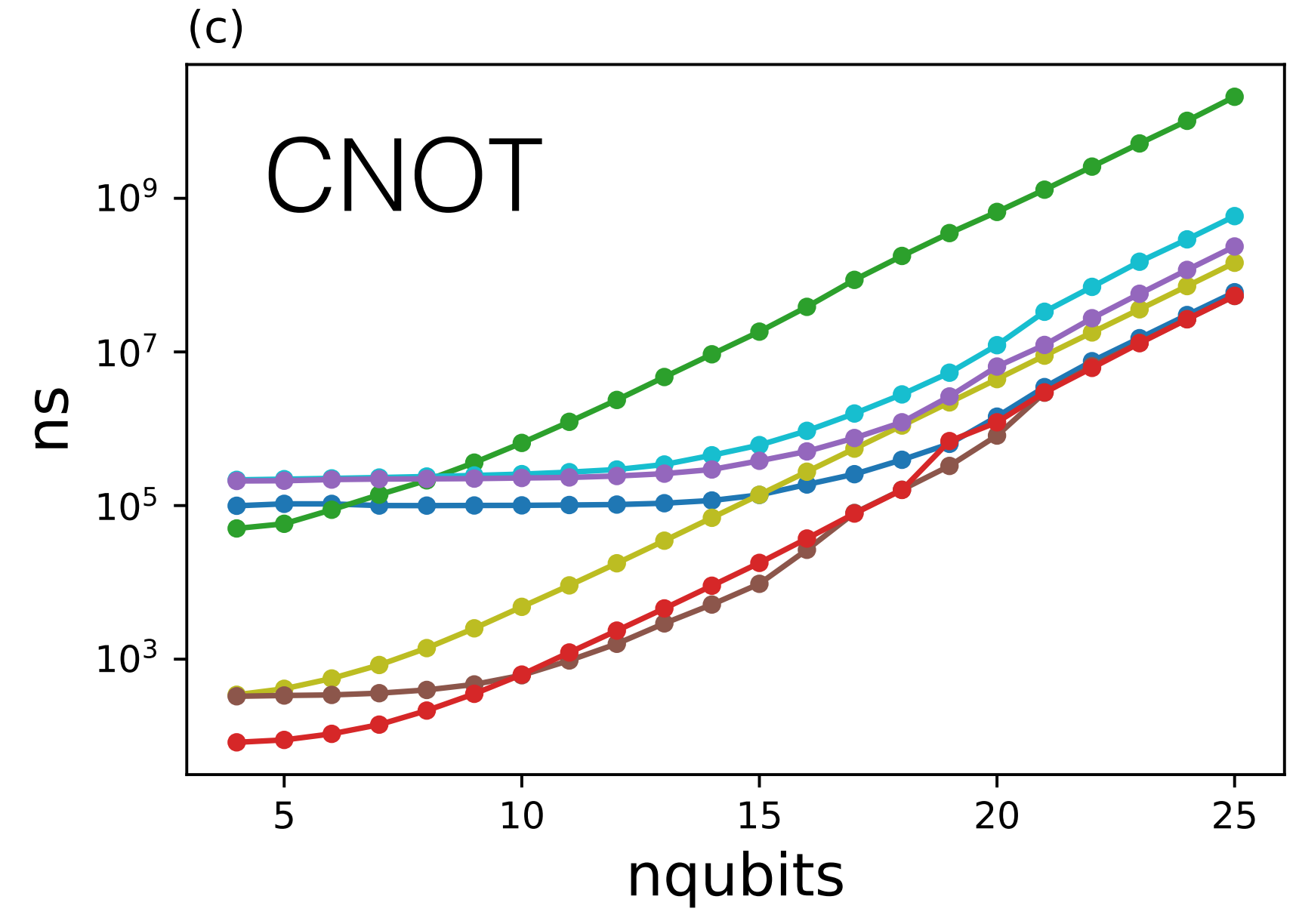
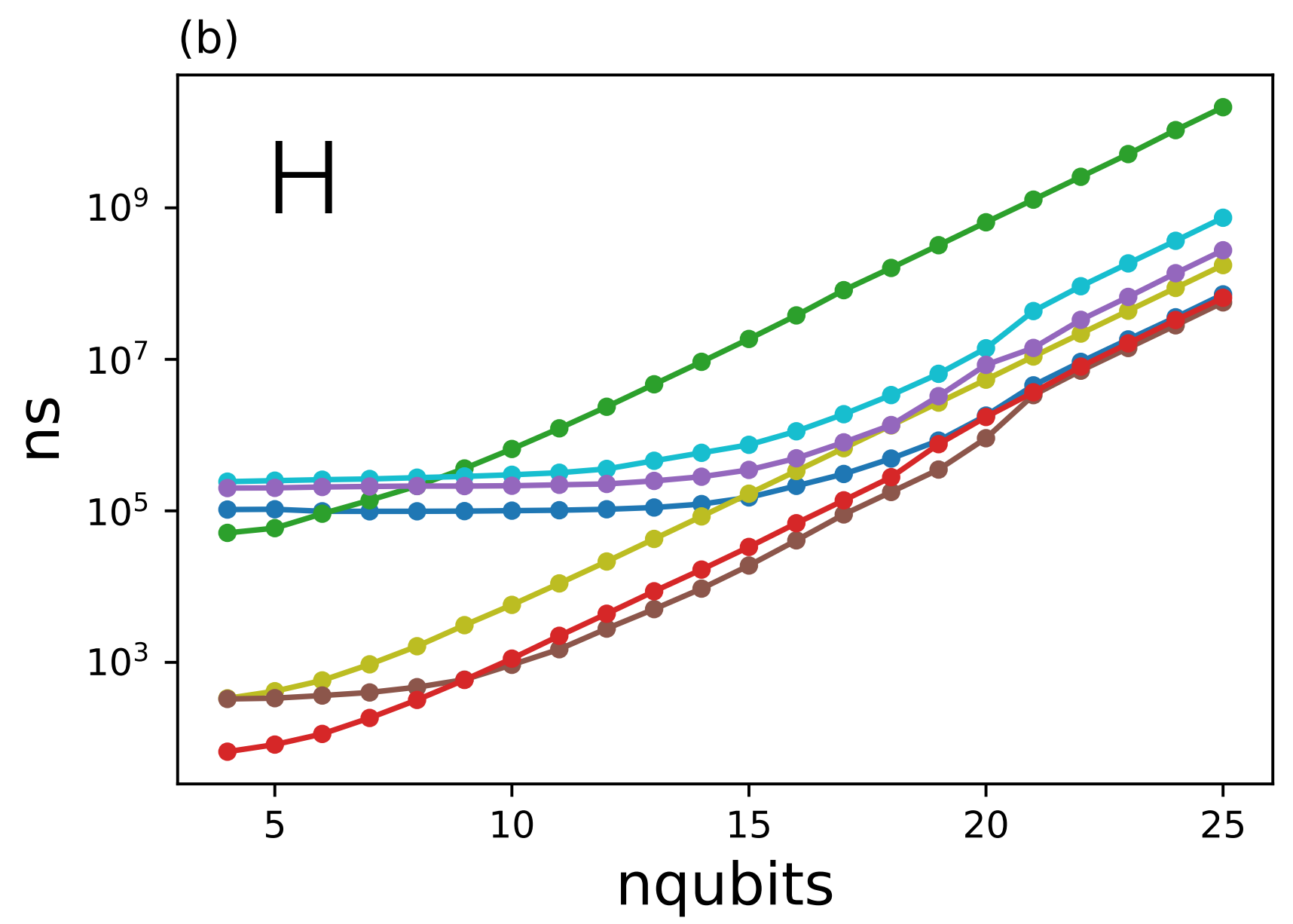
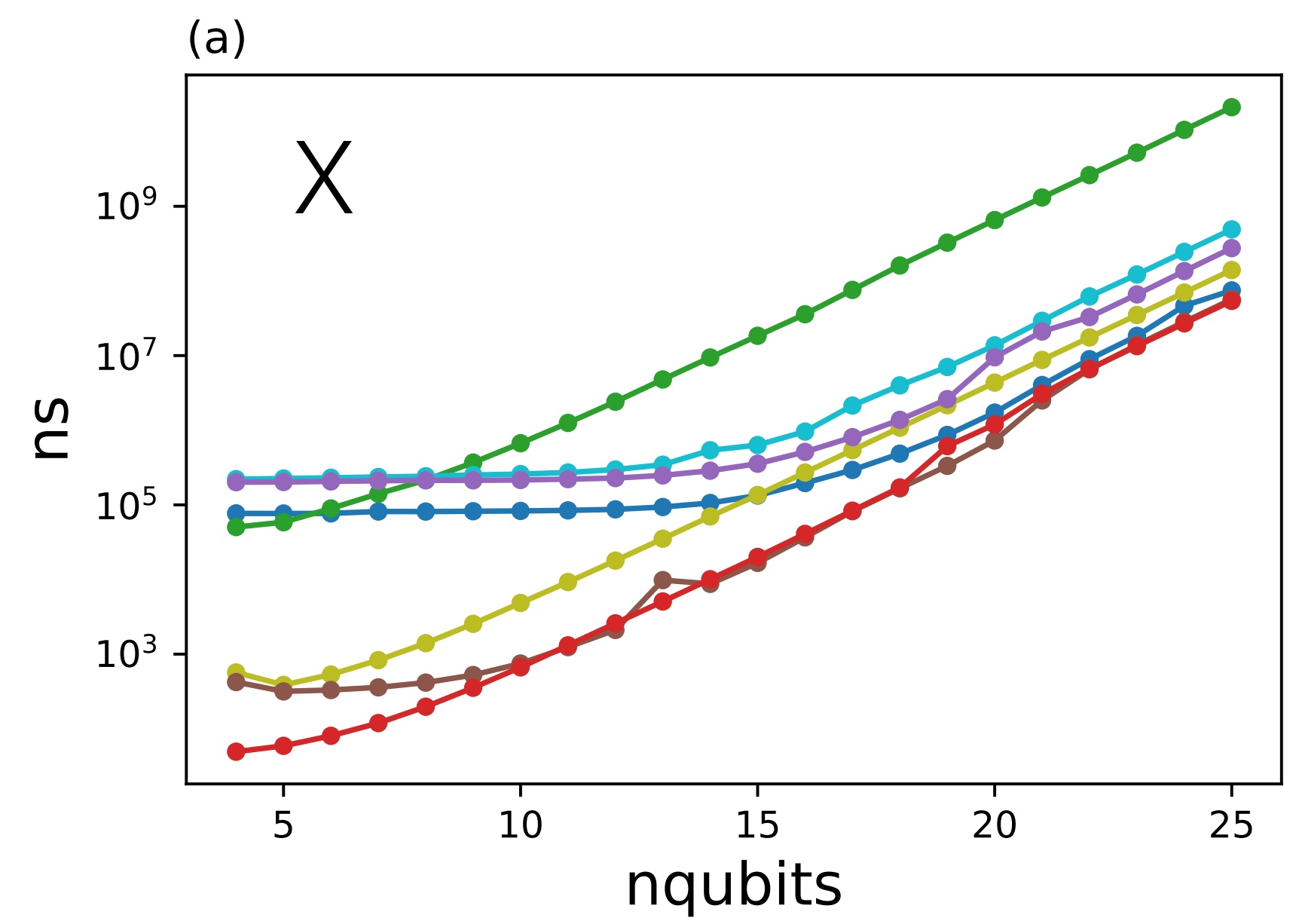
- Julia is fast!
- Generic programming (type system and multiple dispatch)
- Future of technical computing

