Simulation of Hubbard Models in the Era of Synthetic Gauge Field

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Institute of Physics

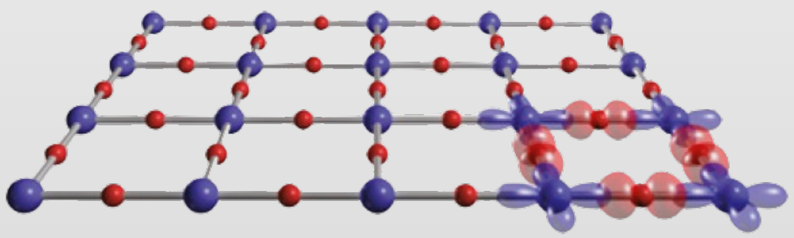
第十届冷原子物理青年学者学术讨论会 2016.7

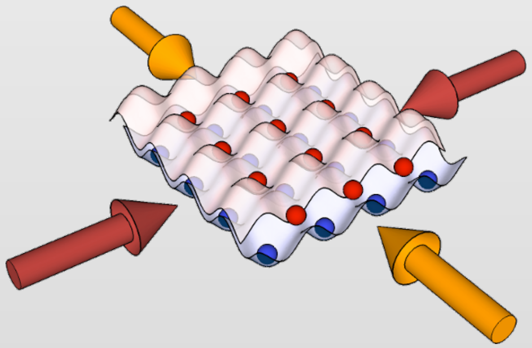
Hubbard Model

$$\hat{H} = -\sum_{i,j,\sigma} t_{ij}\hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma} + U\sum_{i}\hat{n}_{i\uparrow}\hat{n}_{i\downarrow}$$

d-wave SC
Mott physics
Magnetism
BCS-BEC
...

Solid materials





Optical lattices

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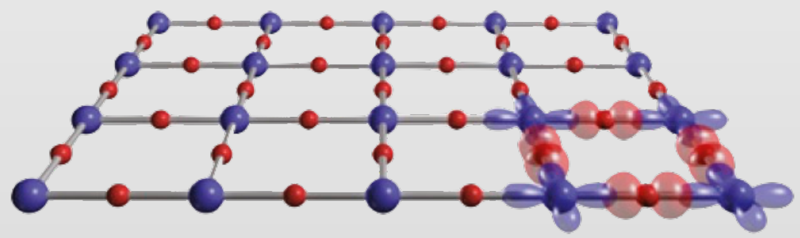
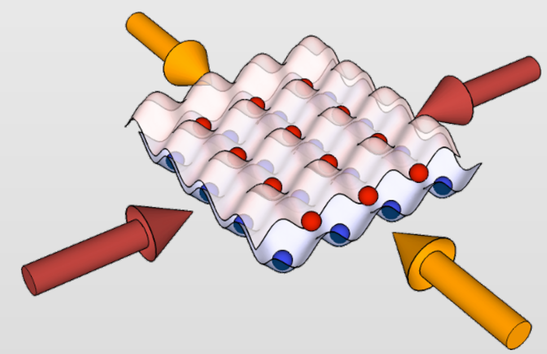
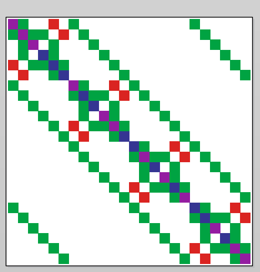




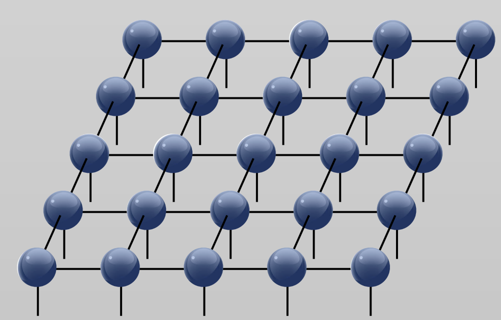
Optical lattices

Algorithms for quantum many body systems

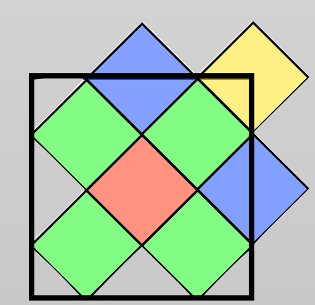


exact diagonalization

quantum Monte Carlo



tensor network states



dynamical mean field theories

Hubbard Model

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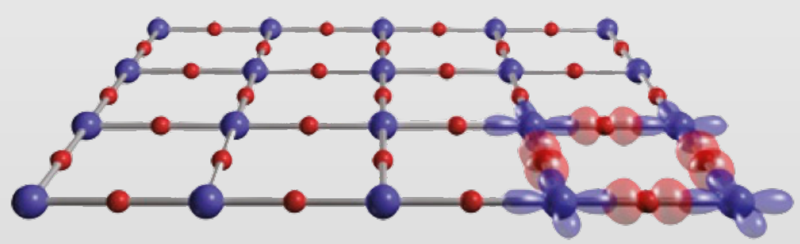
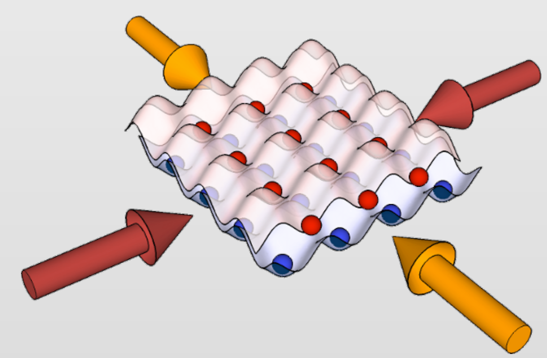
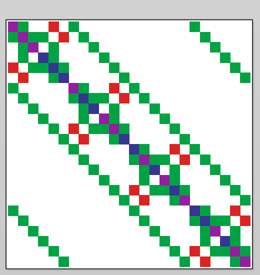




Optical lattices

Algorithms for quantum many body systems



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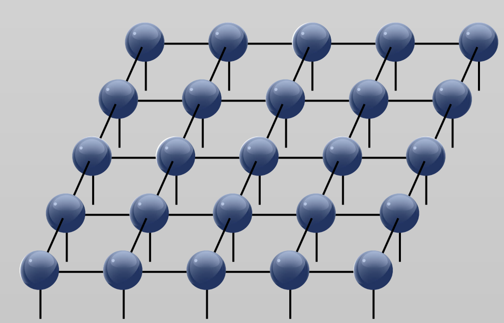

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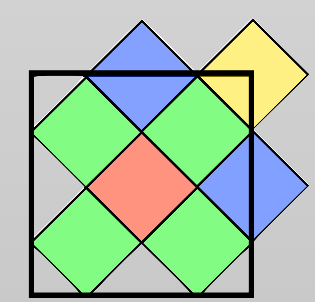

dynamical mean field theories

better scaling

Iazzi and Troyer, PRB 2015
LW, Iazzi, Corboz and Troyer, PRB 2015
Liu and LW, PRB 2015
LW, Liu and Troyer, PRB 2016

entanglement & fidelity

LW and Troyer, PRL 2014
LW, Liu, Imriška, Ma and Troyer, PRX 2015
LW, Shinaoka and Troyer, PRL 2015
Huang, Wang, LW and Werner, arXiv 2016

sign problem

Huffman and Chandrasekharan, PRB 2014
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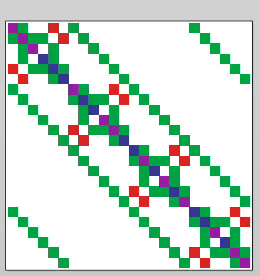
Wei, Wu, Li, Zhang and Xiang, PRL 2016



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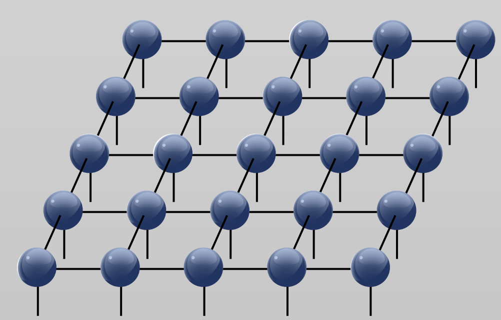

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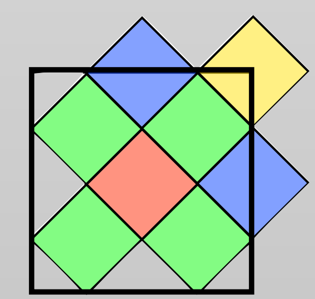

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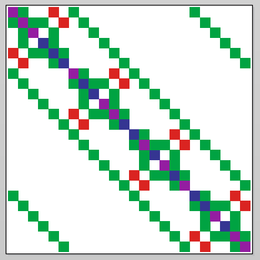

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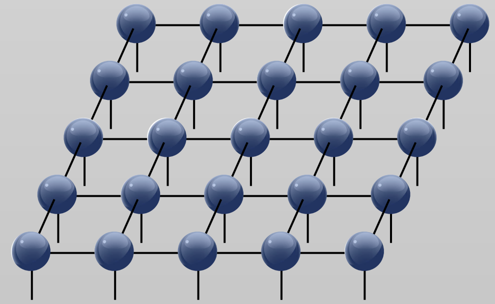

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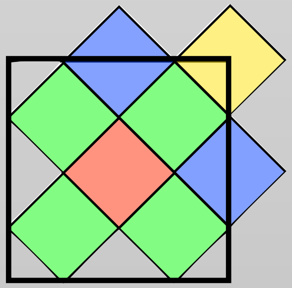
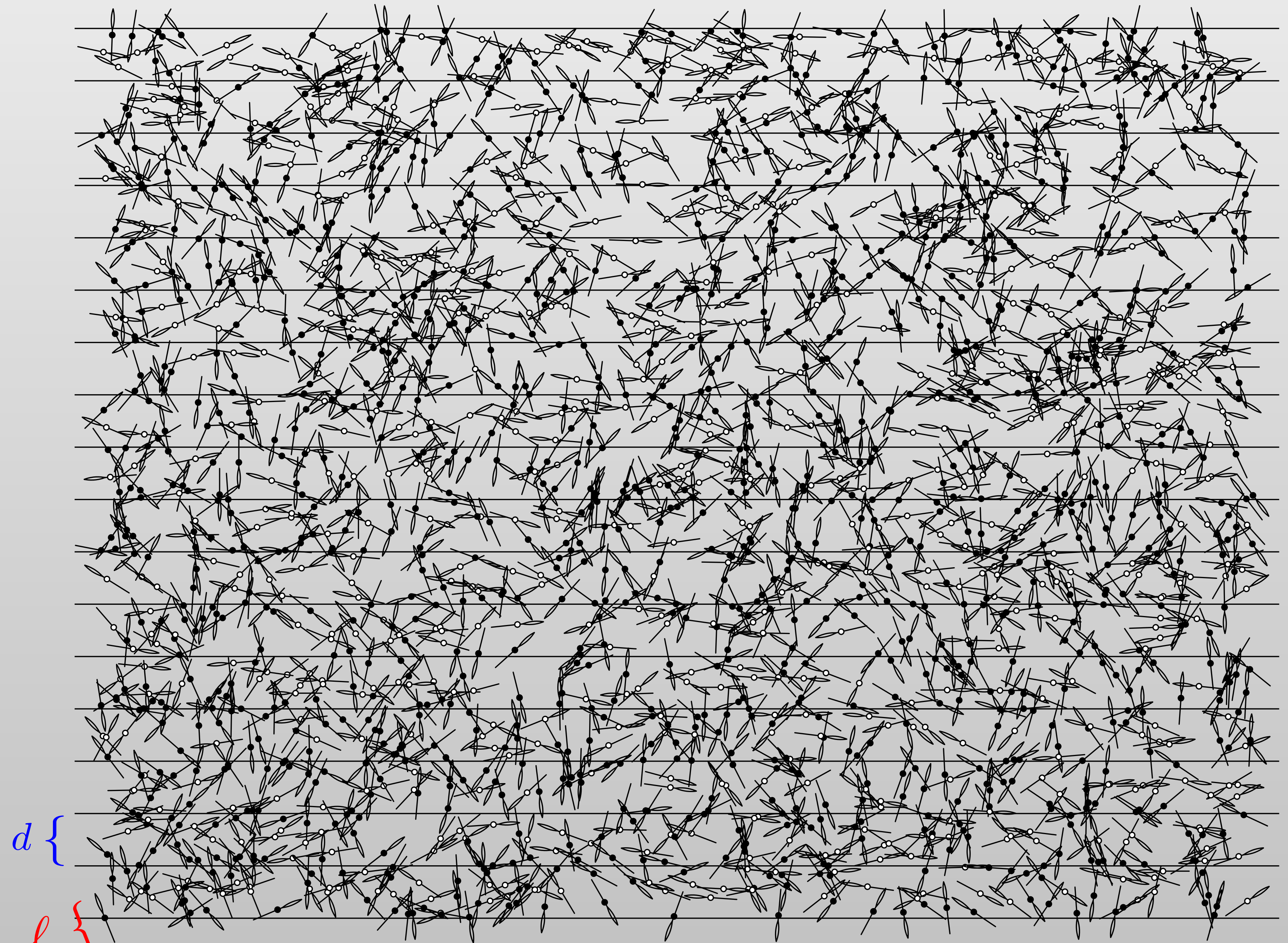


dynamical mean field theories

The first recorded Monte Carlo simulation

$$\langle N_{\text{hits}} \rangle = \frac{2}{\pi} \frac{\ell}{d}$$



Buffon 1777

Statistical Mechanics:
Algorithms and Computations
Werner Krauth

THE JOURNAL OF CHEMICAL PHYSICS VOLUME 21, NUMBER 6 JUNE, 1953

Equation of State Calculations by Fast Computing Machines

Nicholas Metropolis, Arianna W. Rosenbluth, Marshall N. Rosenbluth, and Augusta H. Teller,
Los Alamos Scientific Laboratory, Los Alamos, New Mexico