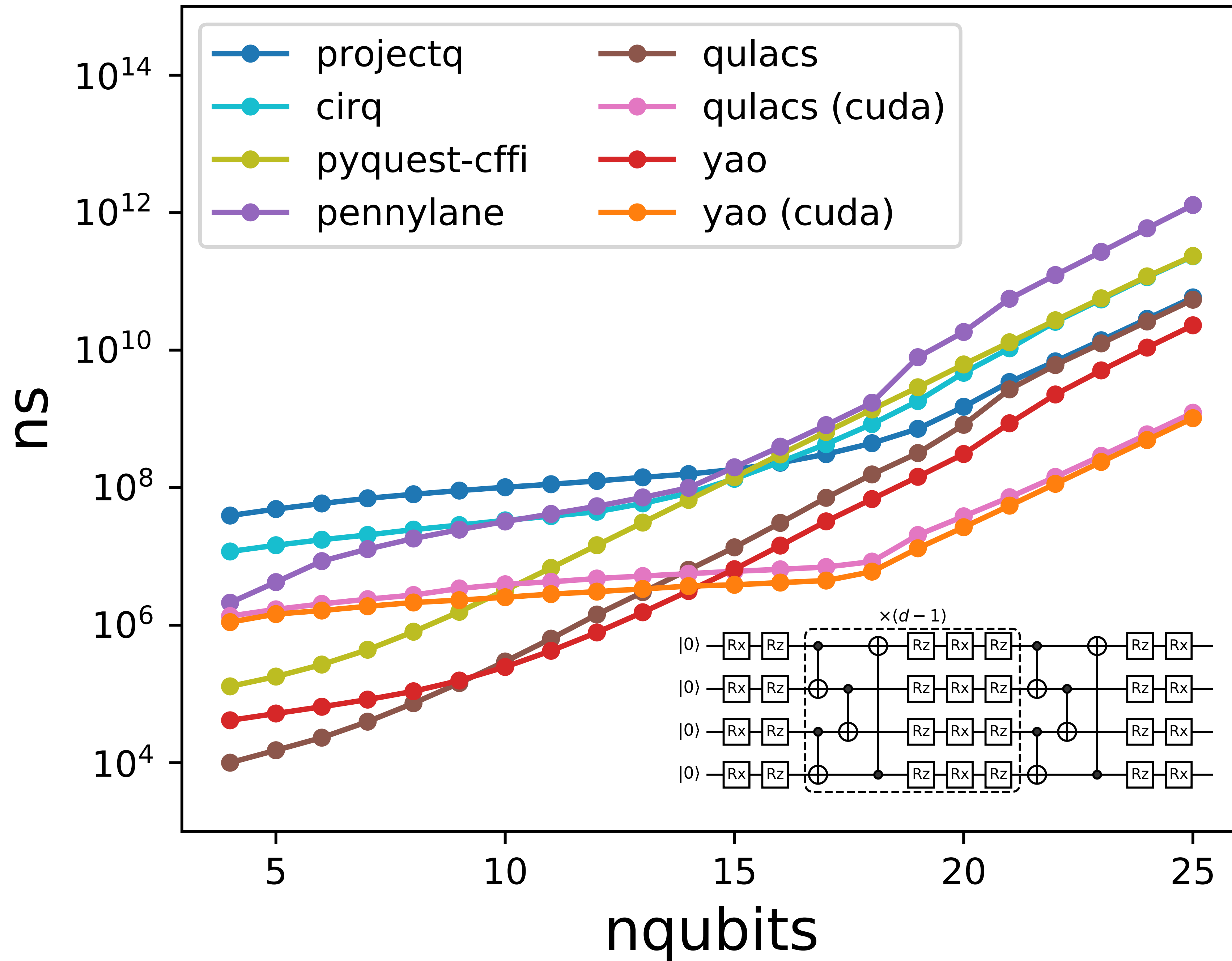


# Demo 1

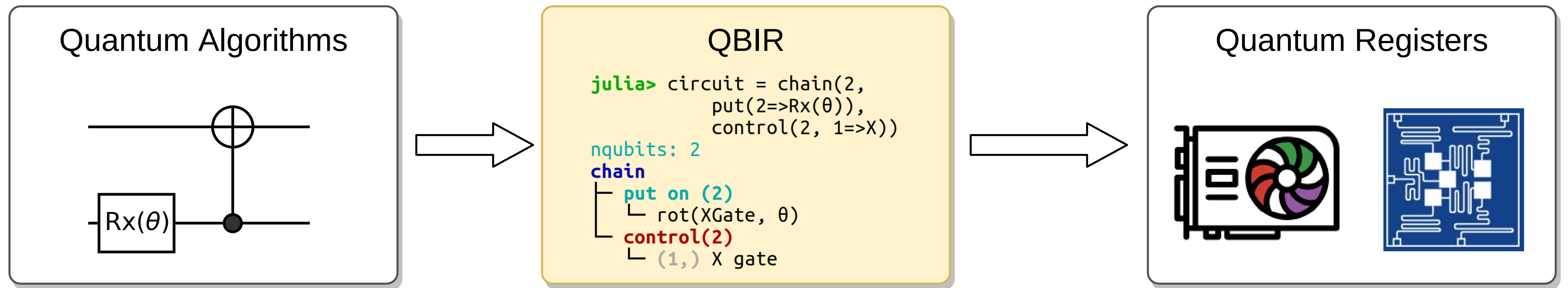
<https://github.com/wangleiphy/YaoTutorial>







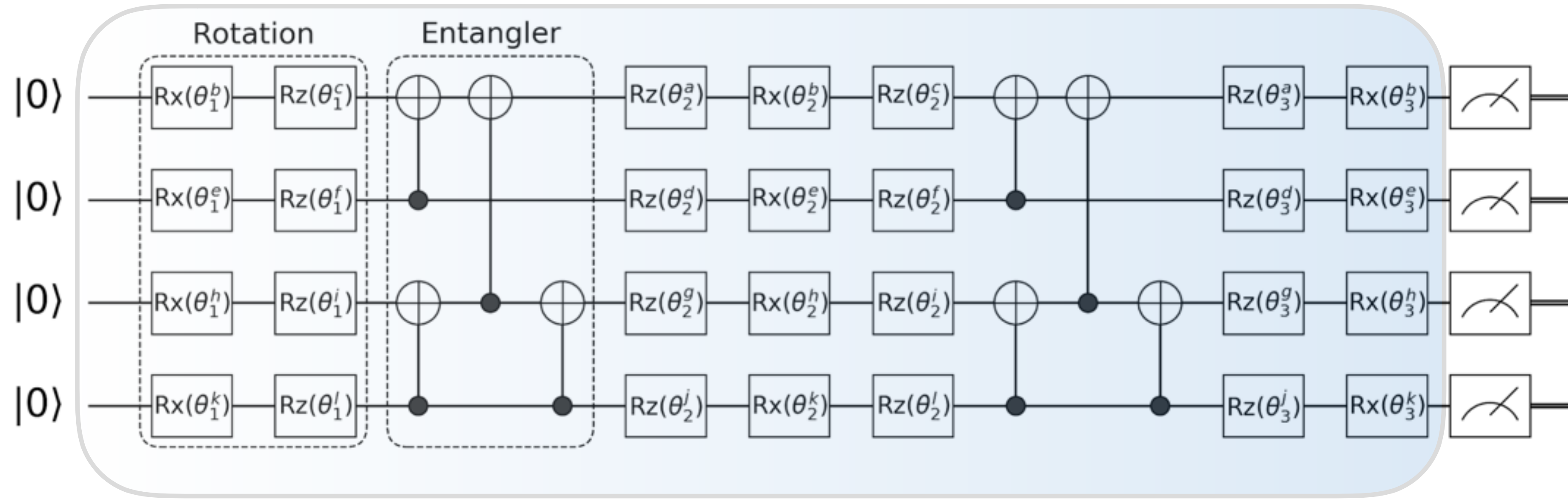
# Quantum Block Intermediate Representation