AND

Edward Teller,* *Department of Physics, University of Chicago, Chicago, Illinois*

(Received March 6, 1953)

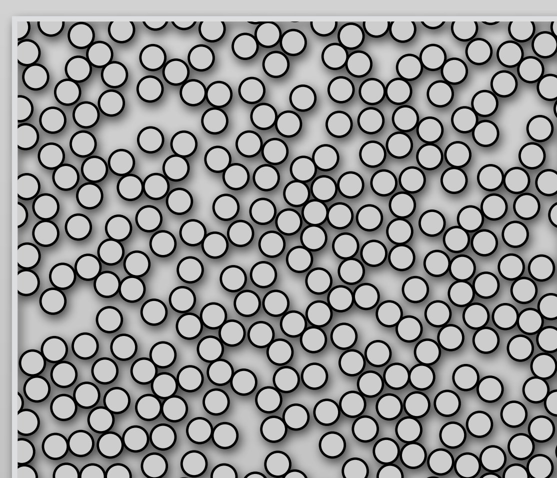
A general method, suitable for fast computing machines, for investigating such properties as equations of state for substances consisting of interacting individual molecules is described. The method consists of a modified Monte Carlo integration over configuration space. Results for the two-dimensional rigid-sphere system have been obtained on the Los Alamos MANIAC and are presented here. These results are compared to the free volume equation of state and to a four-term virial coefficient expansion.

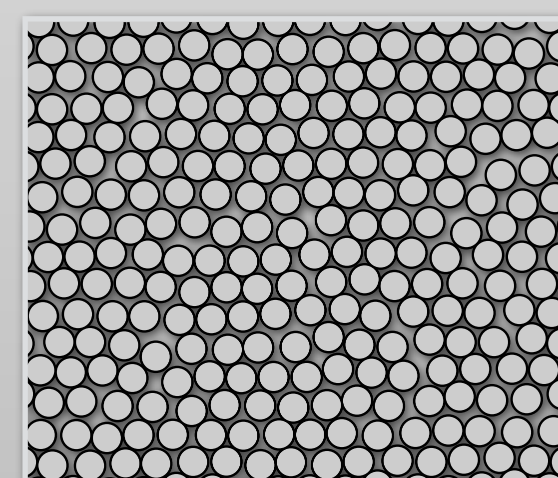
I. INTRODUCTION

THE purpose of this paper is to describe a general method, suitable for fast electronic computing machines, of calculating the properties of any substance which may be considered as composed of interacting individual molecules. Classical statistics is assumed,

II. THE GENERAL METHOD FOR AN ARBITRARY POTENTIAL BETWEEN THE PARTICLES

In order to reduce the problem to a feasible size for numerical work, we can, of course, consider only a finite number of particles. This number N may be as high as several hundred. Our system consists of a square† con-





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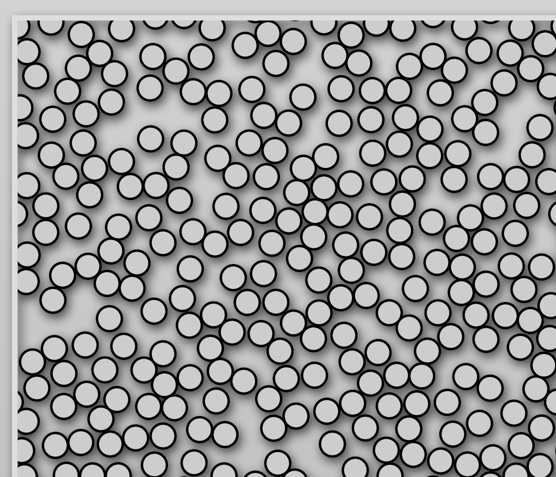
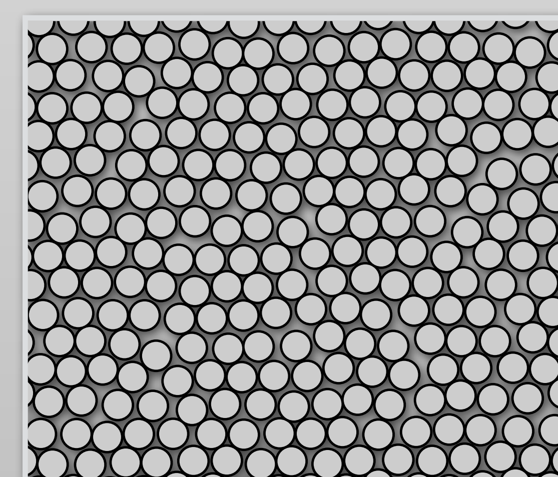


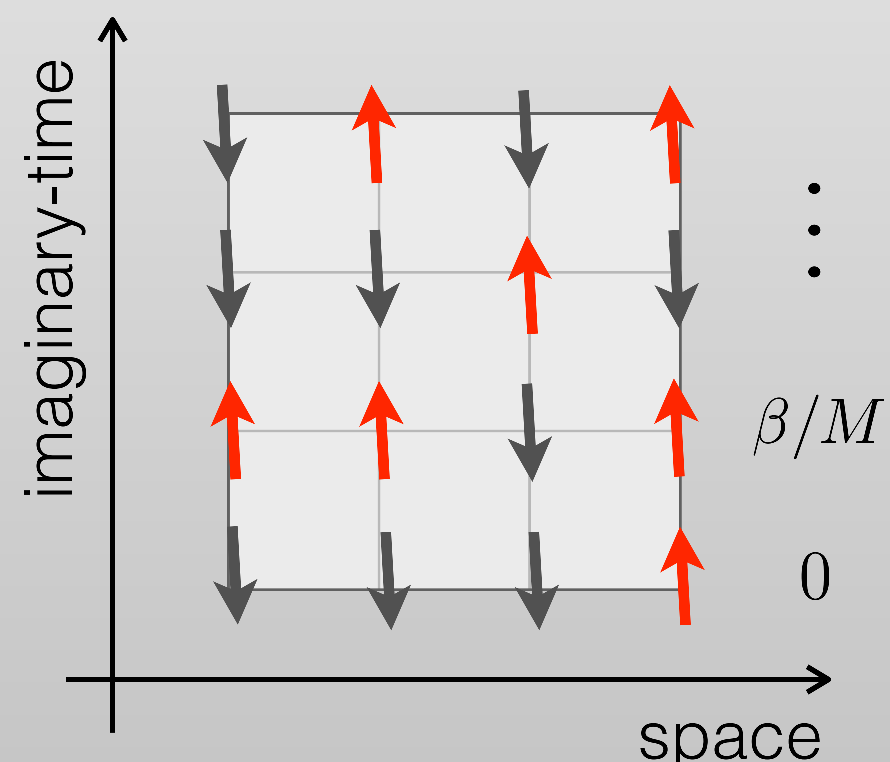


Quantum to classical mapping

$$Z = \mathrm{Tr}\left(e^{-\beta \hat{H}}\right) \qquad \hat{H} = \hat{H}_0 + \lambda \hat{H}_1$$

Trotterization

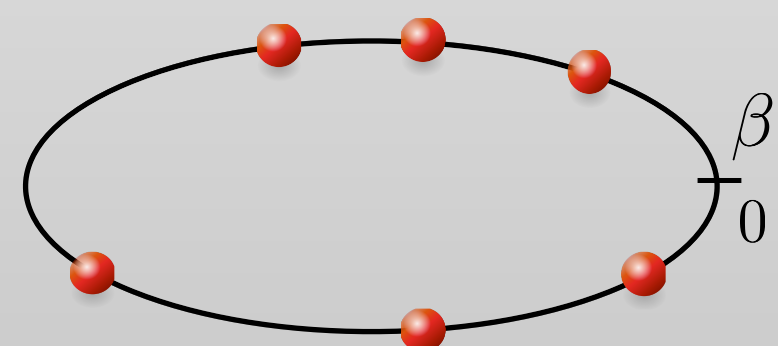
$$Z = \mathrm{Tr}\left(e^{-\frac{\beta}{M}\hat{H}} \ldots e^{-\frac{\beta}{M}\hat{H}}\right)$$



Diagrammatic approach

$$Z = \sum_{k=0}^{\infty} \lambda^k \int_0^{\beta} d\tau_1 \ldots \int_{\tau_{k-1}}^{\beta} d\tau_k \times$$

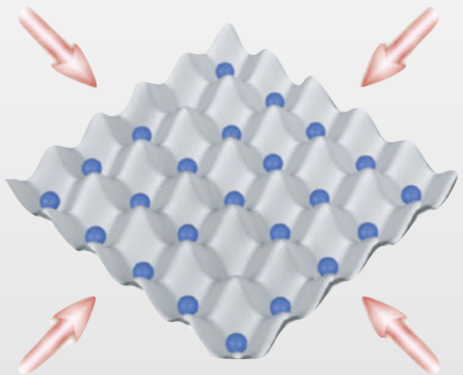
$$\mathrm{Tr}\left[(-1)^k e^{-(\beta-\tau_k)\hat{H}_0} \hat{H}_1 \ldots \hat{H}_1 e^{-\tau_1 \hat{H}_0}\right]$$

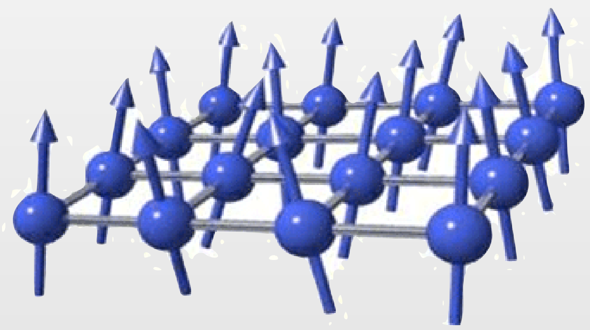


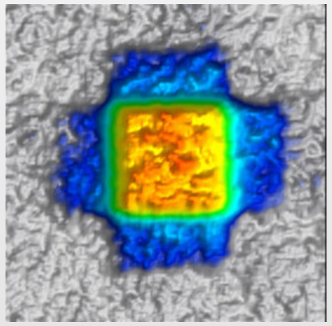
imaginary-time axis

Beard and Wiese, 1996
Prokof'ev, Svistunov, Tupitsyn,1996

Diagrammatic approaches







bosons

World-line Approach

Prokof'ev et al, JETP, **87**, 310 (1998)

quantum spins

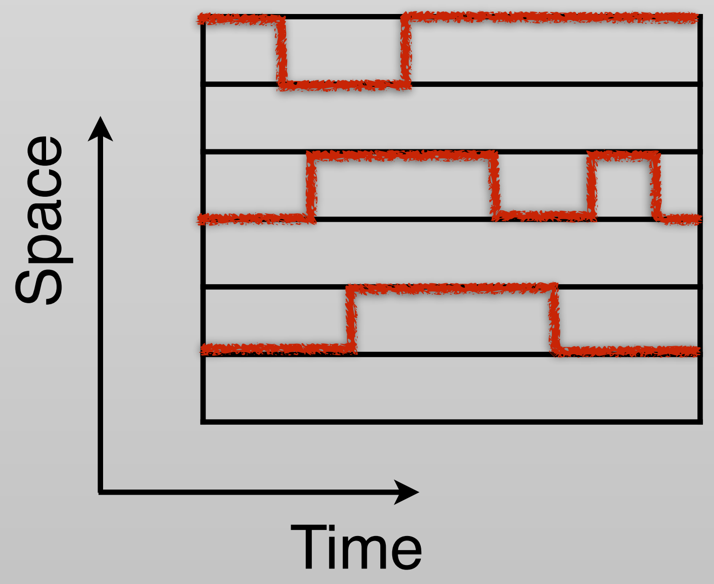
Stochastic Series Expansion

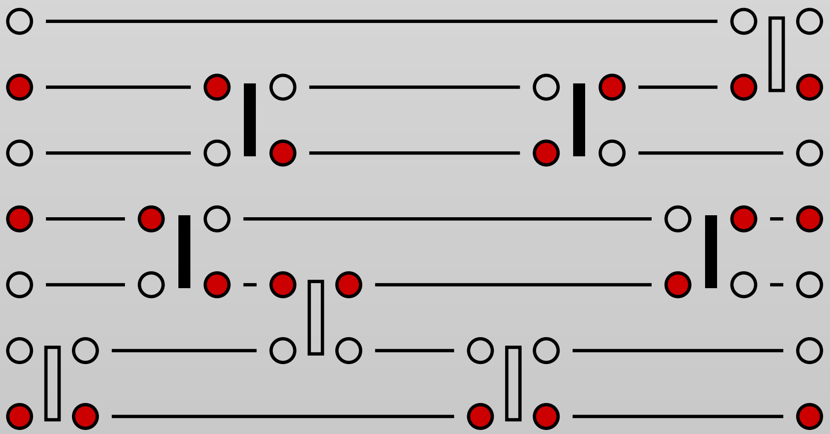
Sandvik et al, PRB, **43**, 5950 (1991)

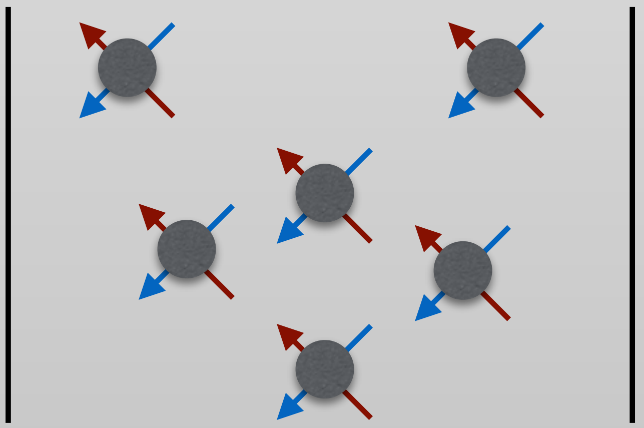
fermions

Determinantal Methods

Gull et al, RMP, **83**, 349 (2011)







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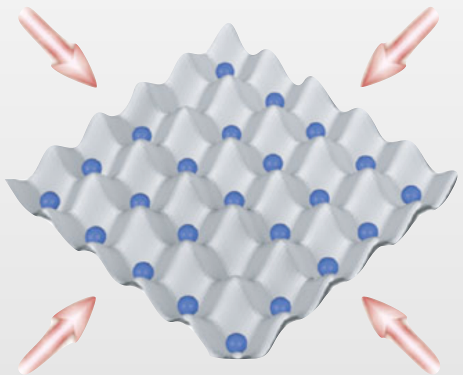

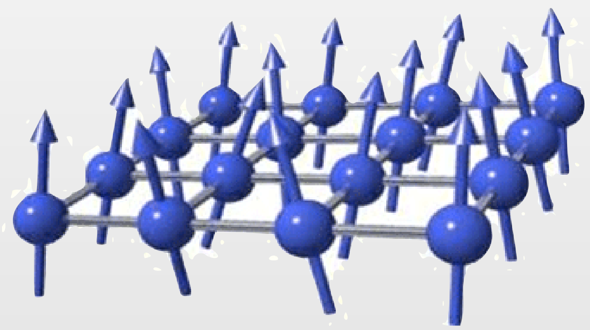

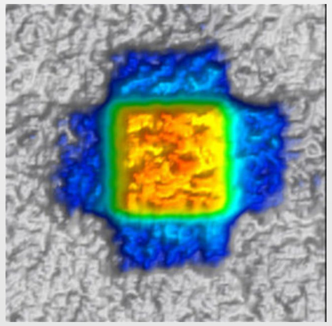

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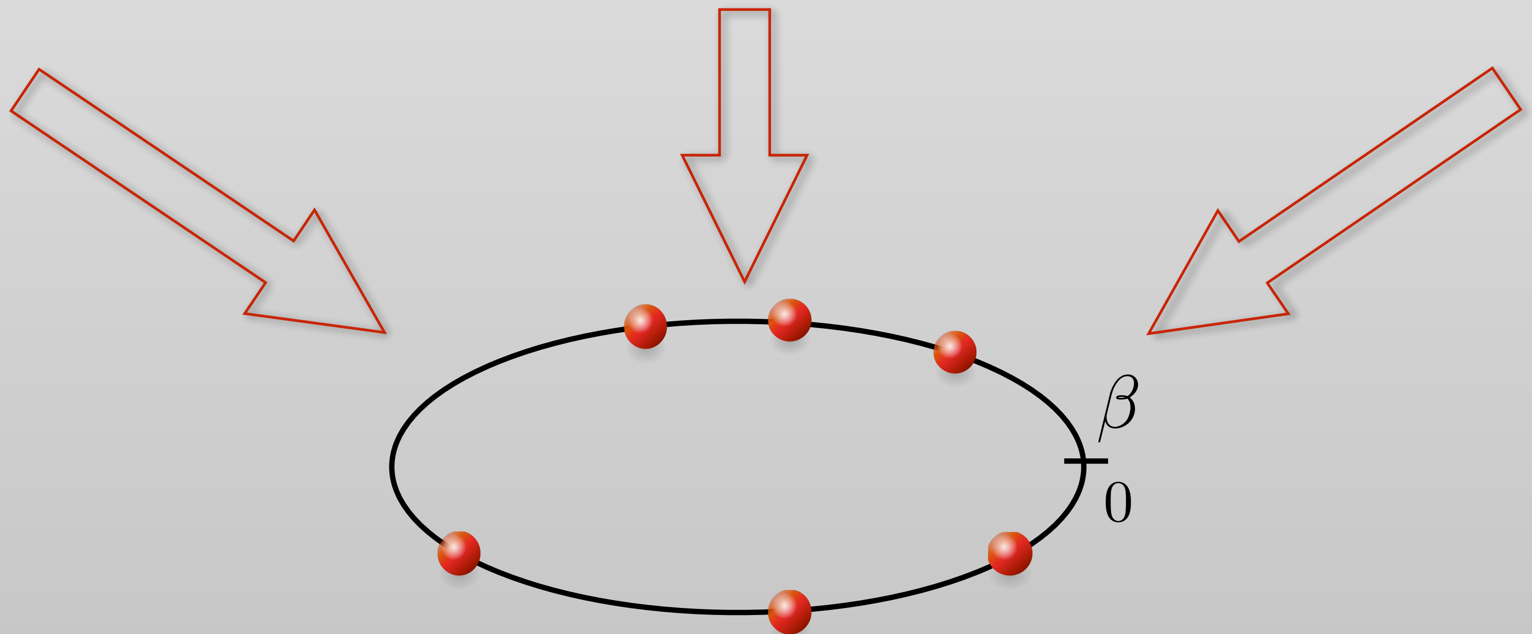


Aspects of QMC

- Unbiased method with statistical error more accurate if you run it longer

- Quite flexible in terms of temperature, dimension and range of interactions

- Frontier: compute quantum information quantities LW and Troyer, PRL 2014
  LW, Liu, Imriška, Ma and Troyer, PRX 2015   LW, Shinaoka and Troyer, PRL 2015

- Can simulate millions of bosons/quantum spins on a PC, thousands of fermions on a cluster

...if there is no sign problem!

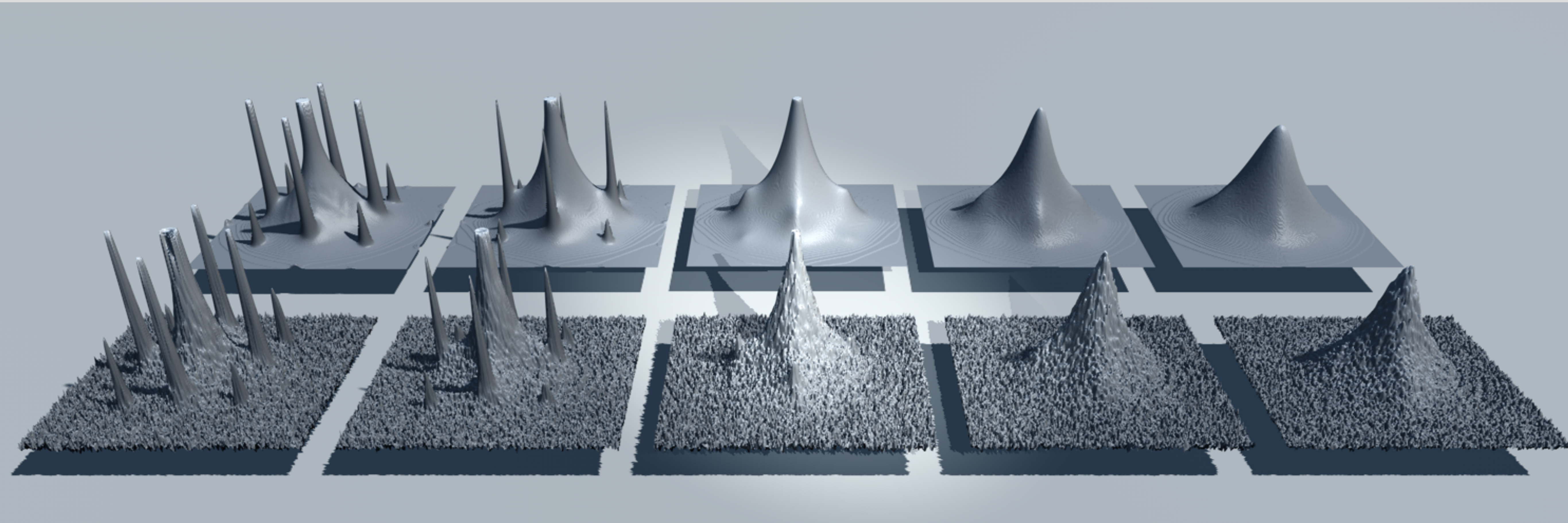
Calibrator

Model all details of the experiment

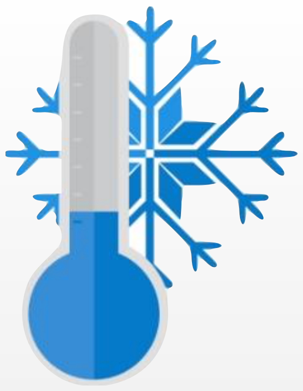
- Accurate microscopic model (including the trap)
- Actual size simulation (~300,000 bosons)
- Calculate what the experiment should see