# Demo 2

<https://github.com/wangleiphy/YaoTutorial>

# *Differentiable*<sup>1</sup> quantum circuits



**Write your simulator as a machine learning model  
Isn't that obvious ?**

# Differentiable programming tools

HIPS/autograd

theano

 PyTorch

  
TensorFlow

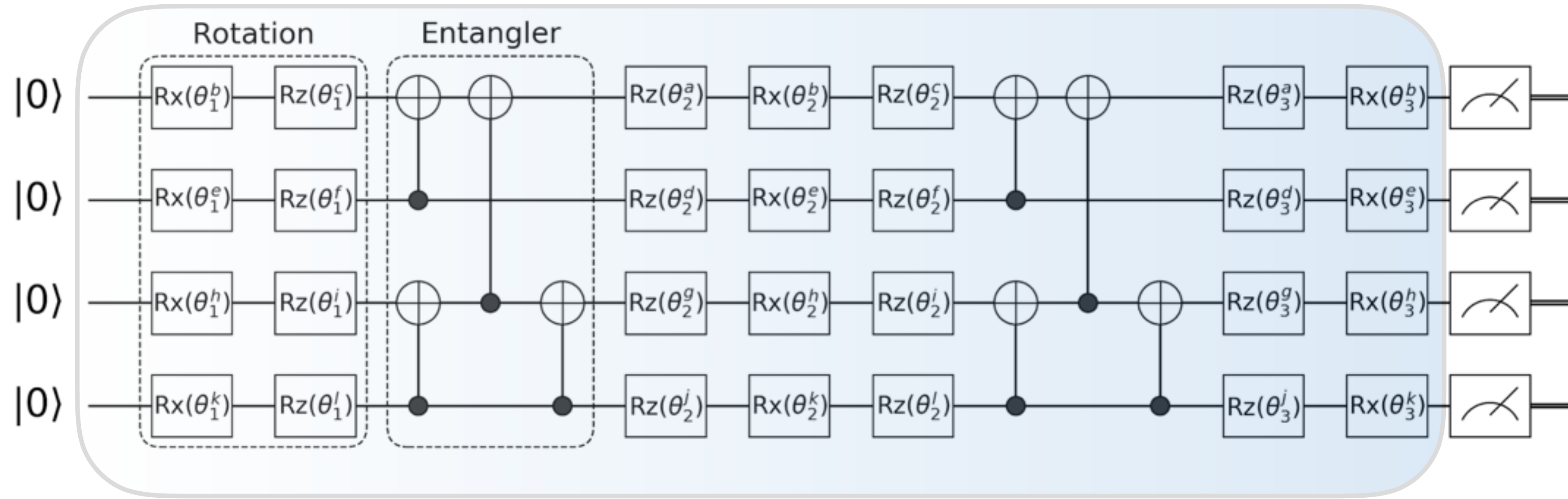
 *flux*



 Keras

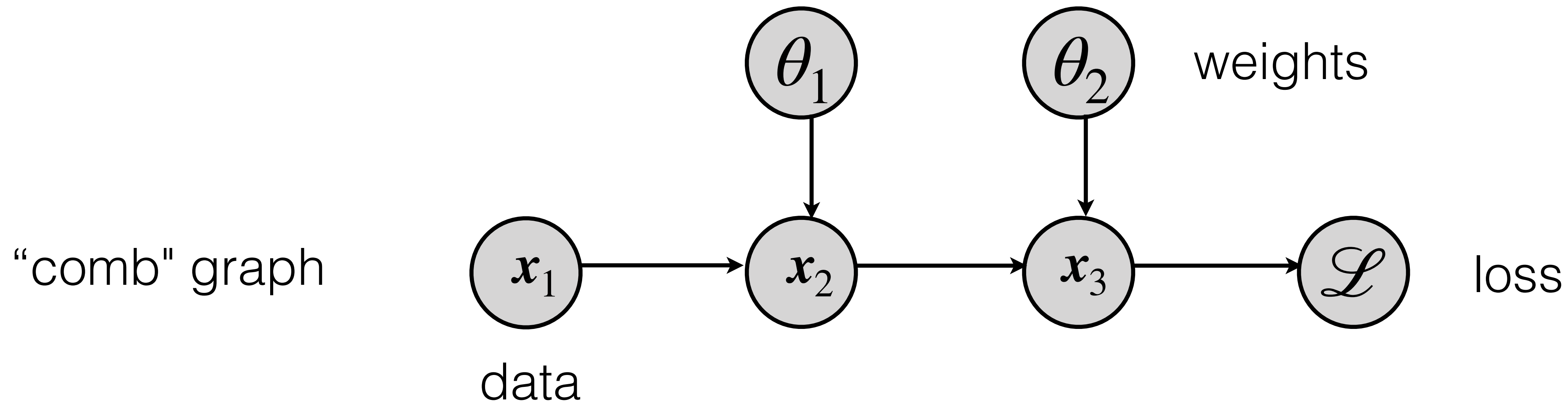
 *Zygote*

# *Differentiable*<sup>1</sup> quantum circuits



**Even better: quantum computing is reversible!**  
**Backpropagation with  $O(1)$  memory in classical simulation**

# Automatic differentiation on computation graph

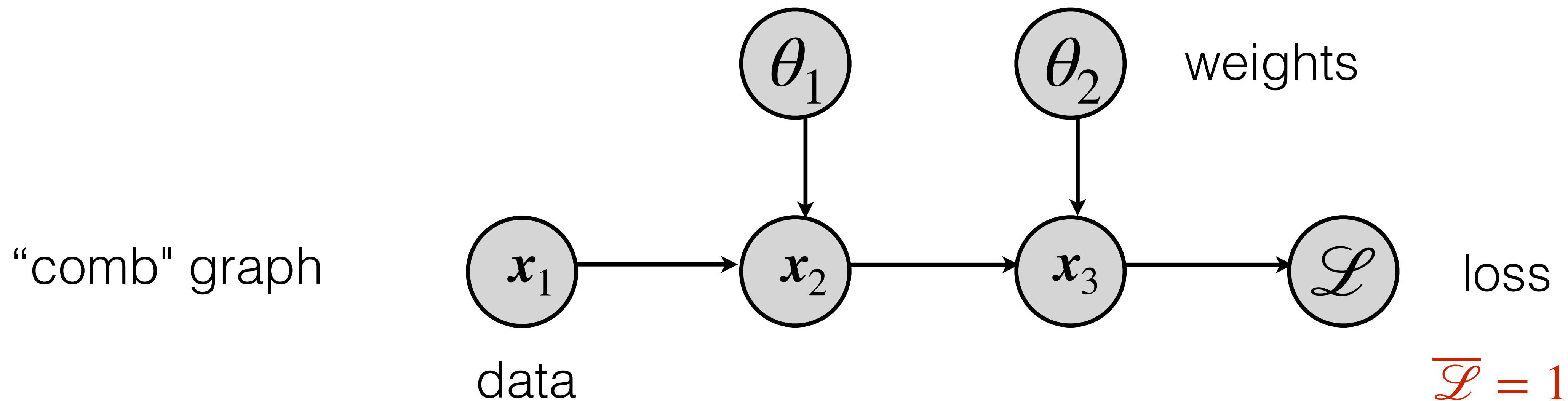


Define “adjoint”  $\bar{x} = \frac{\partial \mathcal{L}}{\partial x}$

**Pullback the adjoint through the graph**



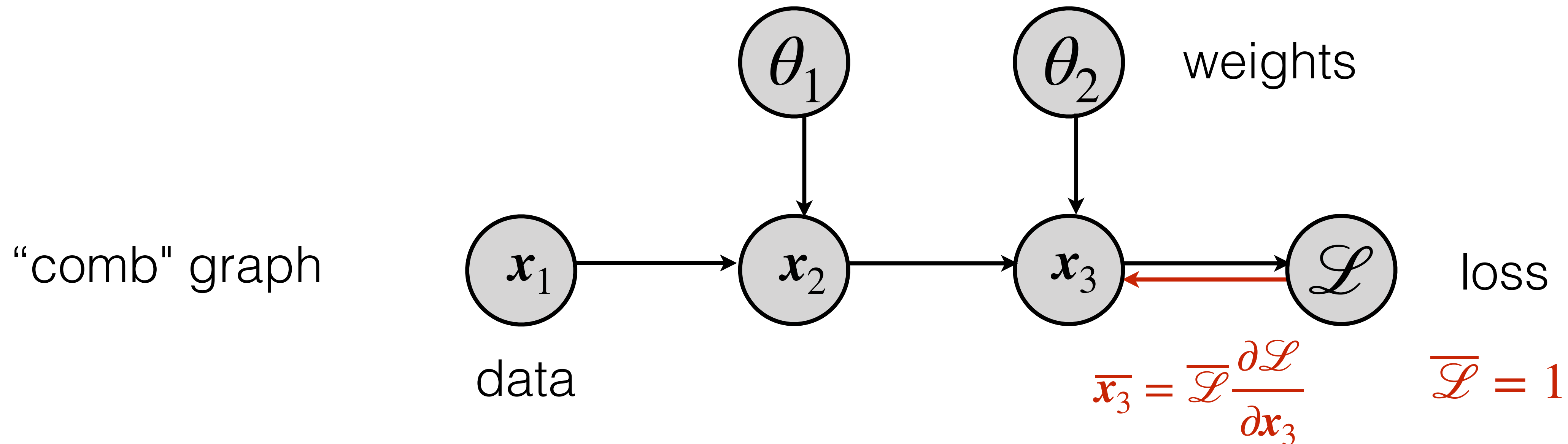
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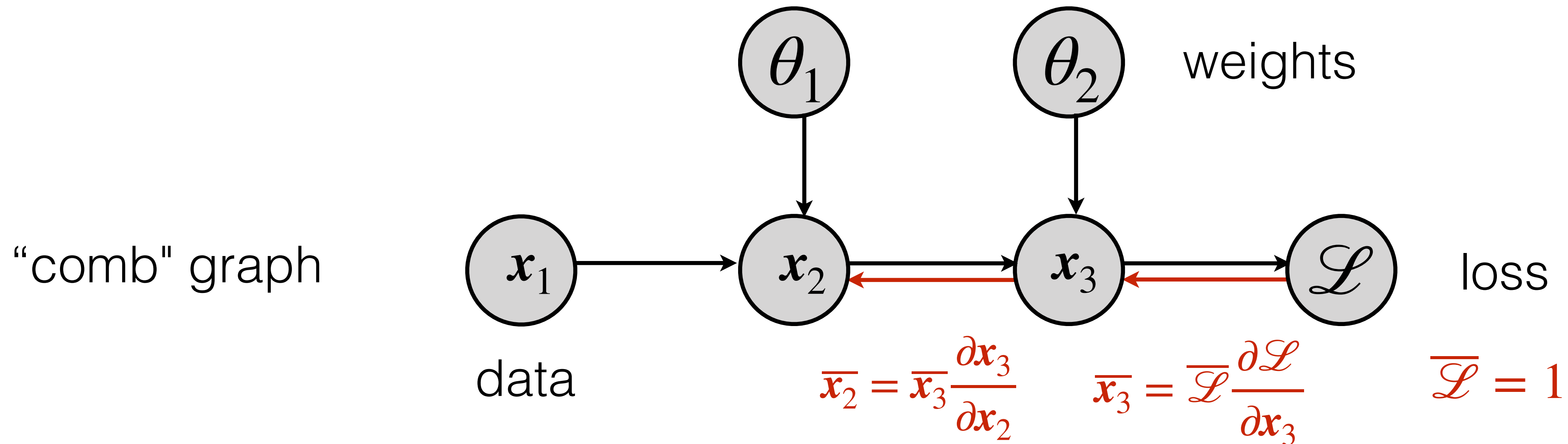
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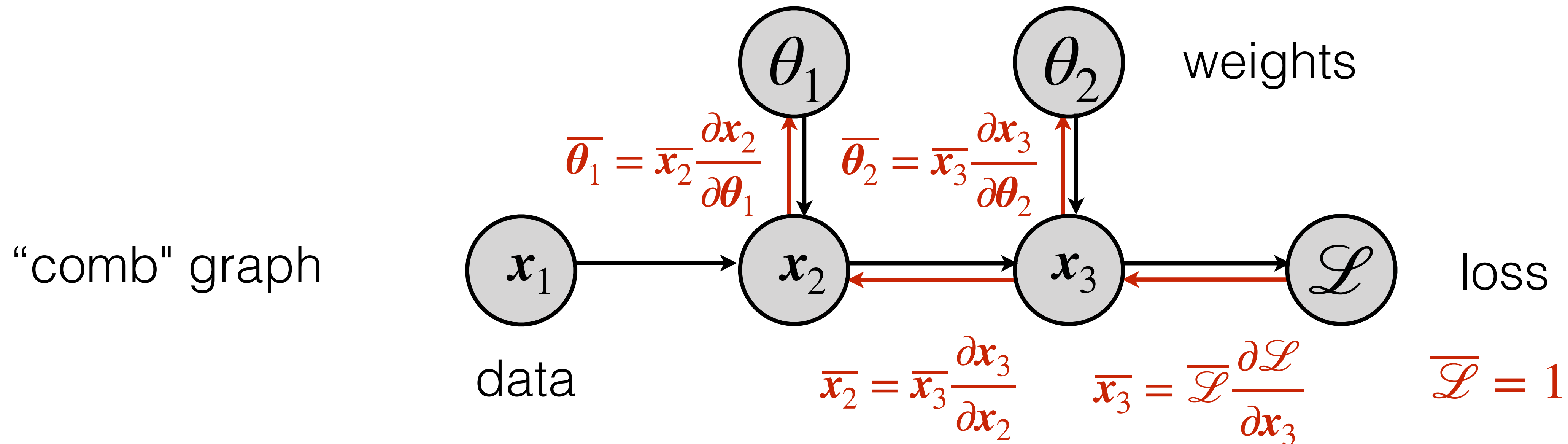
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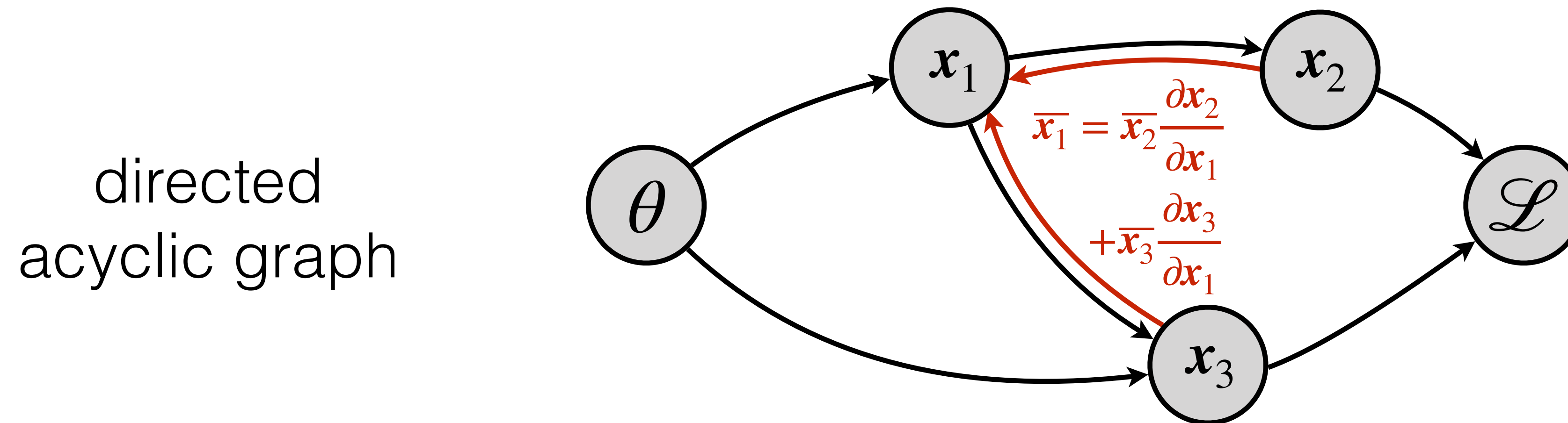
# Automatic differentiation on computation graph



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# Automatic differentiation on computation graph

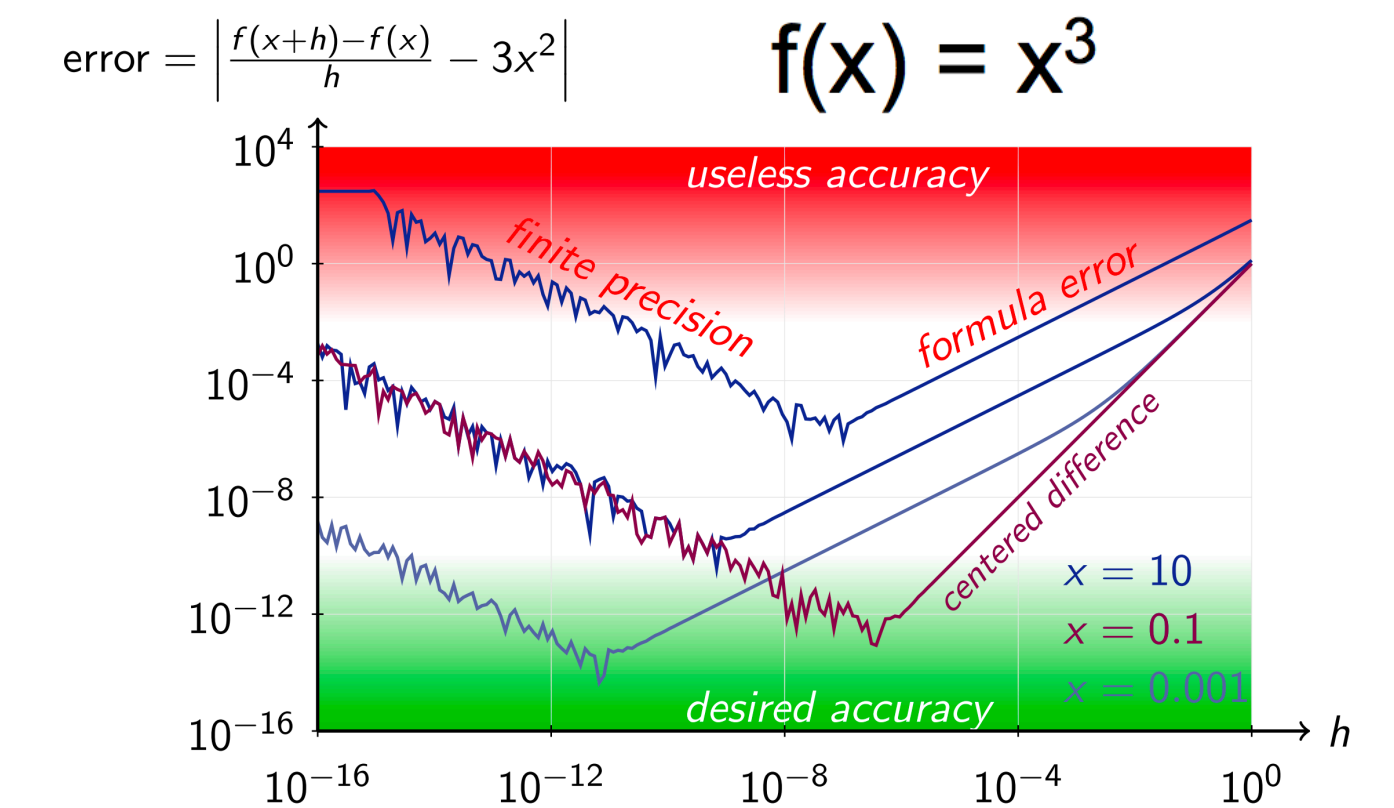


$$\bar{x}_i = \sum_{j: \text{child of } i} \bar{x}_j \frac{\partial x_j}{\partial x_i} \quad \text{with } \bar{L} = 1$$