Time of flight image

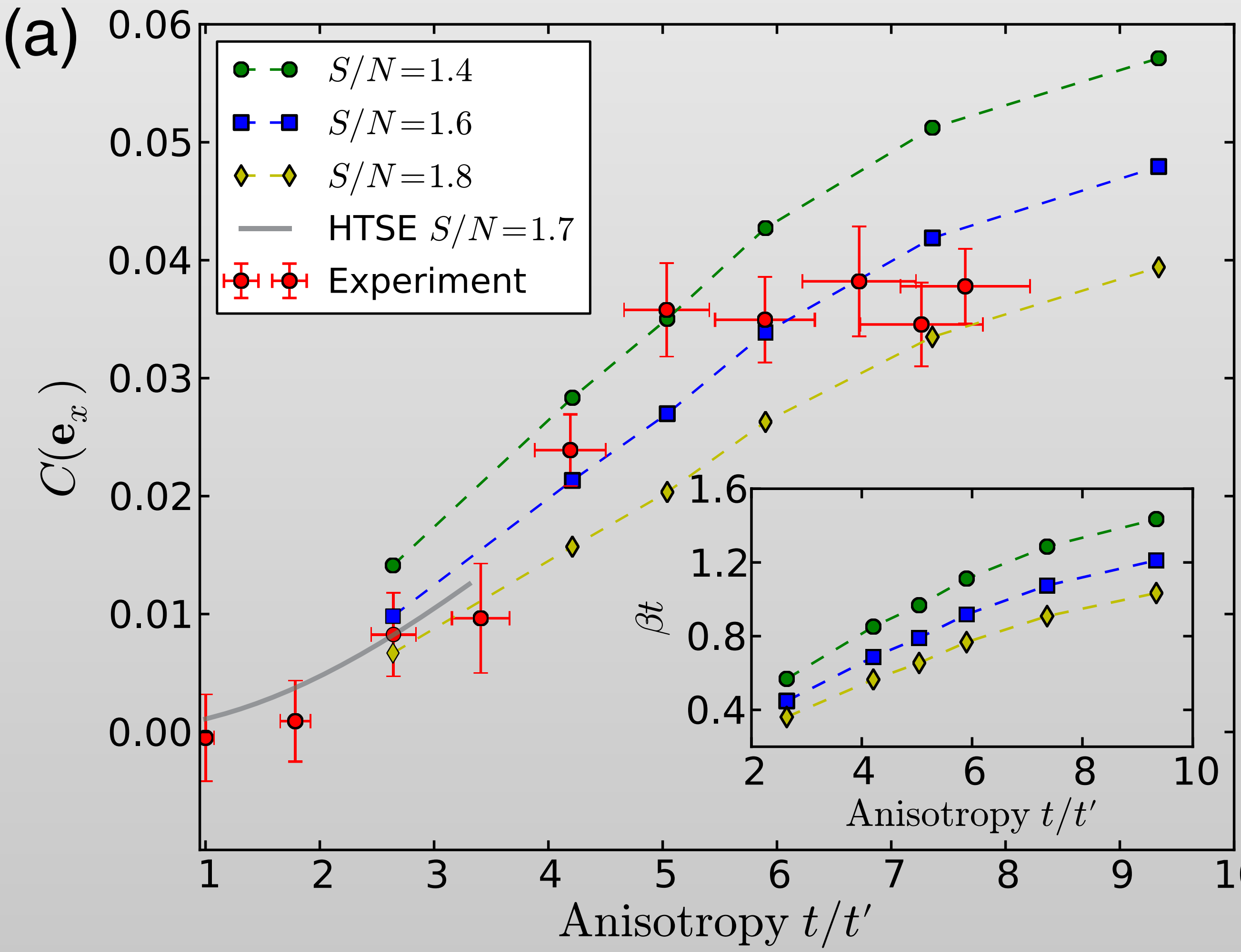
Trotzky, Pollet et al, Nat. Phys, 2010



Thermometer

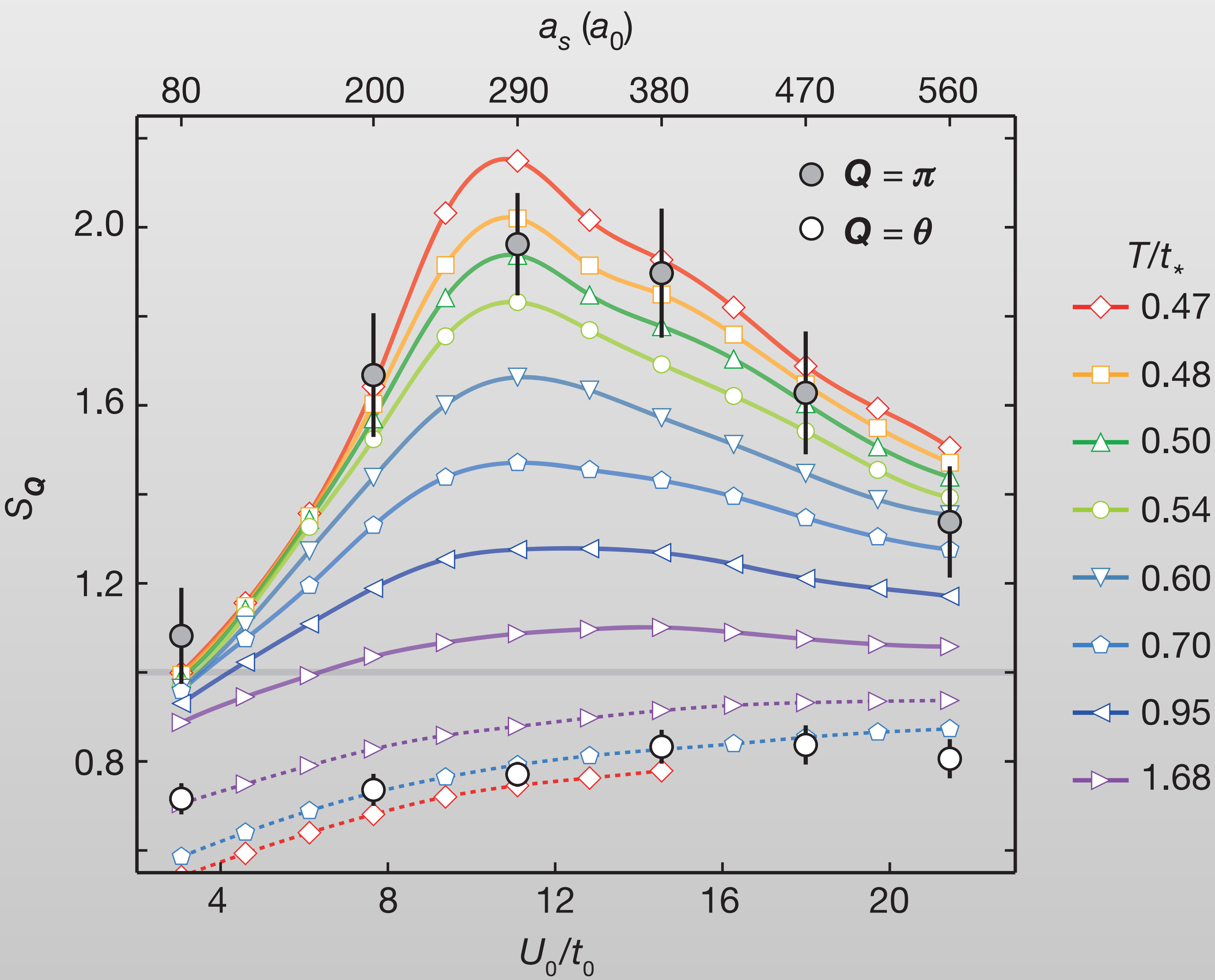


n.n. spin correlation



Imriska et al, PRL 2014

spin structure factor



Hart et al, Nature 2015

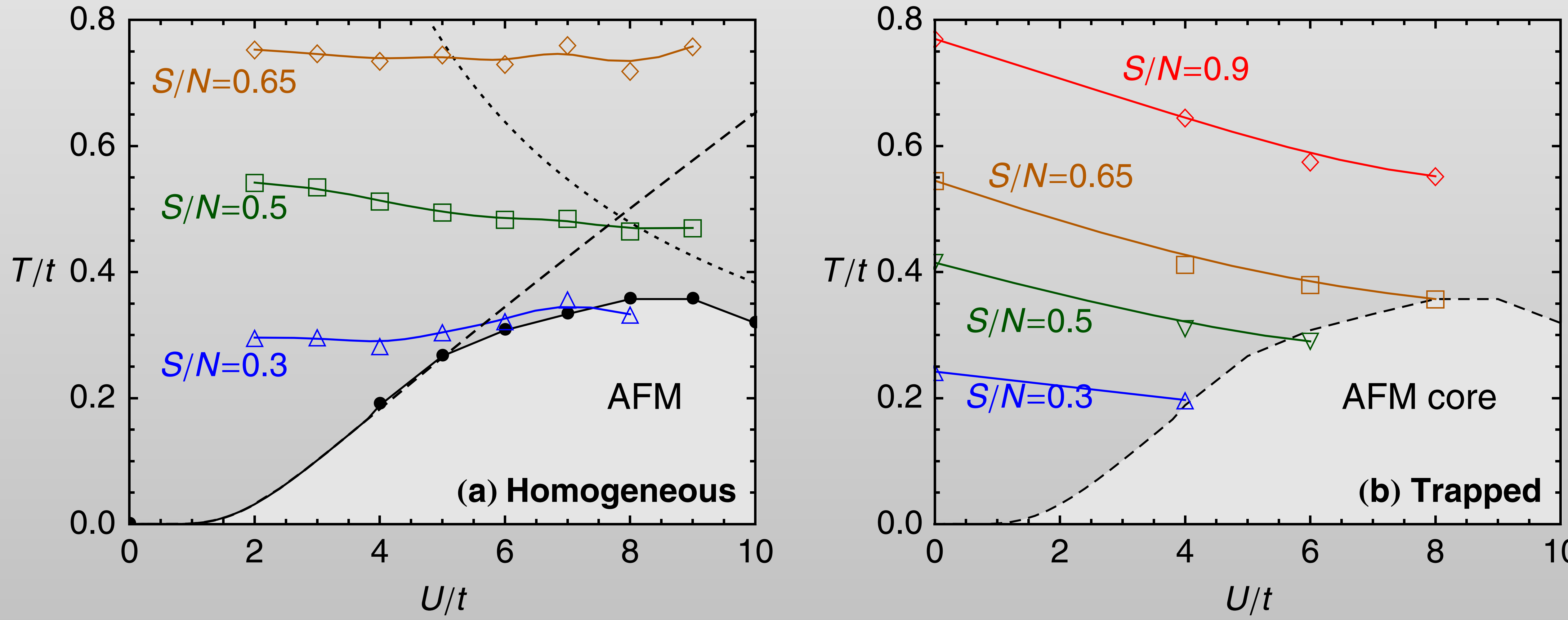
Theoretical guidance

Critical temperatures — Staudt, Kent, Kozik...

Equation of states — Fuchs, LeBlanc, Rigol, Scalettar ...

Isentropic curves — Pollet, Cai, Wang...

Paiva et al, PRL 2011



What about the sign problem ?

$$Z = \sum_{k=0}^{\infty} \int_0^{\beta} d\tau_1 \ldots \int_{\tau_{k-1}}^{\beta} d\tau_k \, \mathrm{Tr}\left[e^{-(\beta-\tau_k)\hat{H}_0} (-\lambda \hat{H}_1) \ldots (-\lambda \hat{H}_1) e^{-\tau_1 \hat{H}_0} \right]$$

$$= \sum_{k=0}^{\infty} \sum_{\mathcal{C}_k} w(\mathcal{C}_k)$$

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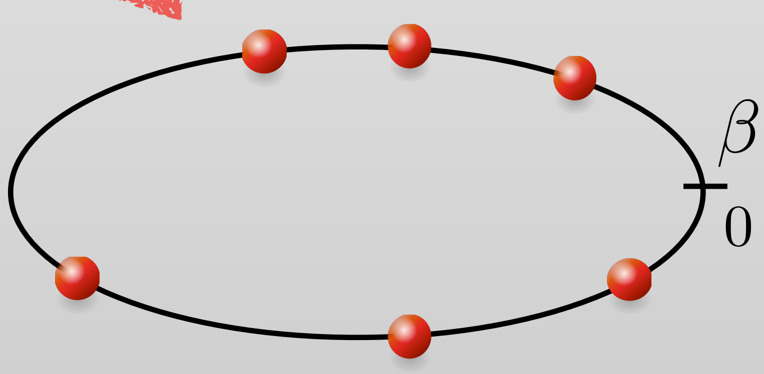

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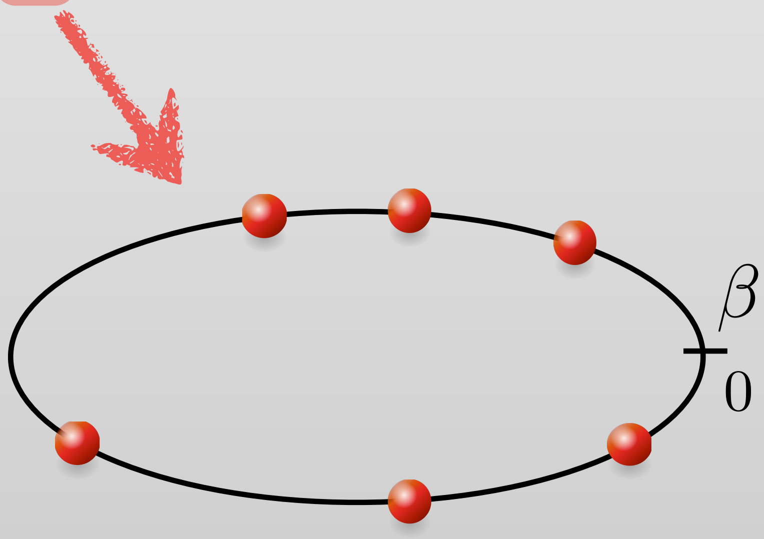

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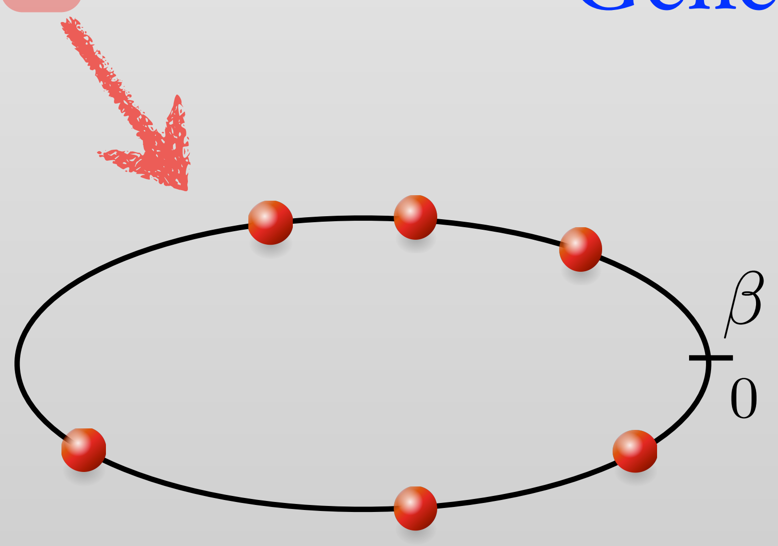
Is it all positive ?