**Message passing for the adjoint at each node**

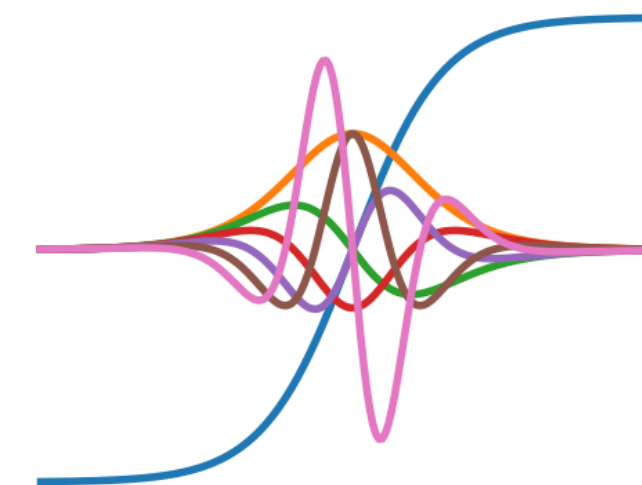
# Advantages of automatic differentiation

- Accurate to the machine precision



- Same computational complexity as the function evaluation:  
Baur-Strassen theorem '83

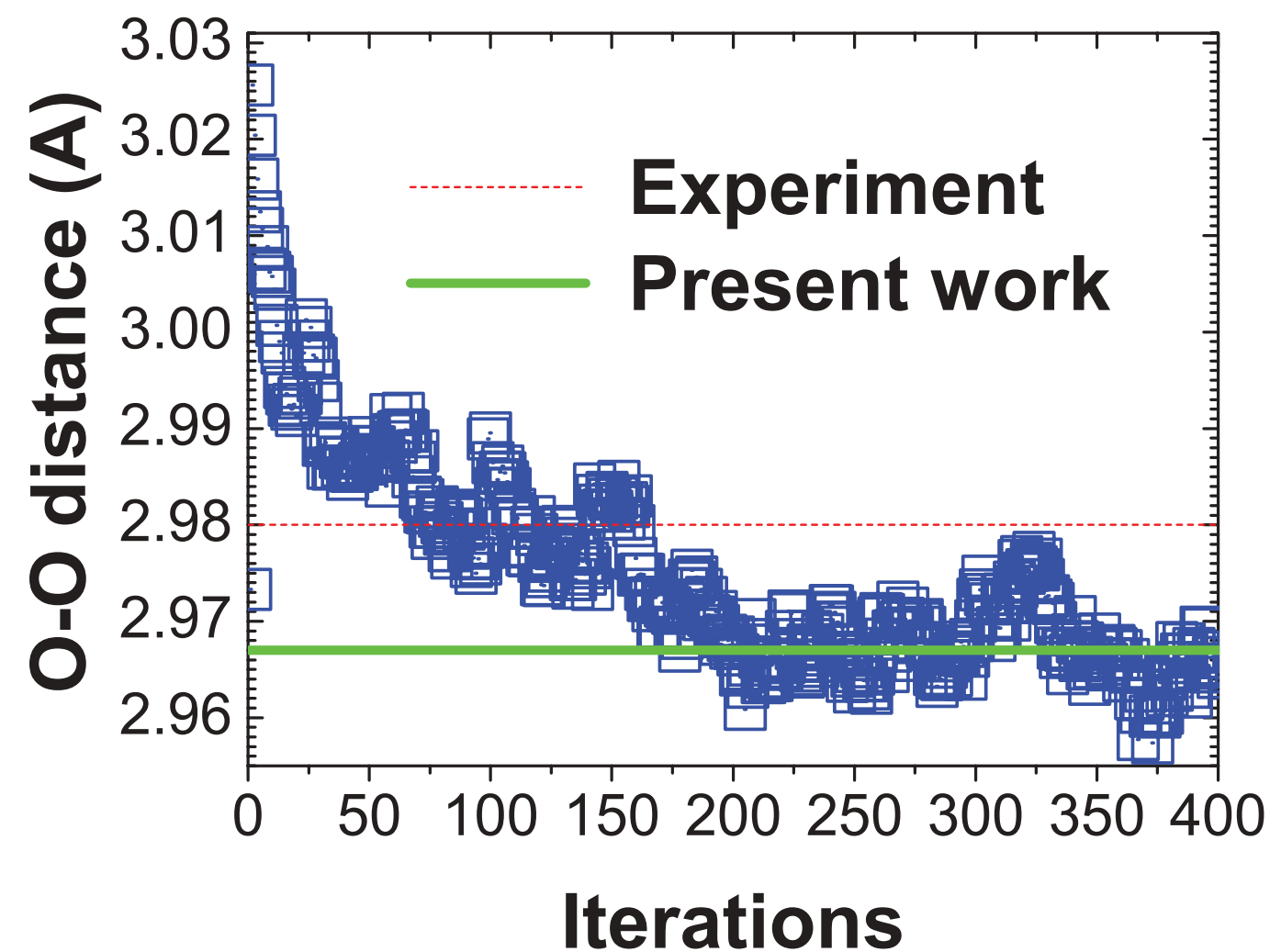
- Supports higher order gradients



```
>>> from autograd import elementwise_grad as egrad # for functions that vectorize over inputs
>>> import matplotlib.pyplot as plt
>>> x = np.linspace(-7, 7, 200)
>>> plt.plot(x, tanh(x),
...         x, egrad(tanh)(x), # first derivative
...         x, egrad(egrad(tanh))(x), # second derivative
...         x, egrad(egrad(egrad(tanh)))(x), # third derivative
...         x, egrad(egrad(egrad(egrad(tanh)))(x), # fourth derivative
...         x, egrad(egrad(egrad(egrad(egrad(tanh)))(x), # fifth derivative
...         x, egrad(egrad(egrad(egrad(egrad(egrad(tanh)))(x)) # sixth derivative
>>> plt.show()
```

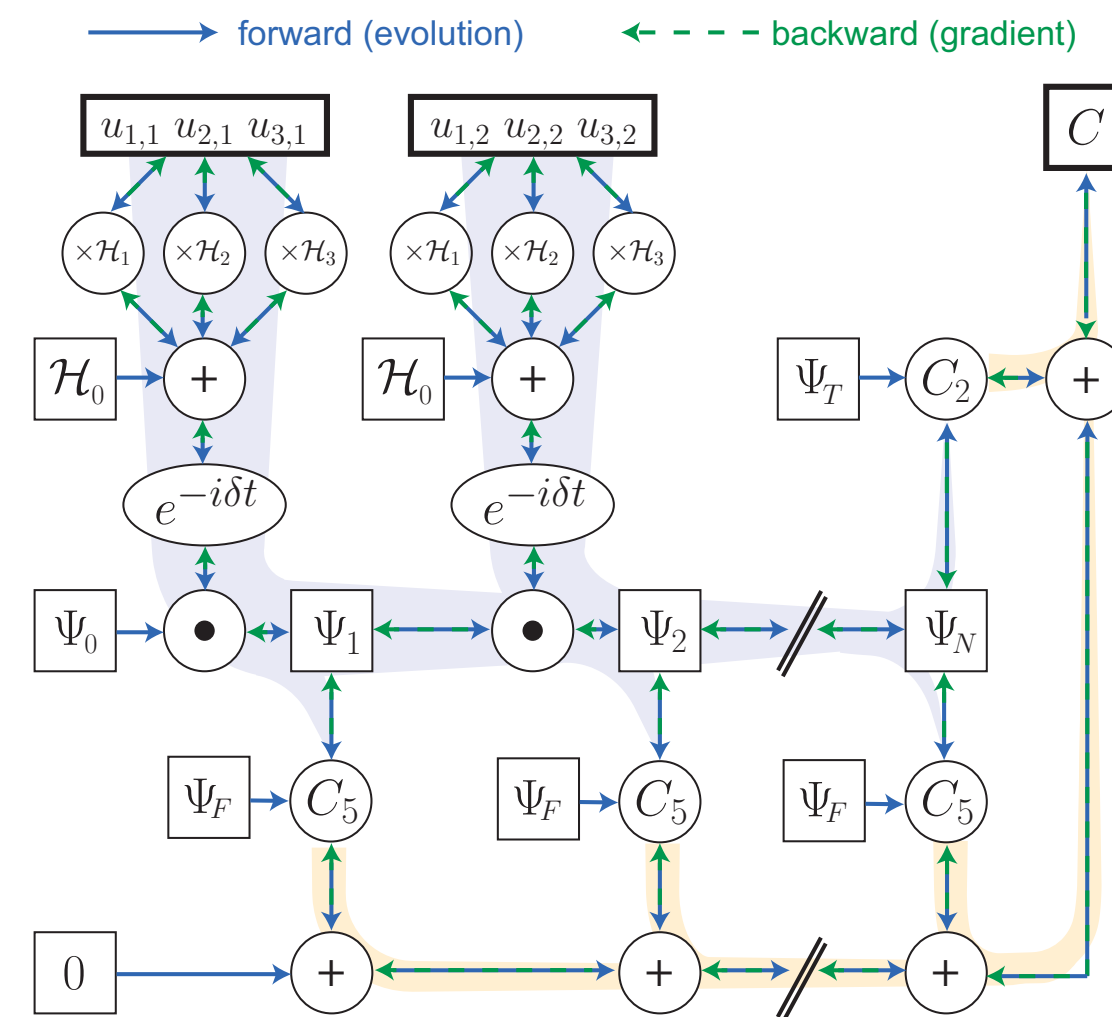
# Applications of AD

## Computing force



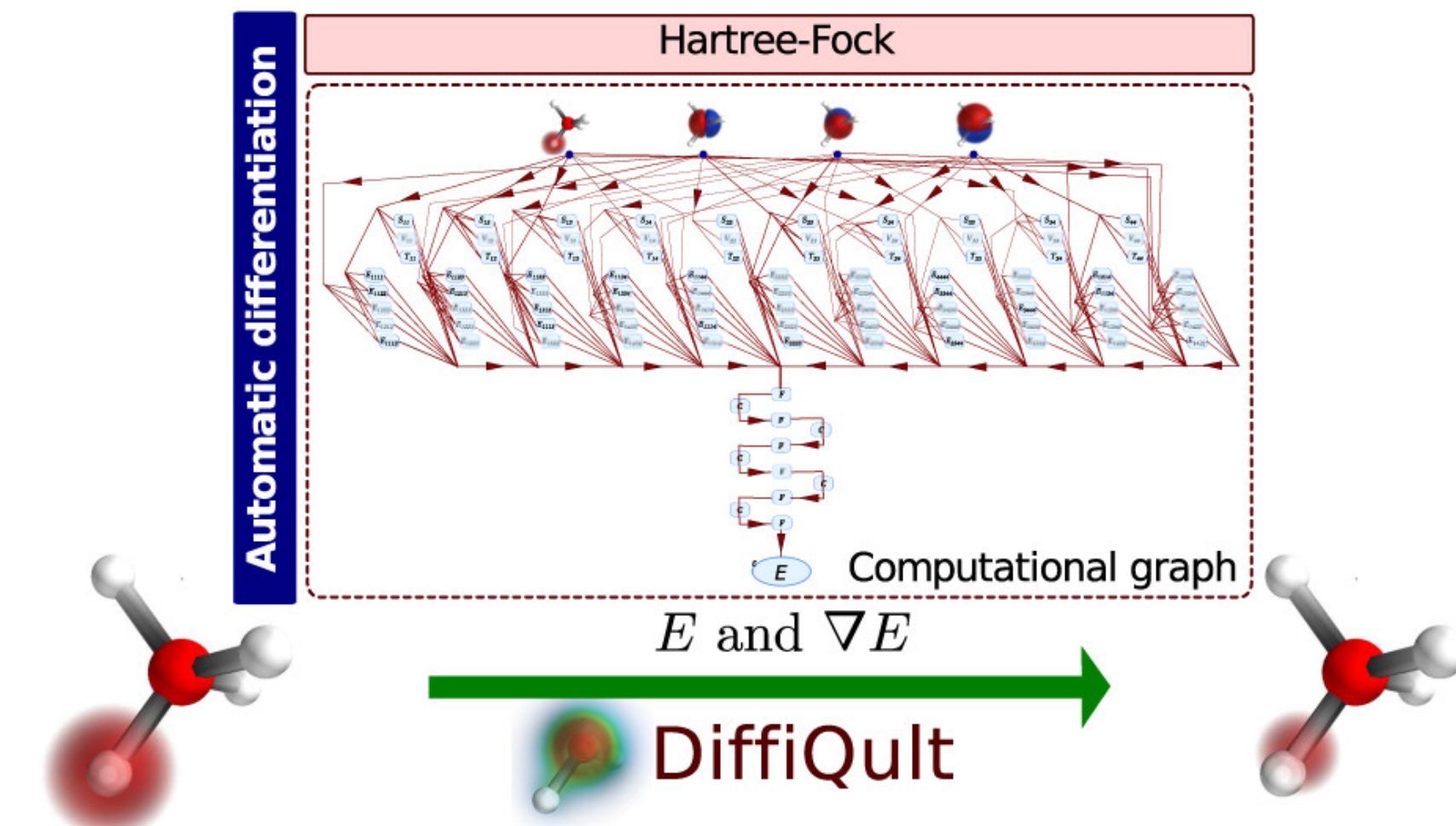
Sorella and Capriotti  
J. Chem. Phys. '10