General solution implies P=NP ! Troyer and Wiese, 2005



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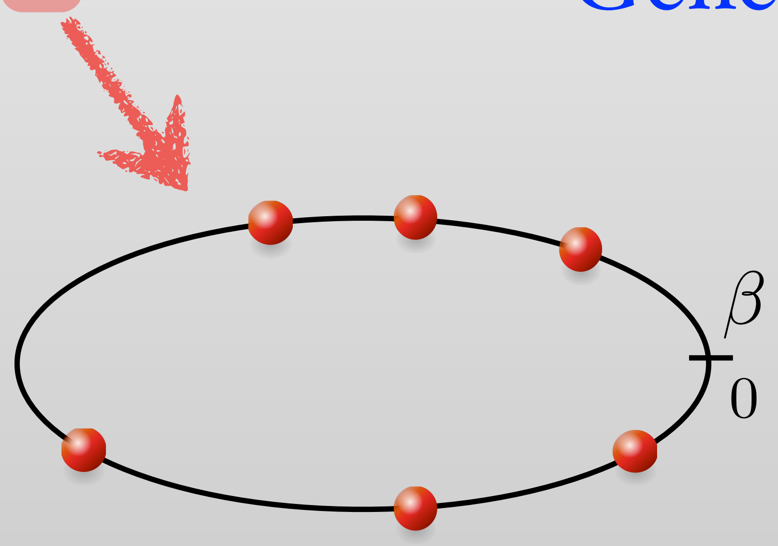

Bosons: no sign problem if there is no frustration

✓ QMC works for any filling, any lattice and any interactions

How about fermions ?

$$Z = \sum_{k=0}^{\infty} \int_0^{\beta} d\tau_1 \ldots \int_{\tau_{k-1}}^{\beta} d\tau_k \, \mathrm{Tr}\left[e^{-(\beta-\tau_k)\hat{H}_0}(-\lambda\hat{H}_1)\ldots(-\lambda\hat{H}_1)e^{-\tau_1\hat{H}_0}\right]$$

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Rubtsov et al, PRB 2005   Gull et al, RMP 2011

$$= \sum_{k=0}^{\infty} \sum_{\mathcal{C}_k} w(\mathcal{C}_k)$$

→

$$\det \begin{pmatrix} \text{Noninteracting} \\ \text{Green's functions} \end{pmatrix}_{k\times k}$$

$\langle k \rangle \sim \beta\lambda N,$ scales as $\mathcal{O}(\beta^3\lambda^3N^3)$

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$$Z = \sum_{k=0}^{\infty} \int_0^{\beta} d\tau_1 \ldots \int_{\tau_{k-1}}^{\beta} d\tau_k \, \mathrm{Tr}\left[e^{-(\beta-\tau_k)\hat{H}_0} (-\lambda \hat{H}_1) \ldots (-\lambda \hat{H}_1) e^{-\tau_1 \hat{H}_0} \right]$$

Rubtsov et al, PRB 2005   Gull et al, RMP 2011

$$= \sum_{k=0}^{\infty} \sum_{\mathcal{C}_k} w(\mathcal{C}_k)$$

$$\det \left(\begin{array}{c} \text{Noninteracting} \\ \text{Green's functions} \end{array} \right)_{k \times k}$$

$\langle k \rangle \sim \beta\lambda N$, scales as $\mathcal{O}(\beta^3\lambda^3 N^3)$

Rombouts, Heyde and Jachowicz, PRL 1999
Iazzi and Troyer, PRB 2015   LW, Iazzi, Corboz and Troyer, PRB 2015

LCT-QMC Methods

$$\det \left(I + \mathcal{T} e^{-\int_0^{\beta} d\tau H_{\mathcal{C}_k}(\tau)} \right)_{N \times N}$$

thus achieving $\mathcal{O}(\beta\lambda N^3)$ scaling!

Fermion sign problem

Spinful fermions: no sign problem thanks to the time-reversal symmetry

$$M_\uparrow = M_\downarrow^*$$

$$w(\mathcal{C}_k) = \det M_\uparrow \times \det M_\downarrow$$

$$= |\det M_\uparrow|^2 \geq 0$$

Lang et al, Phys. Rev. C, 1993
Koonin et al, Phys. Rep, 1997
Hands et al, EPJC, 2000
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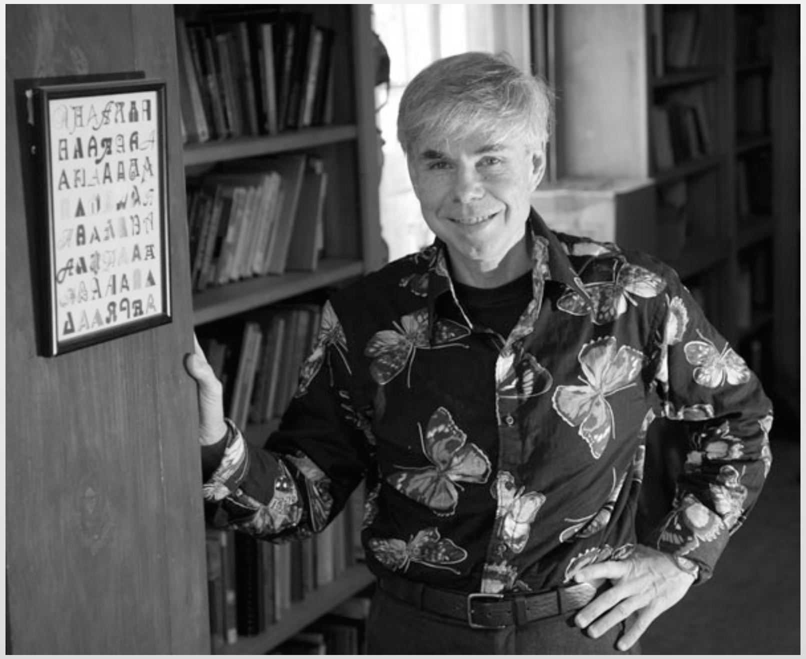
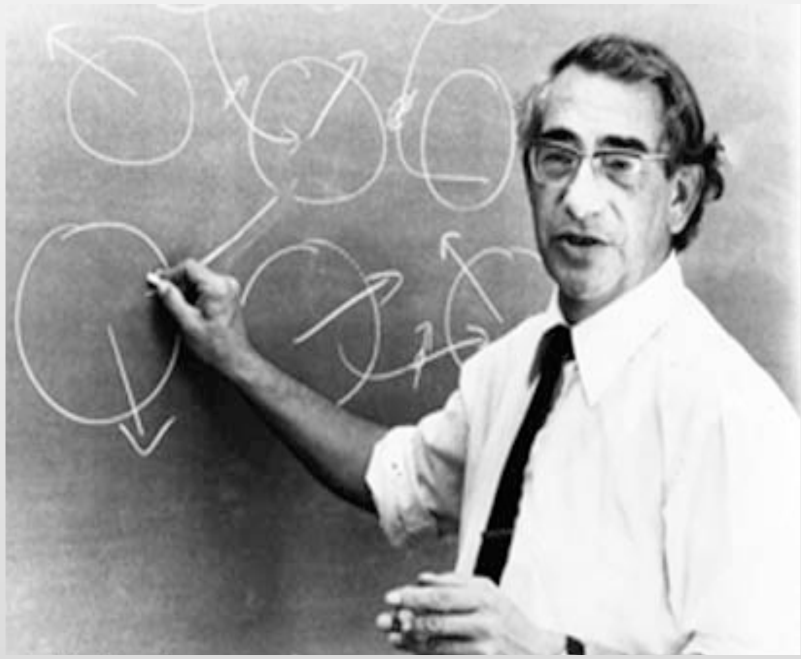
Lang et al, Phys. Rev. C, 1993
Koonin et al, Phys. Rep, 1997
Hands et al, EPJC, 2000
Wu et al, PRB, 2005

- ✓ Attractive interaction at any filling on any lattice
- ✓ Repulsive interaction at half-filling on bipartite lattices
- ✓ Gauge fields are not impossible for fermions !





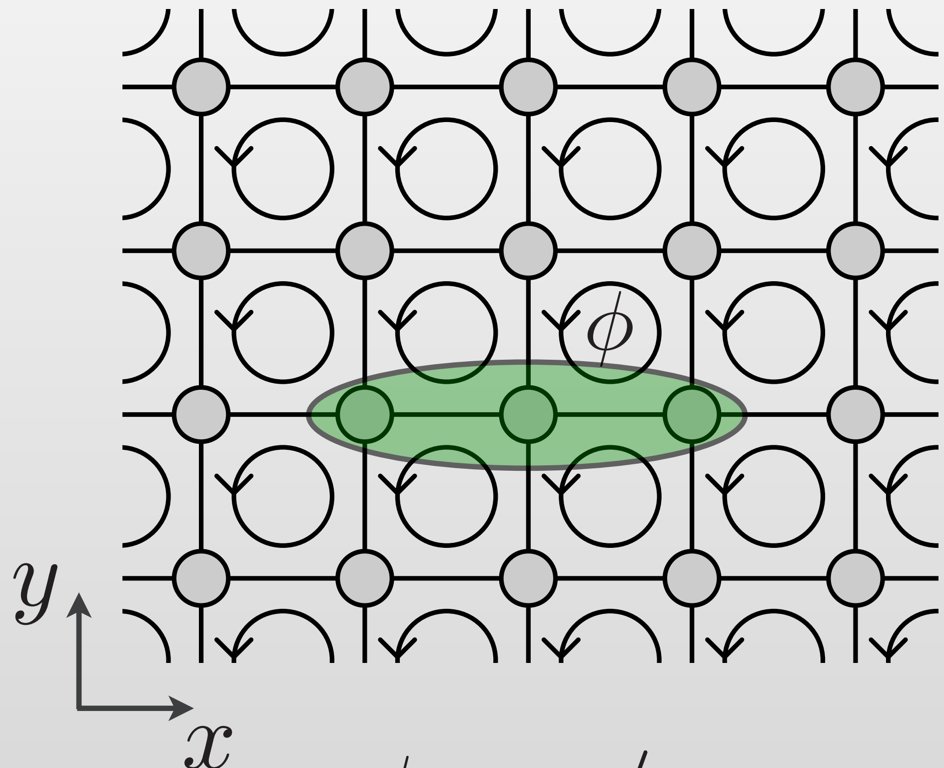
Hofstadter-Hubbard Model

when topology meets interaction

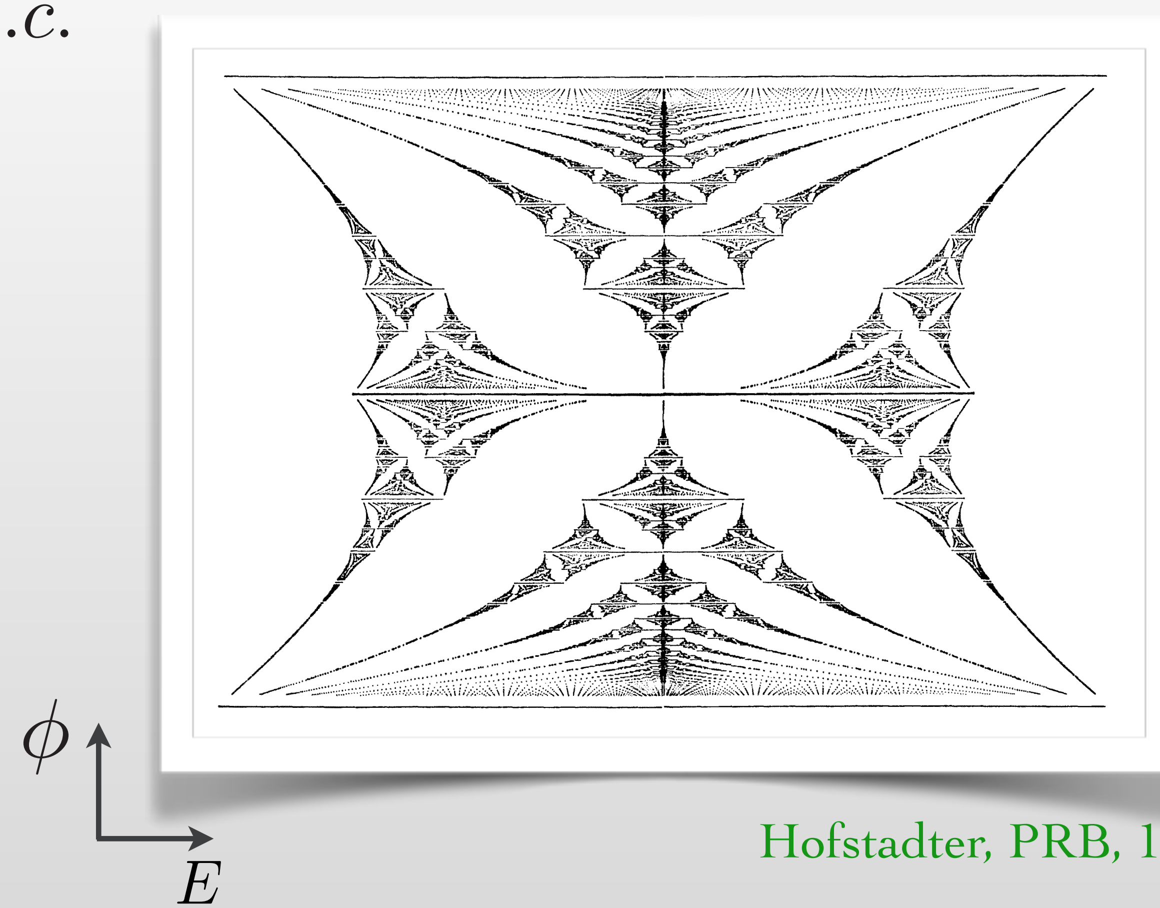
Wang, Hung and Troyer, PRB, 2014

Hofstadter Model

$$\hat{H} = \sum_{x,y} \hat{c}^{\dagger}_{x+1,y}\hat{c}_{x,y} + e^{i2\pi x\phi}\hat{c}^{\dagger}_{x,y+1}\hat{c}_{x,y} + h.c.$$