## Quantum optimal control



Leung et al  
PRA '17

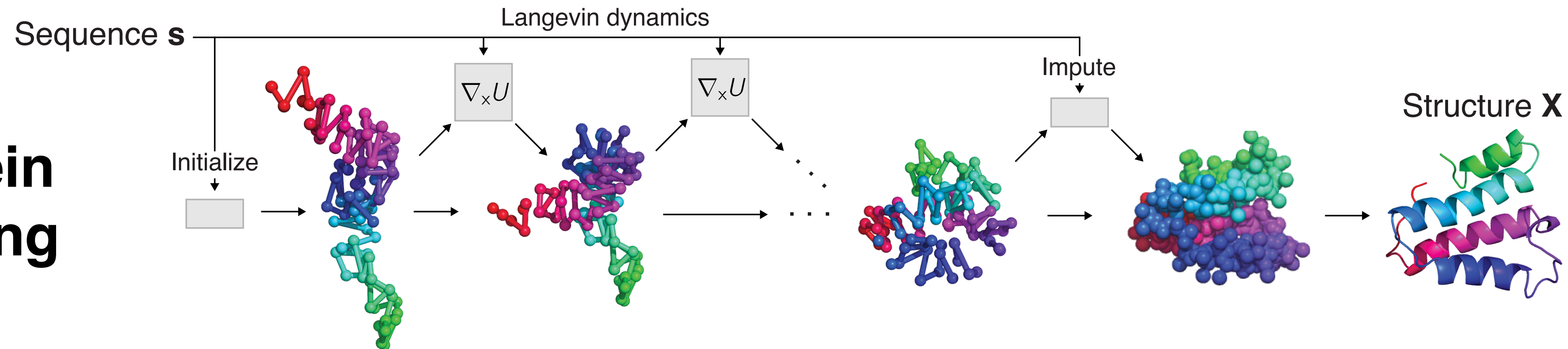
## Variational Hartree-Fock



Tamayo-Mendoza et al  
ACS Cent. Sci. '18

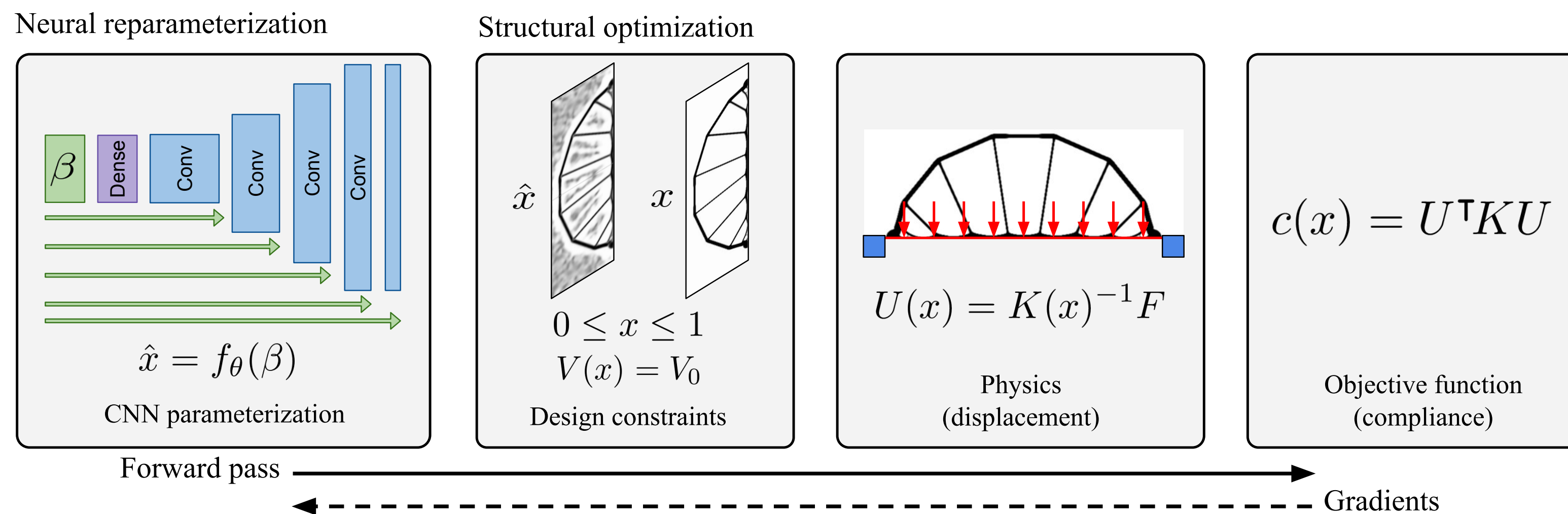
# More Applications...

## Protein Folding



Ingraham et al  
ICLR '19

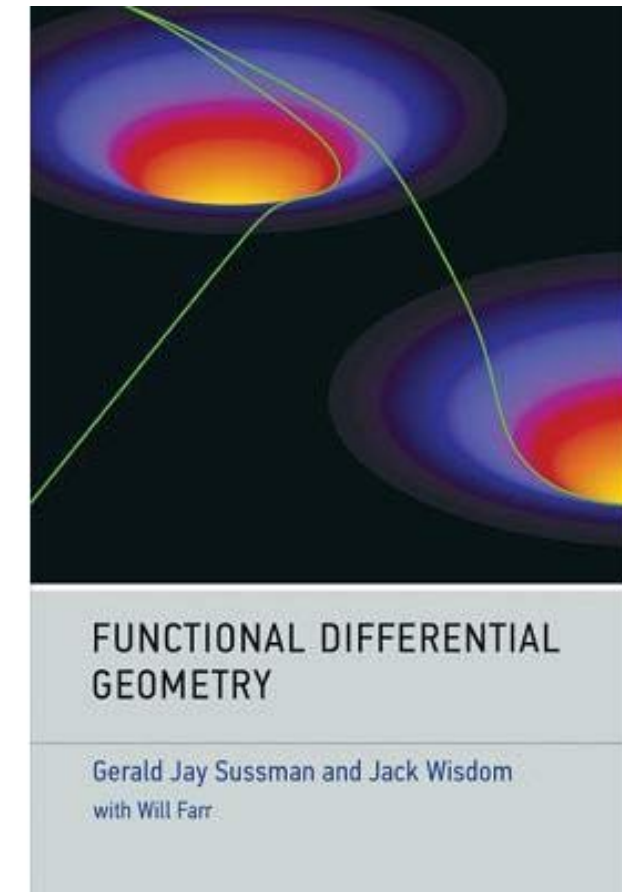
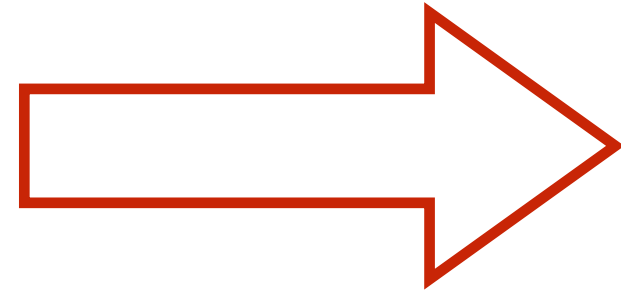
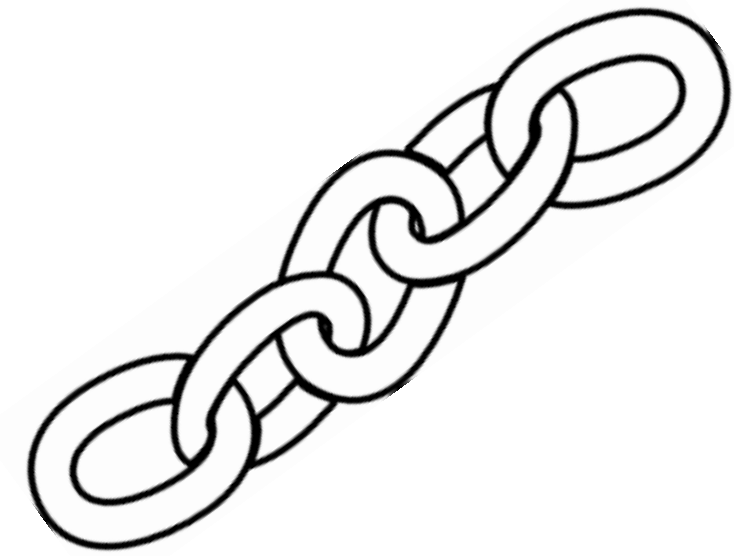
## Structural Optimization



Hoyer et al  
1909.04240



# Understandings of AD



Black  
magic box


Chain  
rule



Functional  
differential geometry

[https://colab.research.google.com/github/google/jax/blob/master/notebooks/autodiff\\_cookbook.ipynb](https://colab.research.google.com/github/google/jax/blob/master/notebooks/autodiff_cookbook.ipynb)

# Reverse versus forward mode


$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{\partial \mathcal{L}}{\partial x_n} \frac{\partial x_n}{\partial x_{n-1}} \dots \frac{\partial x_2}{\partial x_1} \frac{\partial x_1}{\partial \theta}$$


Reverse mode AD: **Vector-Jacobian Product of primitives**

- Backtrace the computation graph  $v_o (J)_{o \times i}$
- Needs to store intermediate results
- Efficient for graphs with large fan-in

**Backpropagation = Reverse mode AD applied to neural networks**

# Reverse versus forward mode

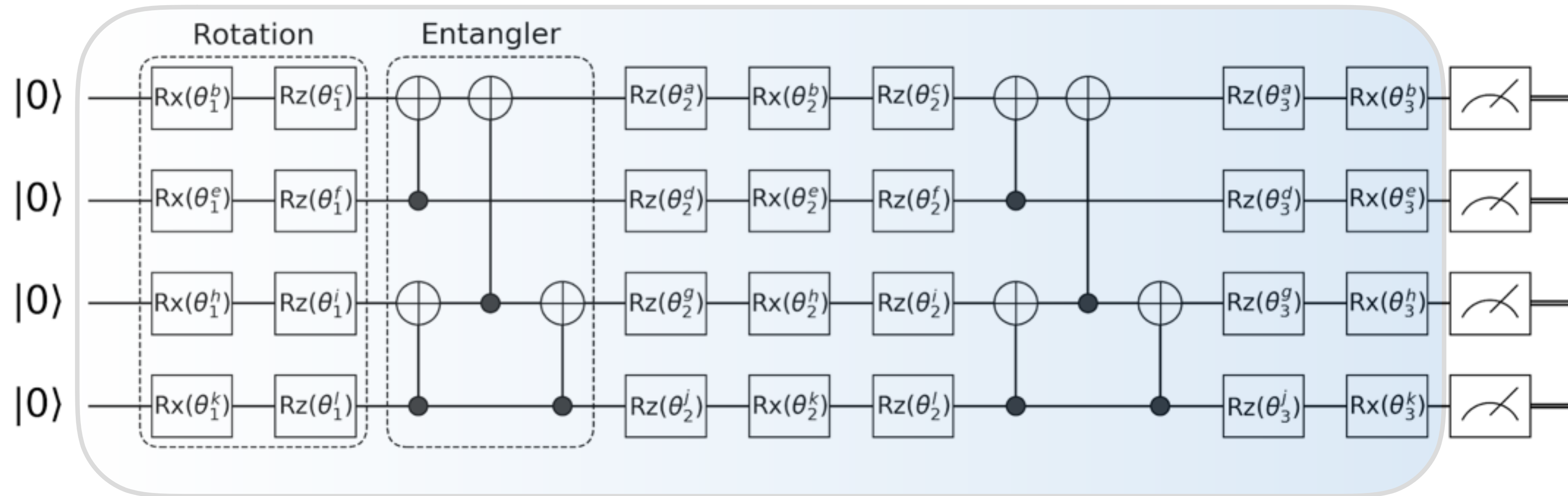
$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{\partial \mathcal{L}}{\partial x_n} \frac{\partial x_n}{\partial x_{n-1}} \dots \frac{\partial x_2}{\partial x_1} \frac{\partial x_1}{\partial \theta}$$


Forward mode AD: **Jacobian-Vector Product of primitives**

- Same order with the function evaluation  $(J)_{o \times i} v_i$
- No storage overhead
- Efficient for graph with large fan-out

**Less efficient for scalar output, but useful for higher-order derivatives**

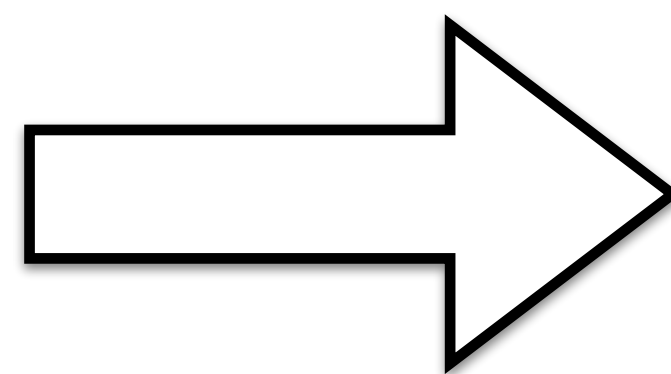
# *Differentiable*<sup>2</sup> quantum circuits



Parametrized gate of the form

$$e^{-\frac{i\theta}{2}\Sigma} \text{ with } \Sigma^2 = 1$$

eg, X, Y, Z, CNOT, SWAP...



Li et al, PRL '17, Mitarai et al, PRA '18  
Schuld et al, PRA '19, Nakanishi et al '19

$$\nabla \langle H \rangle_{\theta} = \left( \langle H \rangle_{\theta+\pi/2} - \langle H \rangle_{\theta-\pi/2} \right) / 2$$

**Unbiased gradient estimator measured on actual quantum circuits**

# Demo 3

<https://github.com/wangleiphy/YaoTutorial>

# Applications of Yao.jl

## Quantum machine learning:

Differentiable Learning of Quantum Circuit Born Machine, 1804.04168

Learning and Inference on Generative Adversarial Quantum Circuits, 1808.03425

...

## Quantum many-body physics:

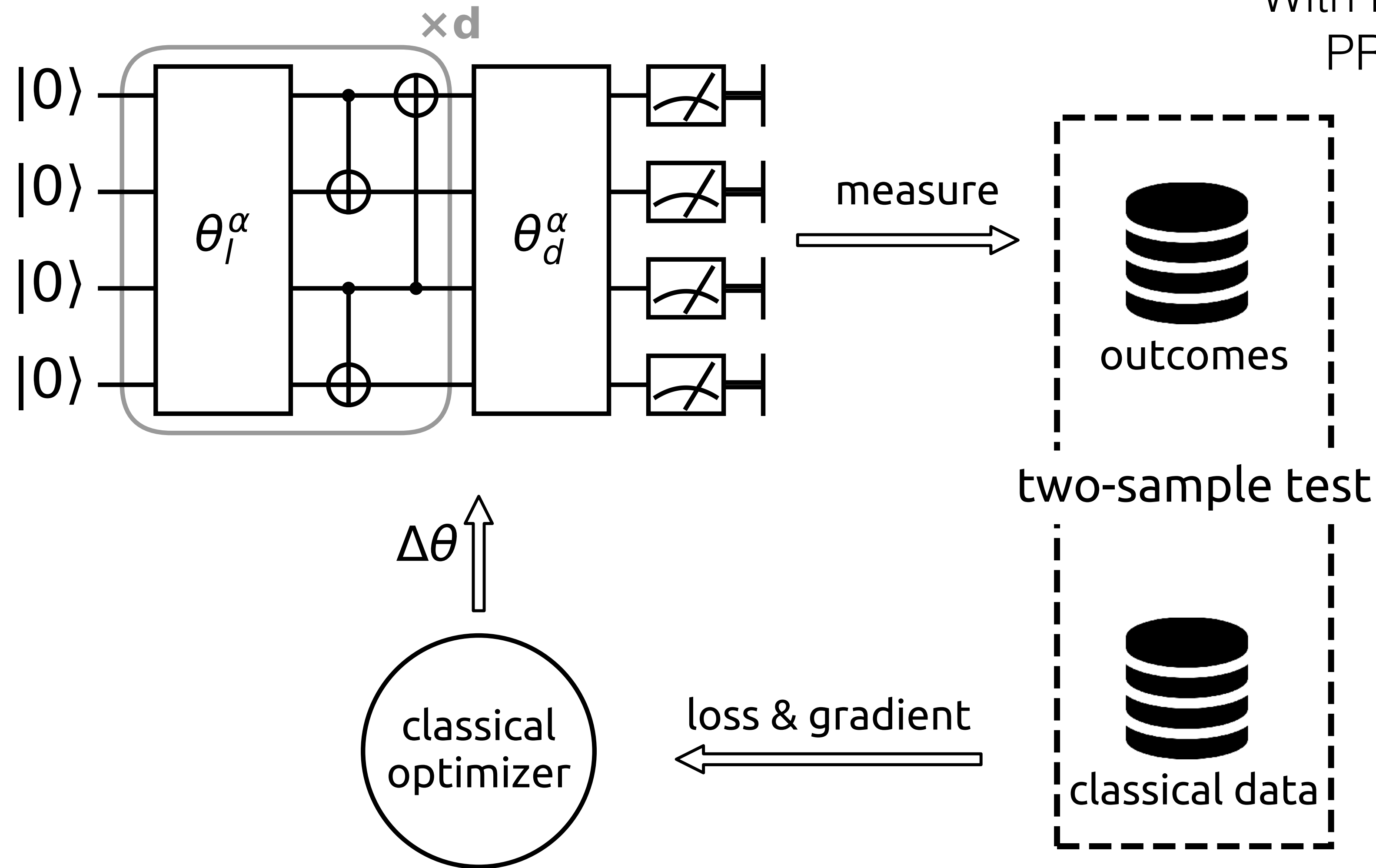
Variational Quantum Eigensolver with Fewer Qubits, 1902.02663

Solving Quantum Statistical Mechanics with VAN + Quantum Circuits, 1912.?????

...

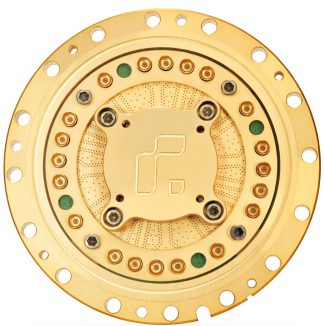
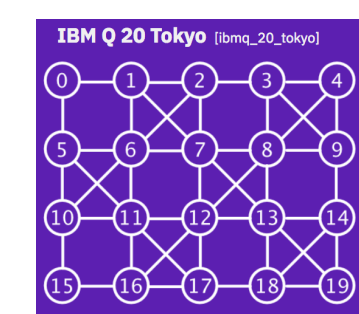
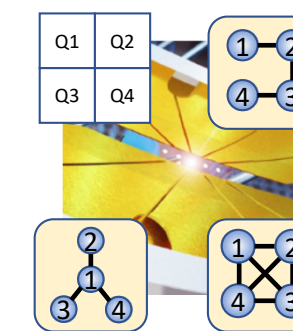
# Quantum Circuit Born Machine

With Liu, Zeng, Wu, Hu  
PRA '18, PRA '19



Experiments:

1801.07686  
1812.08862  
1811.09905  
1901.08047  
1904.02214



**Train quantum circuits as probabilistic generative models with implicit density**  
**Strong expressibility due to quantum sampling complexity**