$$\phi = p/q$$

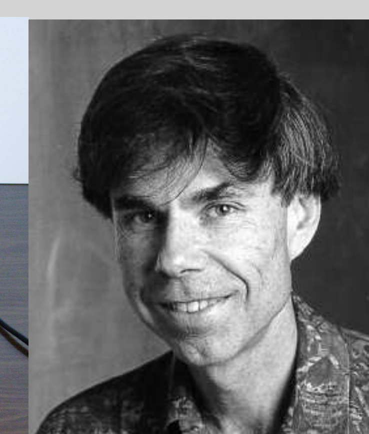


Hofstadter, PRB, 1976

Thouless, Kohmoto, Nightingale and den Nijs, 1982

Chern number of the n-th gap is given by the Diophantine equation

$$n = qs + pC$$



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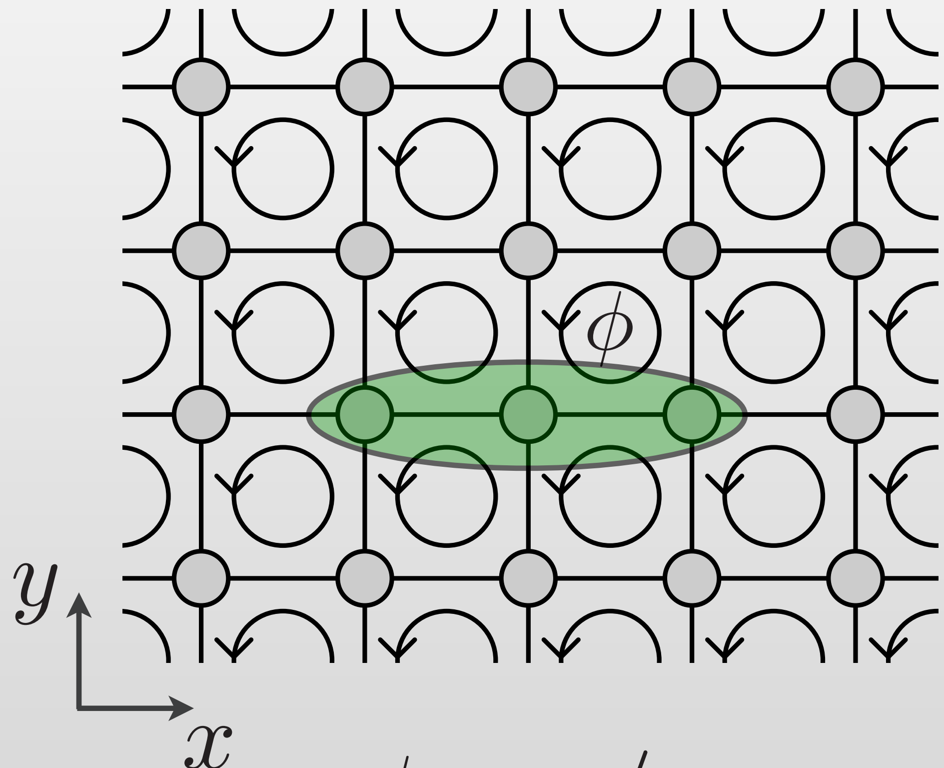

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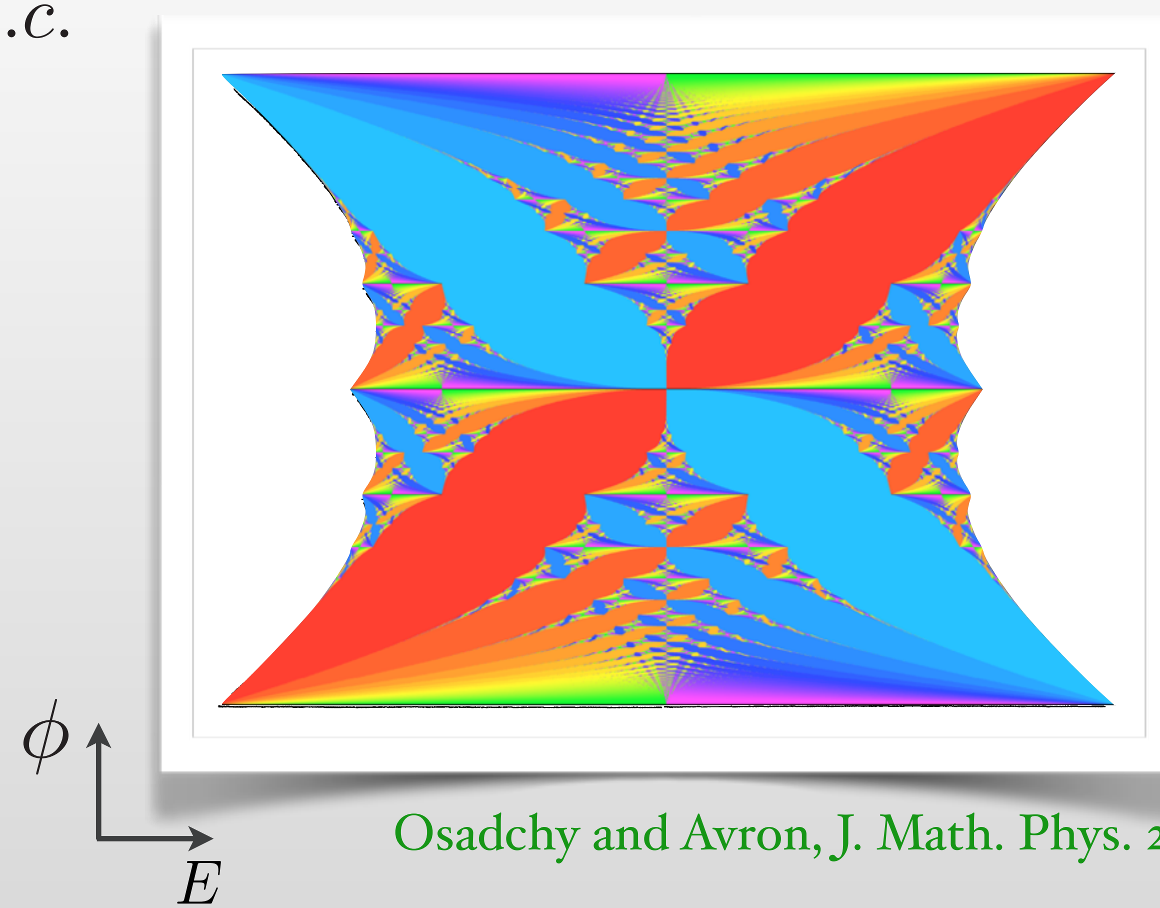

Osadchy and Avron, J. Math. Phys. 2001

Thouless, Kohmoto, Nightingale and den Nijs, 1982

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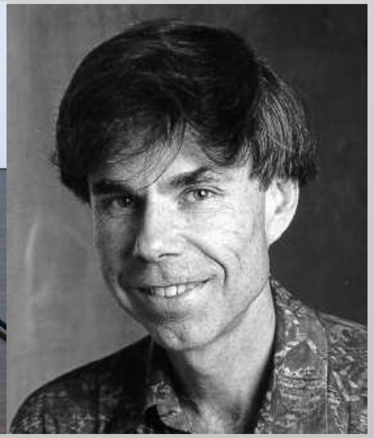
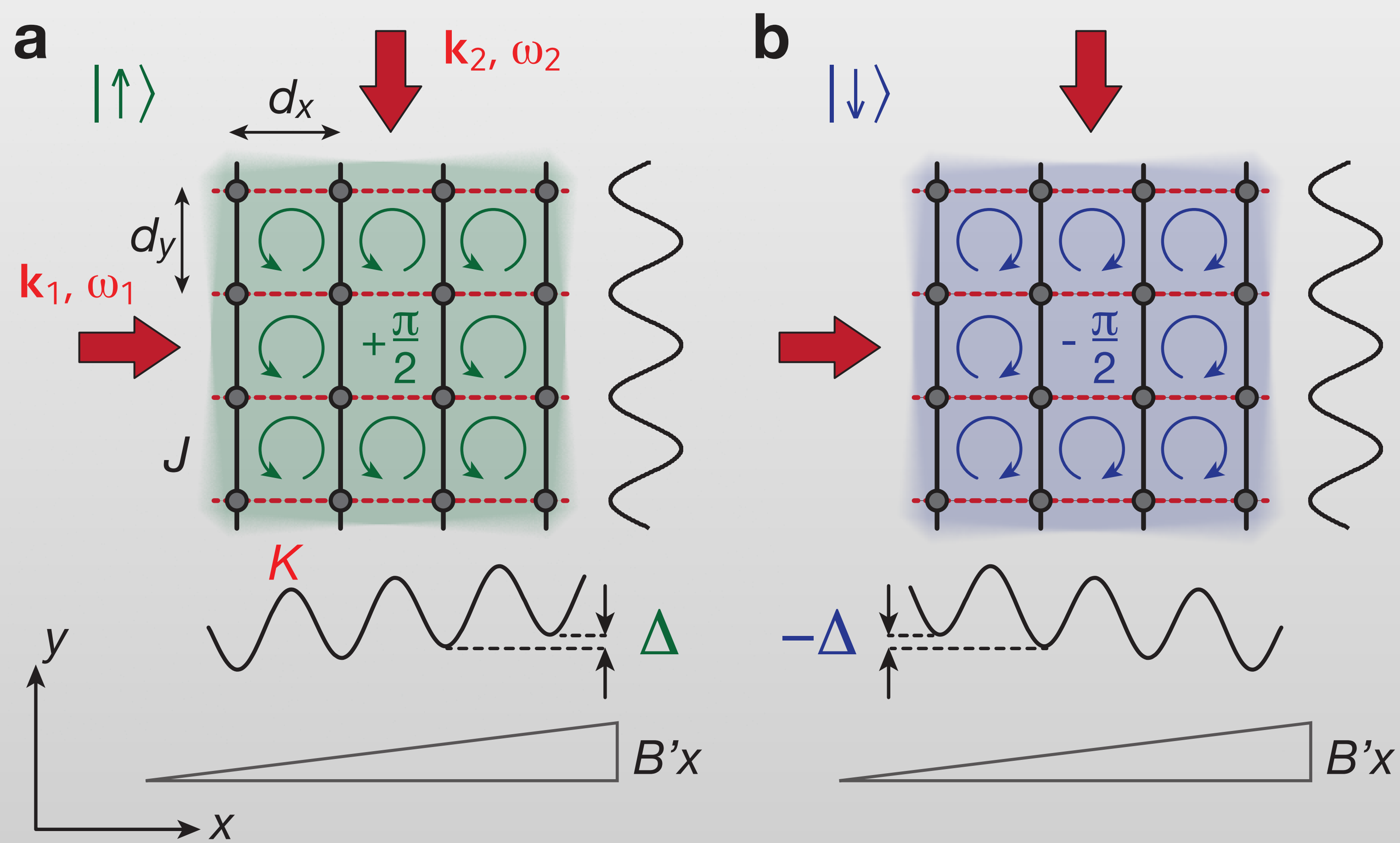


Time-Reversal Invariant Fluxes

Opposite fluxes for the two spin species

Aidelsburger et al, PRL 2013

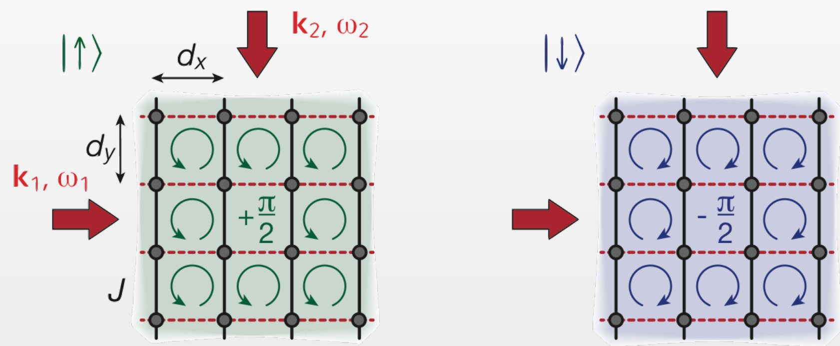


Quantum Spin Hall Insulator

if load fermions into the lowest band

cf Miyake et al, PRL 2013 Kennedy et al, PRL 2013
and experiments in NIST, Hamburg ...