# However, there is a HUGE GAP in the qubit number

What we want to solve



What current technology offers

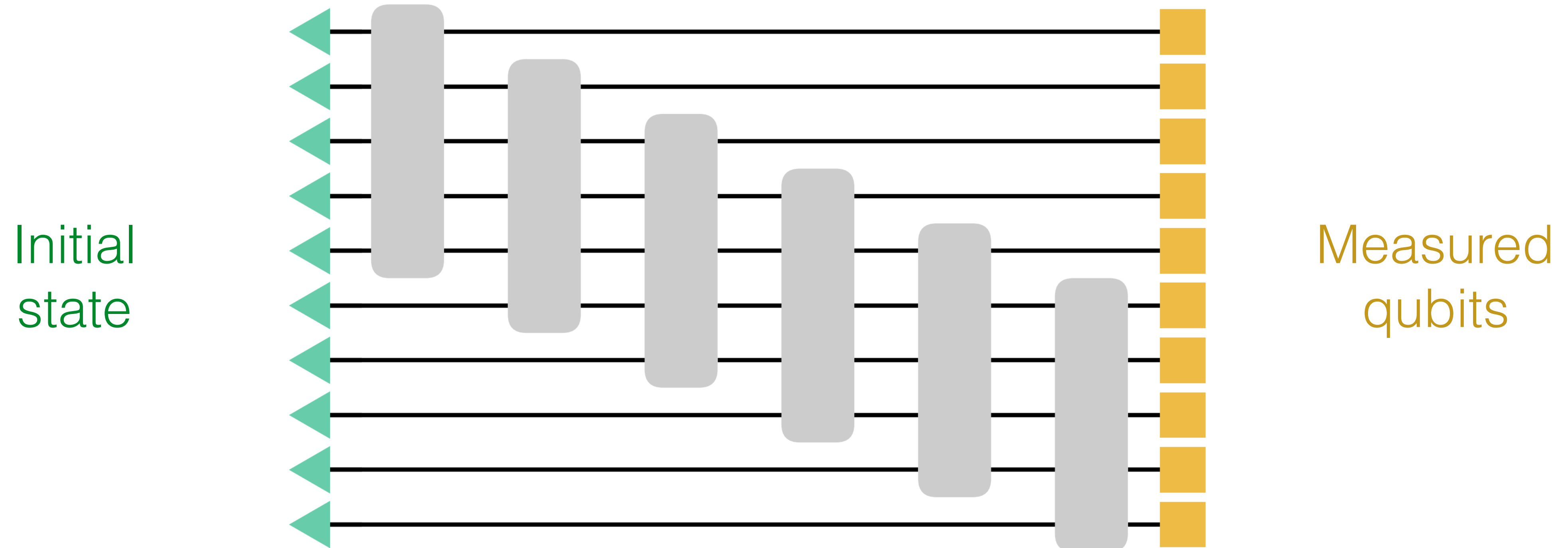


**Variational quantum eigensolver with fewer qubits**

Jin-Guo Liu, Yi-Hong Zhang, Yuan Wan, LW, 1902.02663



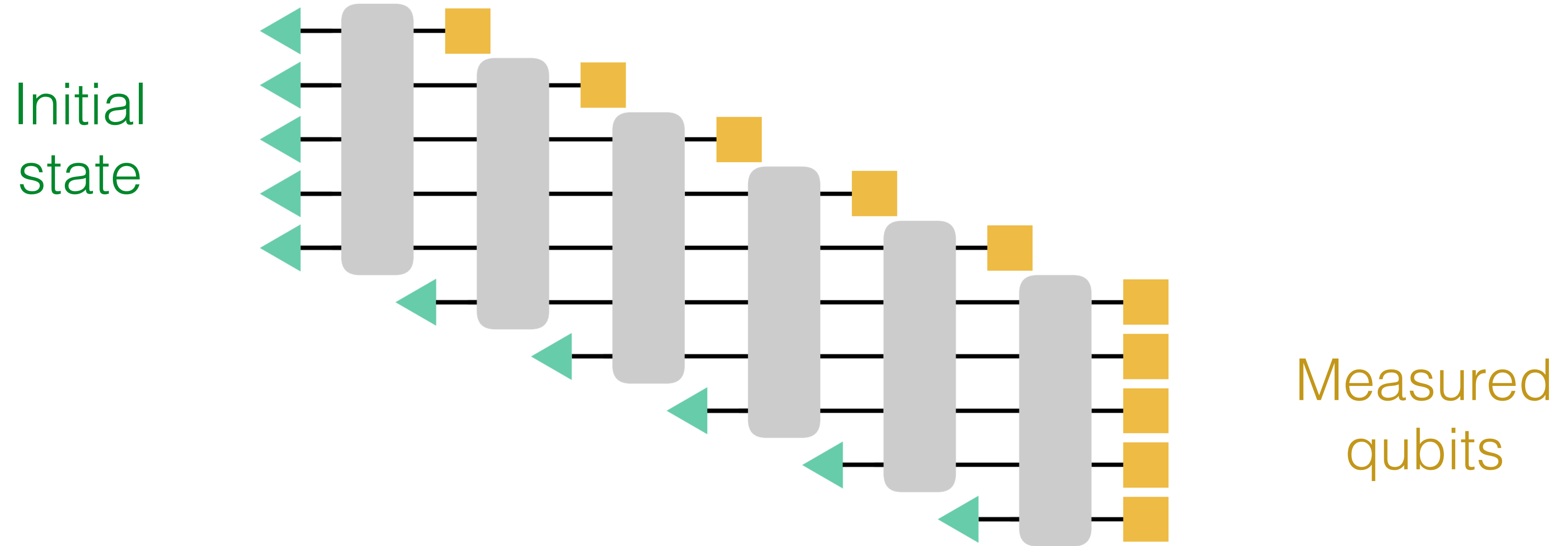
# A qubit efficient variational circuit



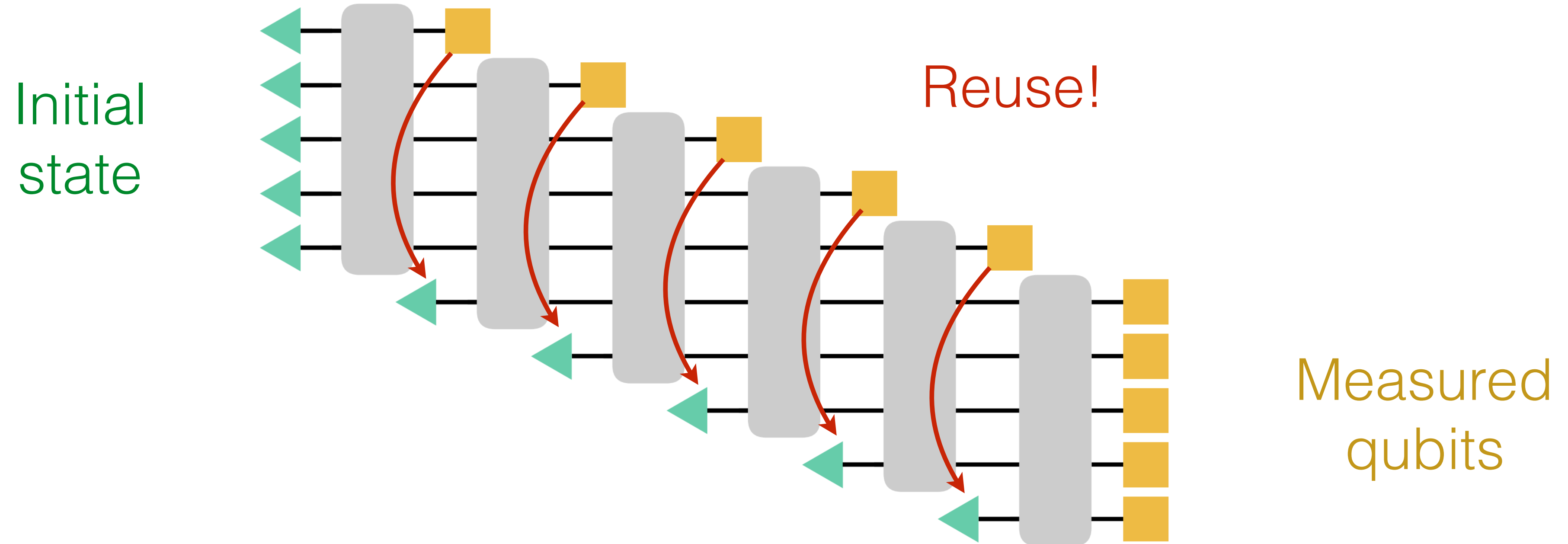
Huggins, Patel, Whaley, Stoudenmire, 1803.11537  
see also Cramer et al, Nat. Comm. '10

**Tensor network inspired quantum circuit architecture**

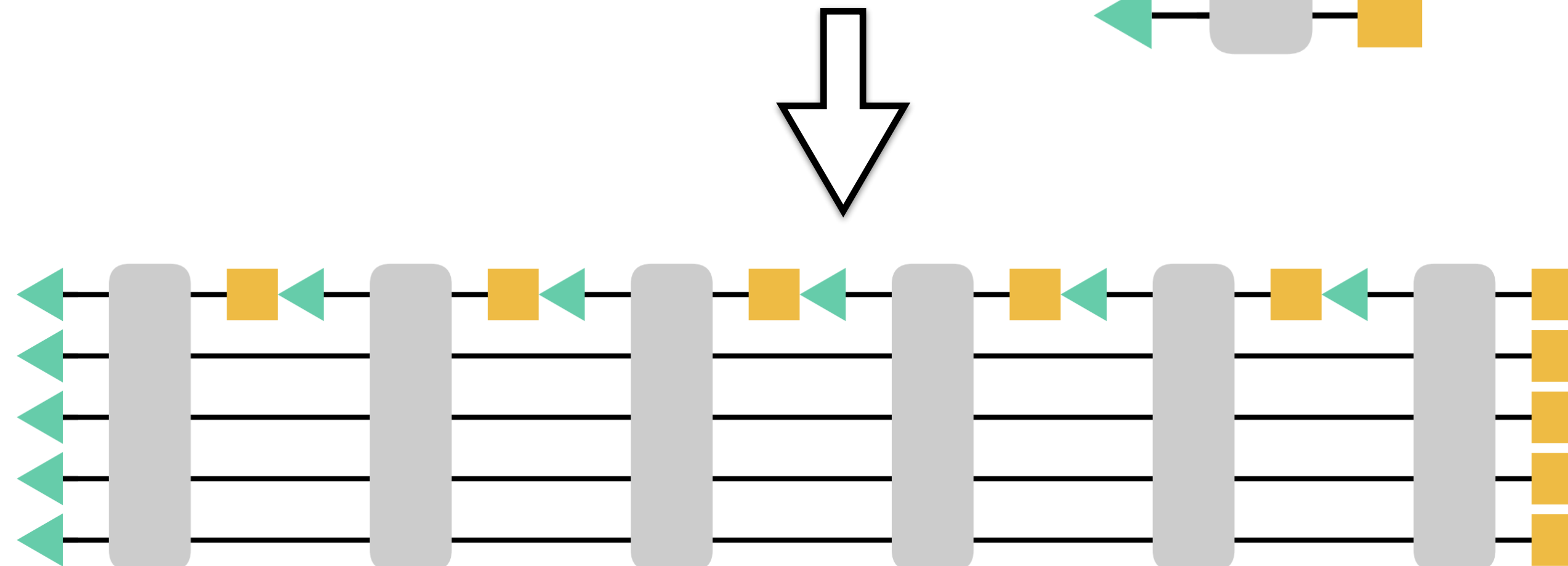
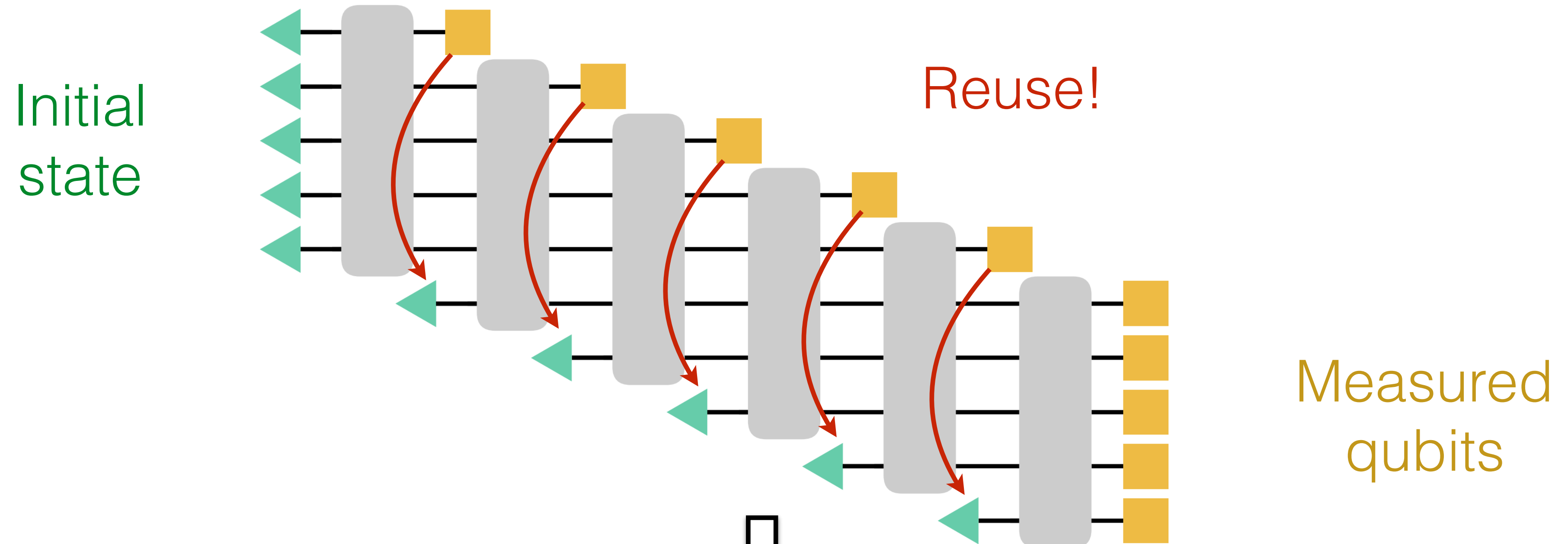
# A qubit efficient variational circuit



# A qubit efficient variational circuit

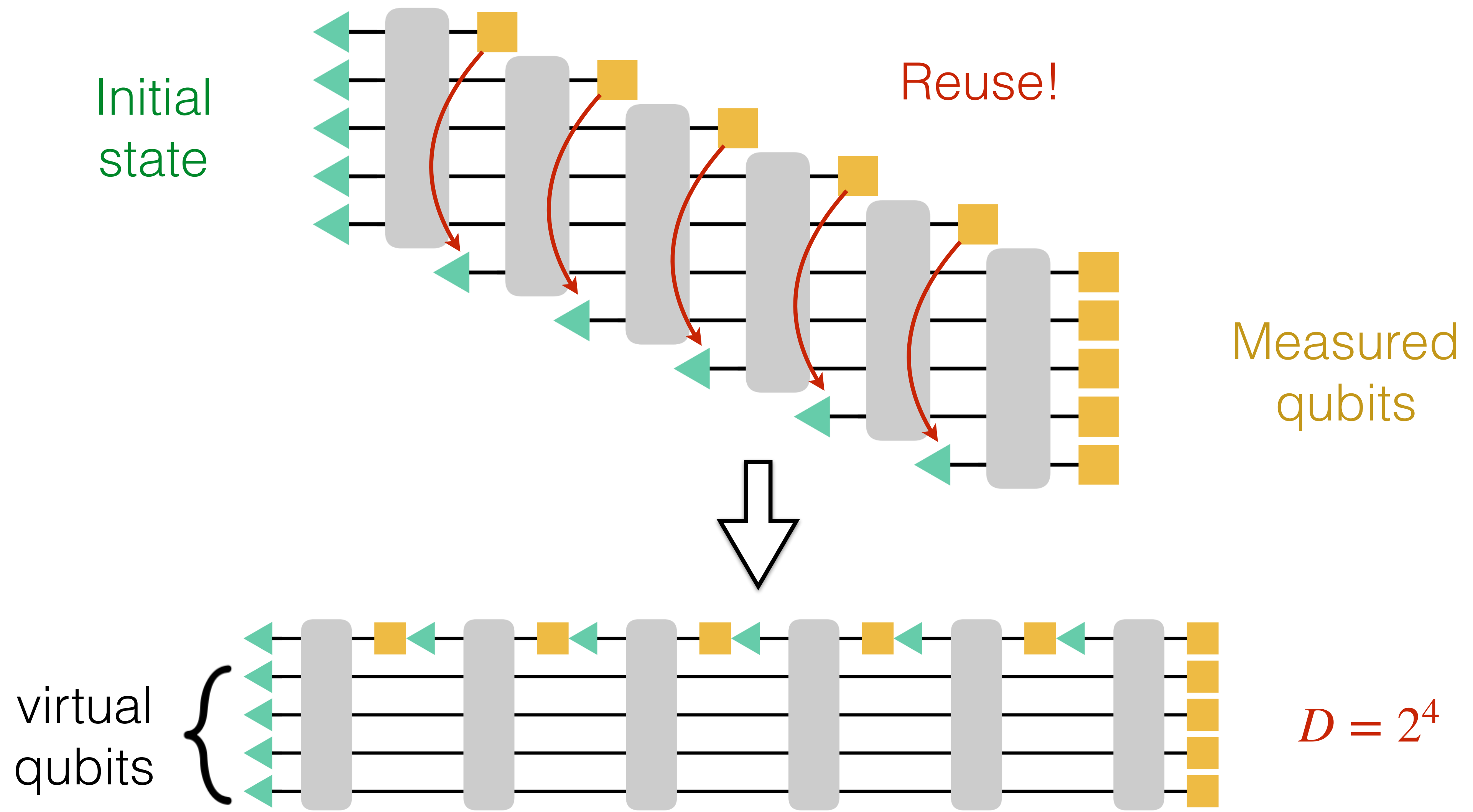


# A qubit efficient variational circuit



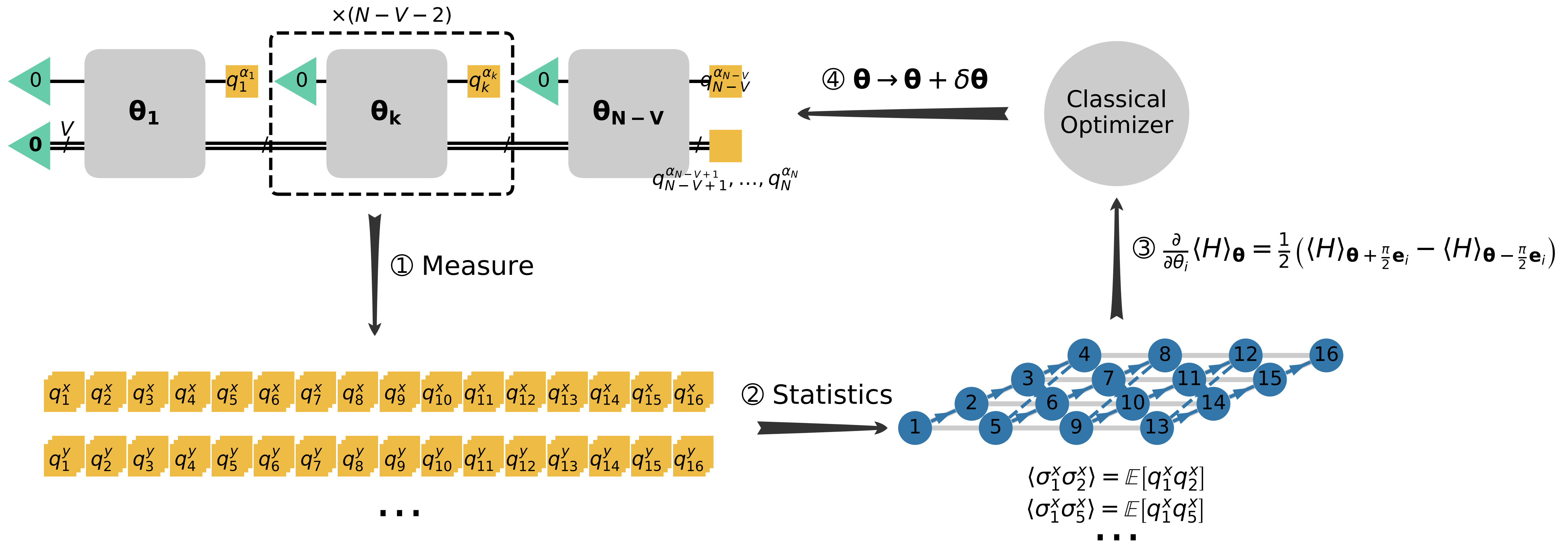
**Matrix Product State with exponentially large bond dimensions**