Hofstadter-Hubbard Model

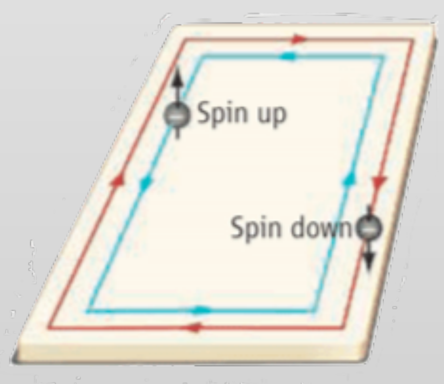


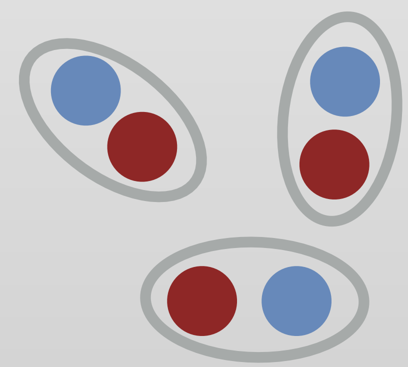
Aidelsburger et al, PRL 2013
Miyake et al, PRL 2013
Kennedy et al, PRL 2013

$$\hat{H} = \hat{H}_{\uparrow}(\phi) + \hat{H}_{\downarrow}(-\phi) - U \sum_i \hat{n}_{i\uparrow}\hat{n}_{i\downarrow}$$

cf. same fluxes
Zhai et al, PRL 2010

cf. repulsive interaction
Cocks et al, PRL 2012



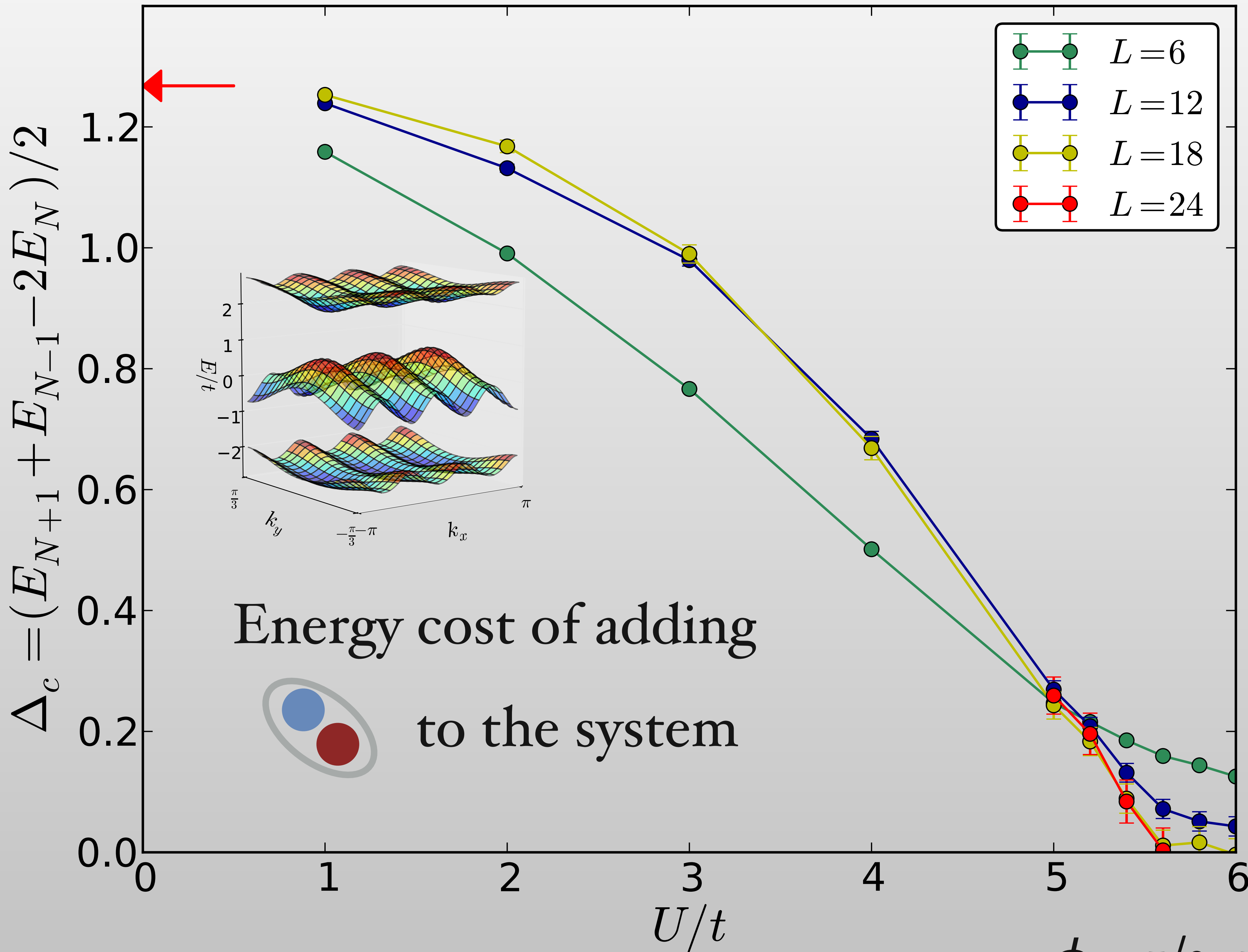


What's the topological signature of the transition ?

LW, Hung and Troyer, PRB 2014

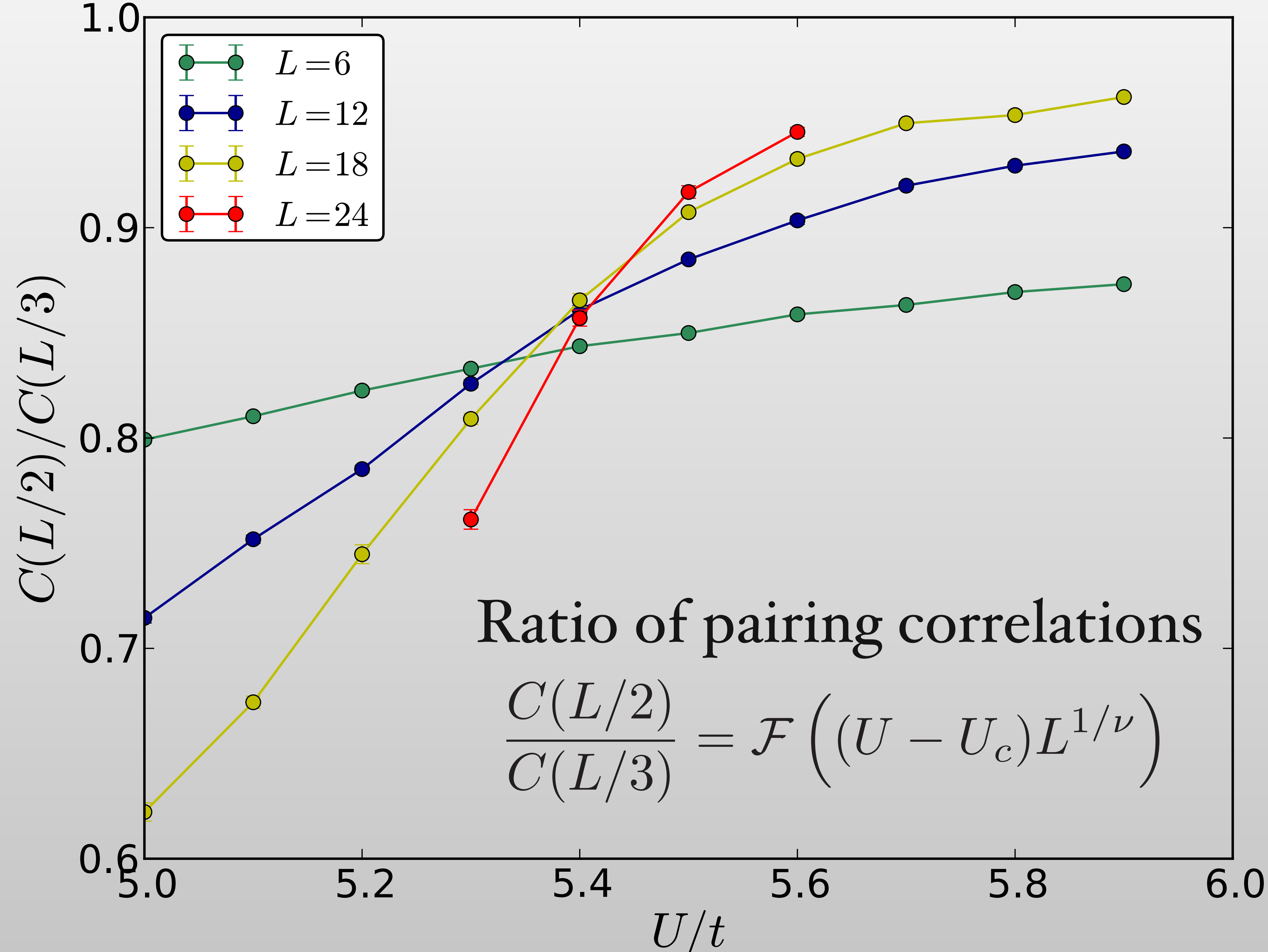
Locate the transition point

LW, Hung and Troyer, PRB 2014



ϕ = 1/3, 1/3 filling

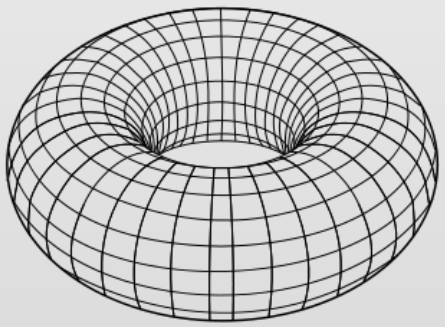
Locate the transition point



What can we say about topology ?

Niu, Thouless, Wu, 1985 Sheng et al, 2006

Twisted boundary conditions

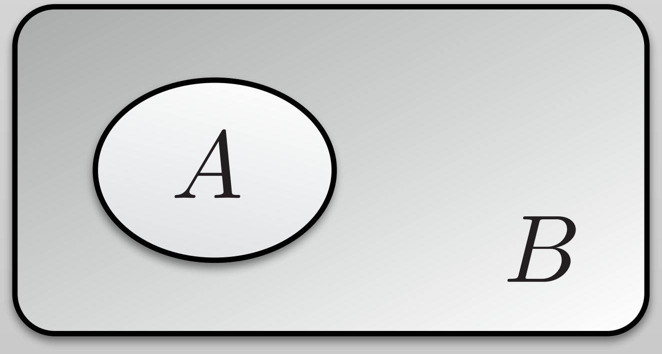


So 1985, Ishikawa et al, 1986, Volovik, 2003
Zhong Wang et al, 2010-2012 Gurarie et al, 2011

Interacting Green's function

$$\nu = \frac{\varepsilon^{\mu\rho\lambda}}{24\pi^2} \int \mathrm{d}\omega\, \mathrm{d}^2 k \times \mathrm{Tr}[G\partial_\mu G^{-1} G\partial_\rho G^{-1} G\partial_\lambda G^{-1}]$$