# A qubit efficient variational circuit

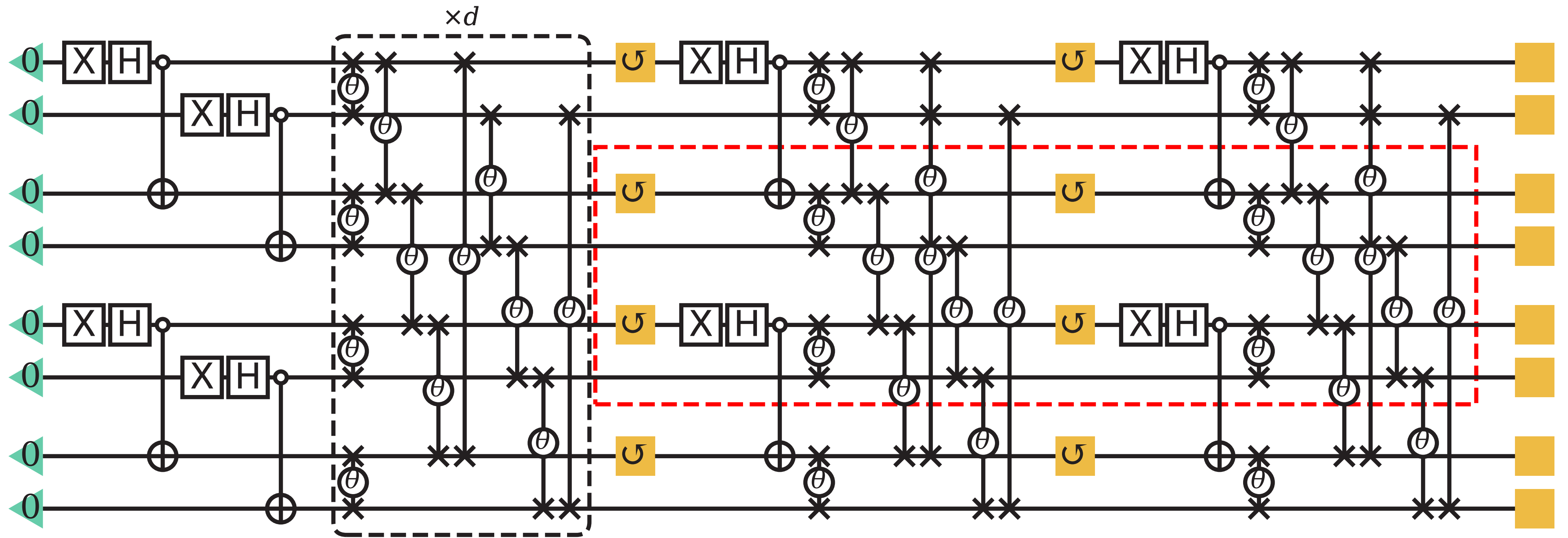


**Matrix Product State with exponentially large bond dimensions**

# Q-MPS



# Q-PEPS



# How to prepare quantum thermal states?

Thermofield Double States

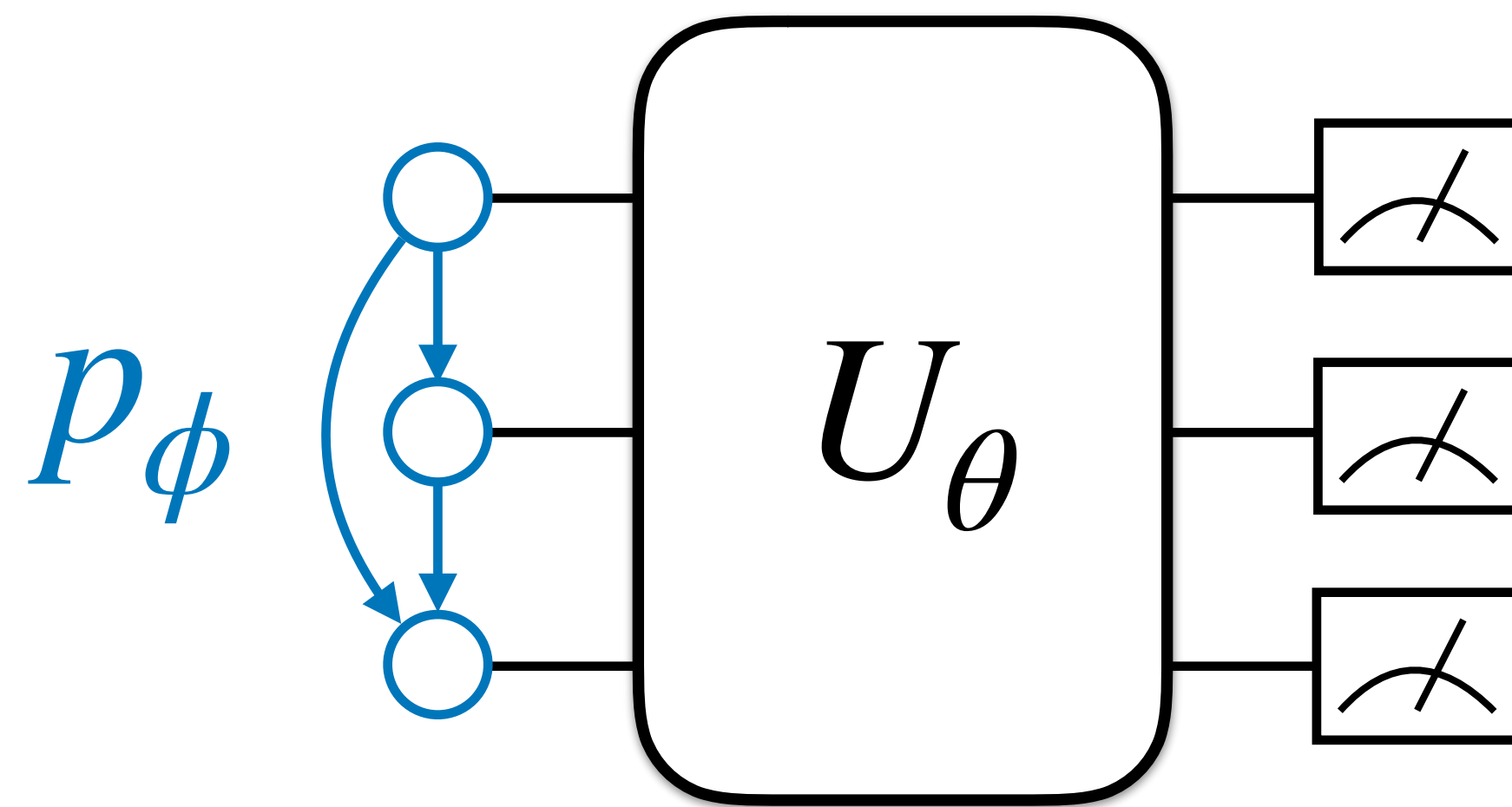
Wu & Hsieh, 1811.11756

Quantum imaginary-time evolution

Motta et al, 1901.07653



# “ $\beta$ ”-VQE

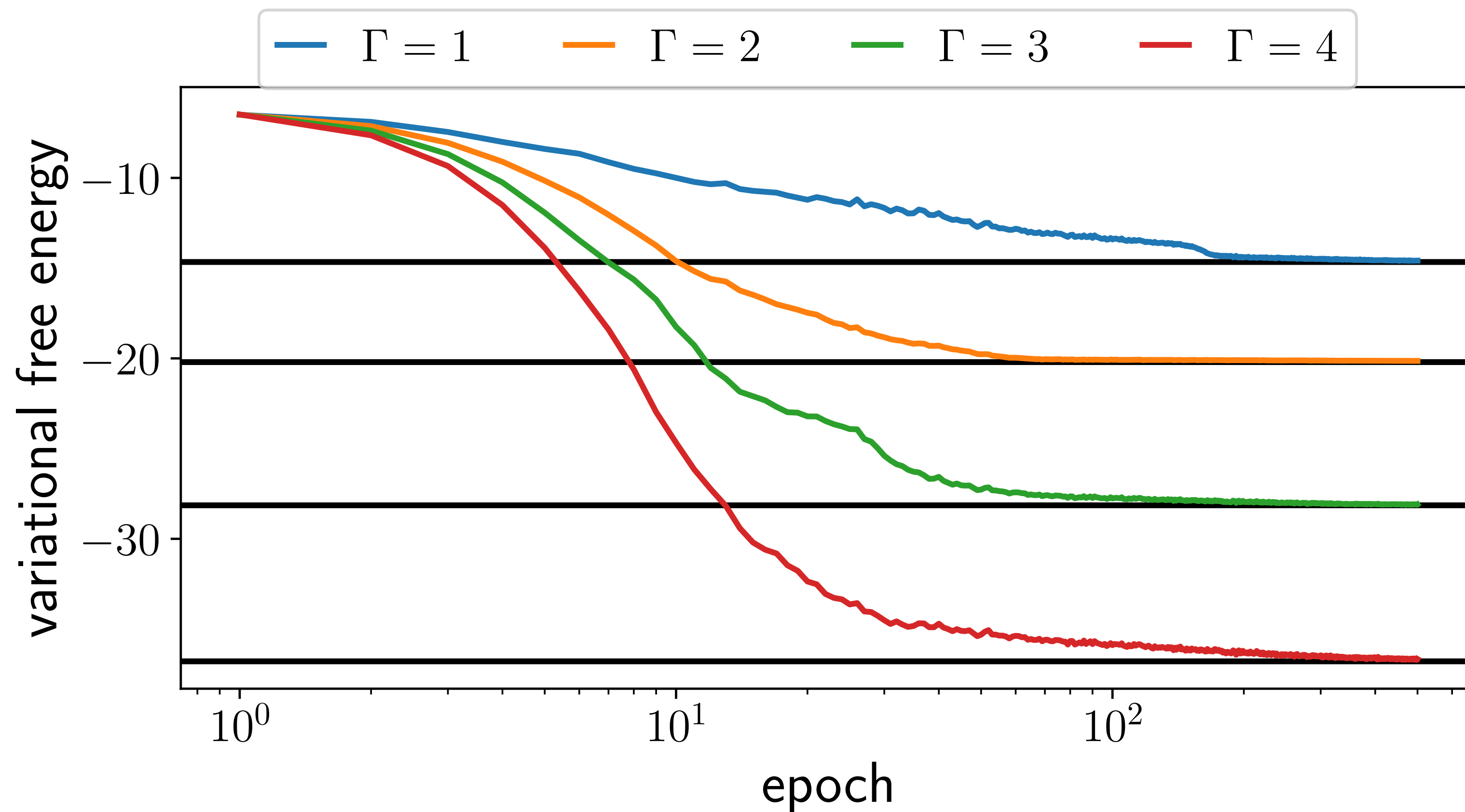


$$\sum_x P_\phi(x) U_\theta |x\rangle \langle x| U_\theta^\dagger = \rho$$

**A classical mixture of quantum states parametrizes density matrices**

# “ $\beta$ ”-VQE

$$\mathcal{L} = \beta \text{Tr}(\rho H) + \text{Tr}(\rho \ln \rho) \geq -\ln Z$$



3x3 quantum  
Ising model @  $\beta=1$

Liu, Mao, Zhang,  
LW, 1912.11381

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Xiu-Zhe Luo, Jin-Guo Liu, Pan Zhang, Lei Wang, [1912.10877](#)

**Thank You!**