Hung et al, PRB, 2013 Lang et al, PRB, 2013



Turner et al, 2010 Assaad et al, 2014 Da Wang et al, 2015

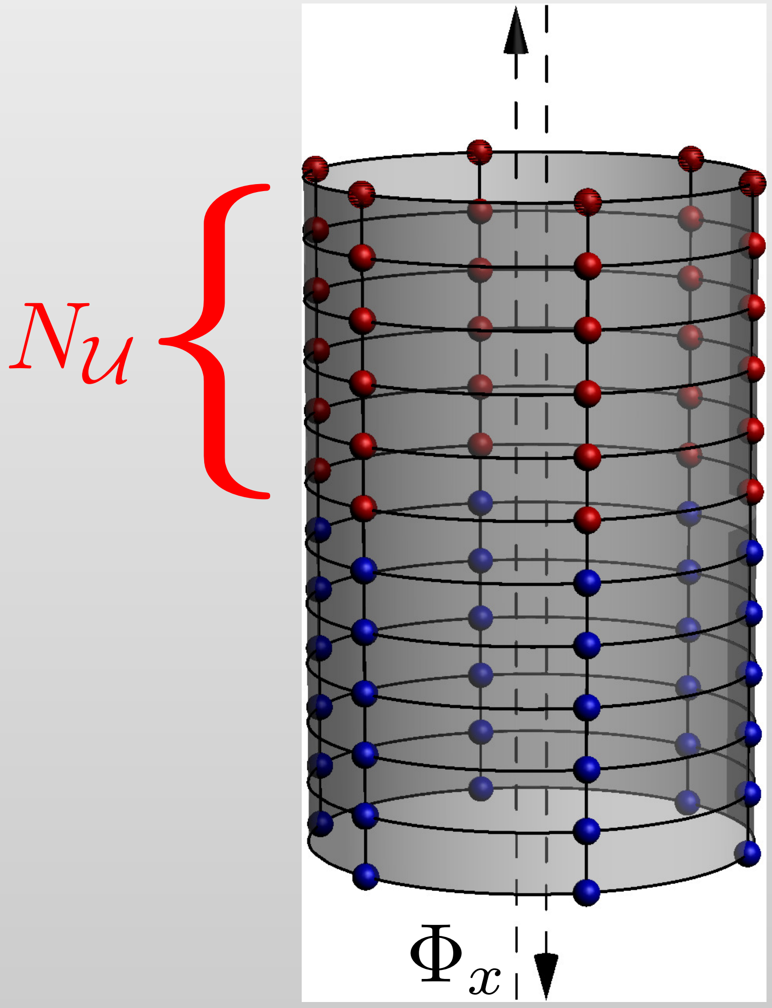
Entanglement entropy & spectra

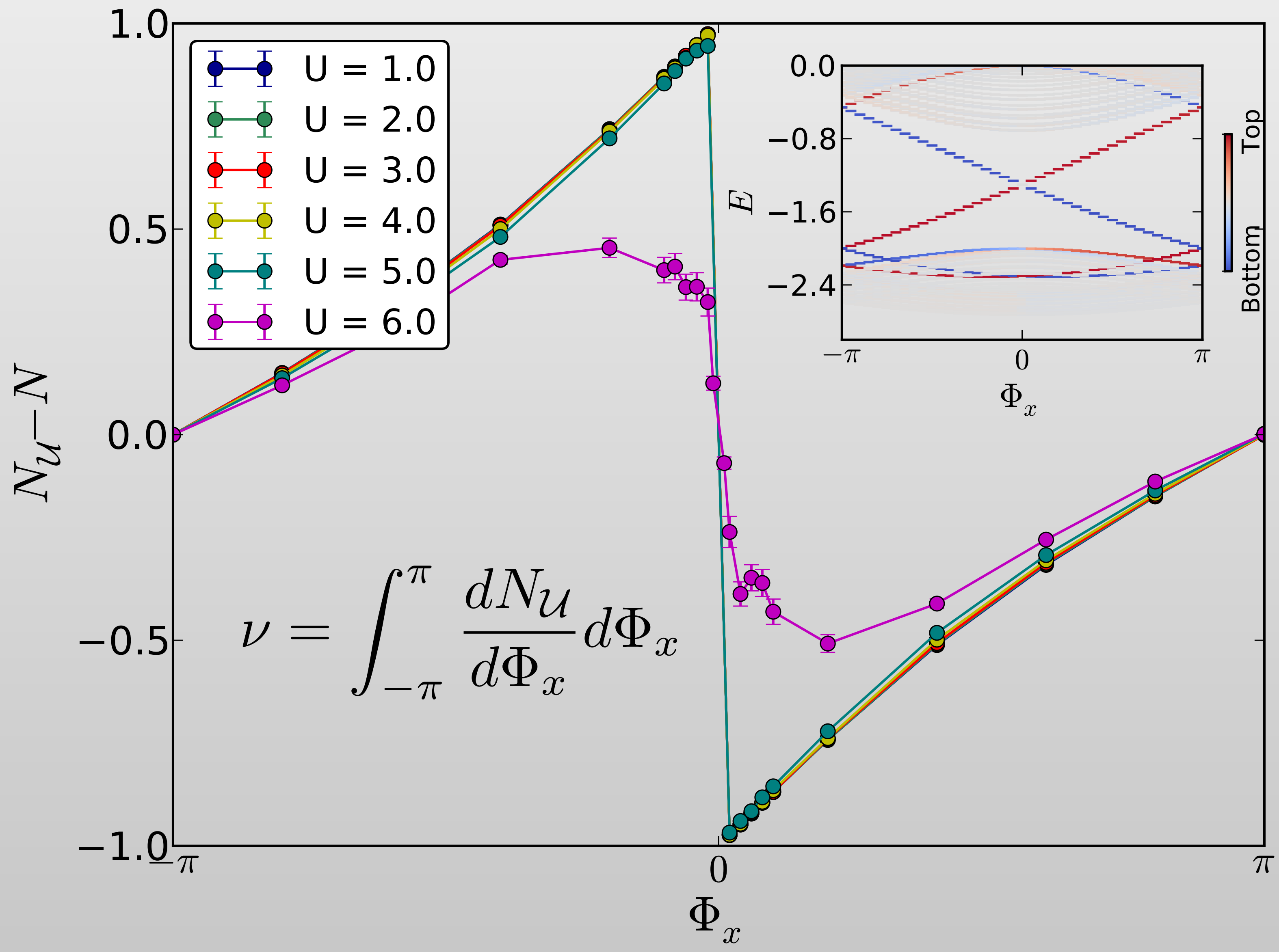
Topological Pumping

Laughlin, PRB 1981 Thouless, PRB 1983

LW, Hung and Troyer, PRB 2014

Flux insertion pumps quantized particle in the QSHI





May even be measured in the experiment !

Mancini et al, Science 2015 Stuhl et al, Science 2015 Cooper and Rey, PRA 2015 Zeng, Wang and Zhai, PRL, 2015

What about the sign problem ?

Sign problem free: Kramers pairs due to the time-reversal symmetry

$$M_{\uparrow} = M_{\downarrow}^{*}$$

$$w(\mathcal{C}_k) = \det M_{\uparrow} \times \det M_{\downarrow}$$
$$= |\det M_{\uparrow}|^2 \geq 0$$

Lang et al, Phys. Rev. C, 1993
Koonin et al, Phys. Rep, 1997
Hands et al, EPJC, 2000
Wu et al, PRB, 2005

But, how about this ?

Spinless fermions $\hat{H} = \sum_{\langle i,j \rangle} -t\left(\hat{c}_i^{\dagger}\hat{c}_j + \hat{c}_j^{\dagger}\hat{c}_i\right) + V\left(\hat{n}_i - \frac{1}{2}\right)\left(\hat{n}_j - \frac{1}{2}\right)$

$$w(\mathcal{C}_k) = \det M$$

Scalapino et al, PRB 1984 Gubernatis et al, PRB 1985
up to 8*8 square lattice and T≥0.3t

Meron cluster approach, Chandrasekharan and Wiese, PRL 1999
solves sign problem for V ≥ 2t

Solutions !

PHYSICAL REVIEW B **89**, 111101(R) (2014)

Solution to sign problems in half-filled spin-polarized electronic systems

Emilie Fulton Huffman and Shailesh Chandrasekharan
Department of Physics, Duke University, Durham, North Carolina 27708, USA
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Solving the fermion sign problem in quantum Monte Carlo simulations by Majorana representation

Zi-Xiang Li,[1] Yi-Fan Jiang,[1,2] and Hong Yao[1,3,*]
[1] *Institute for Advanced Study, Tsinghua University, Beijing 100084, China*
[2] *Department of Physics, Stanford University, Stanford, California 94305, USA*
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Efficient continuous-time quantum Monte Carlo method for the ground state of correlated fermions

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PRL **115,** 250601 (2015) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2015

Split Orthogonal Group: A Guiding Principle for Sign-Problem-Free Fermionic Simulations

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Latest update

Wei, Wu, Li, Zhang, Xiang, PRL 2016

A tale of open science

$$w(\mathcal{C}_k) \sim \det\left(I + \mathcal{T} e^{-\int_0^\beta d\tau H_{\mathcal{C}_k}(\tau)}\right)$$

Free fermions with an effective imaginary-time dependent Hamiltonian

A tale of open science

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Free fermions with an effective imaginary-time dependent Hamiltonian

Let real matrices $A_i = \begin{pmatrix} 0 & B_i \\ B_i^T & 0 \end{pmatrix}$

then $\det\left(I + e^{A_1} e^{A_2} \ldots e^{A_N}\right) \geq 0$

http://mathoverflow.net/questions/204460/how-to-prove-this-determinant-is-positive

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Tao and Paul Erdős in 1985

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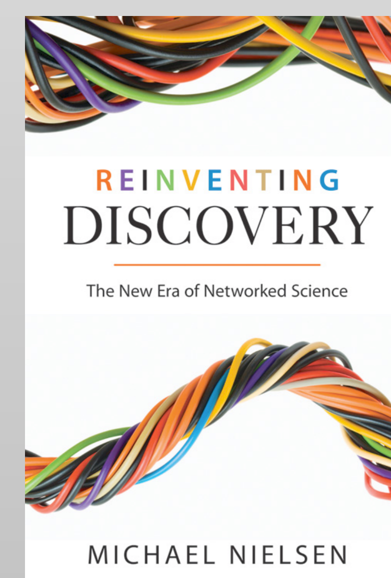

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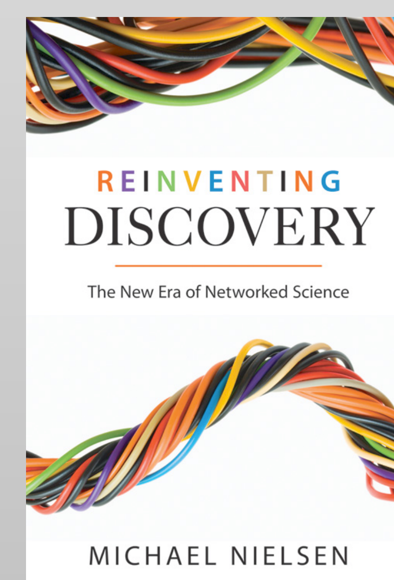

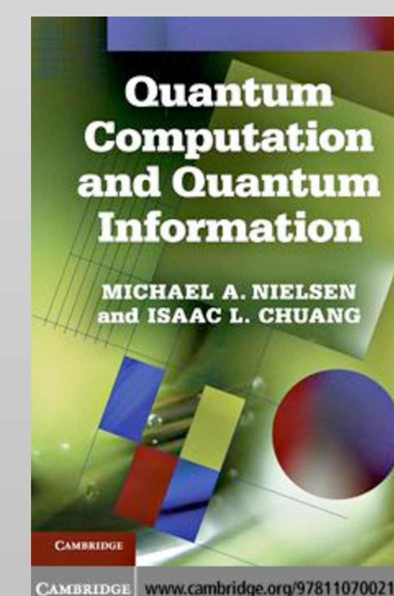

A new "de-sign" principle

LW, Liu, Iazzi, Troyer and Harcos, PRL 2015

If $M^T \eta M = \eta$ where $\eta = \mathrm{diag}(I, -I)$

A new “de-sign” principle

LW, Liu, Iazzi, Troyer and Harcos, PRL 2015

If $M^T \eta M = \eta$ where $\eta = \mathrm{diag}(I, -I)$

Then $M \in O(n,n)$

split orthogonal group

$O^{+-}(n,n)$

$O^{++}(n,n)$

$O^{--}(n,n)$

$O^{-+}(n,n)$

A new “de-sign” principle

LW, Liu, Iazzi, Troyer and Harcos, PRL 2015

If $$M^T \eta M = \eta$$ where $$\eta = \mathrm{diag}(I, -I)$$

Then $$\det\left(I + M\right)$$

has a definite sign

for each component !

$O^{+-}(n,n)$

$\equiv 0$

$O^{++}(n,n)$

≥ 0

$O^{--}(n,n)$

≤ 0

$O^{-+}(n,n)$

$\equiv 0$

A new "de-sign" principle

LW, Liu, Iazzi, Troyer and Harcos, PRL 2015

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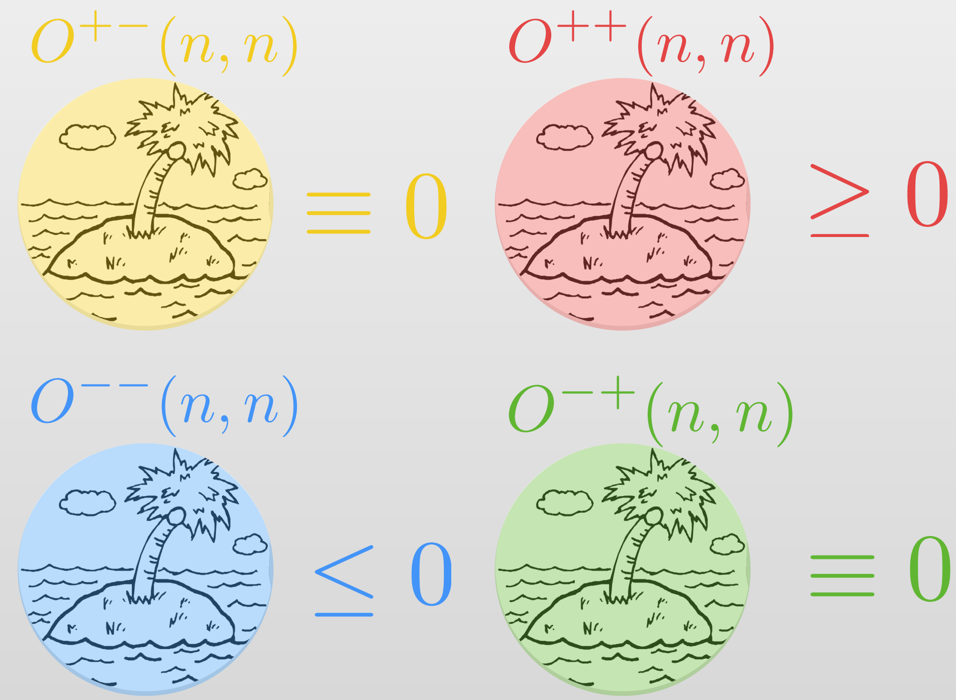

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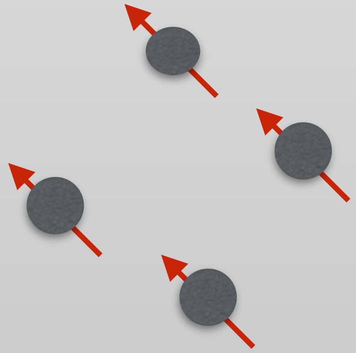


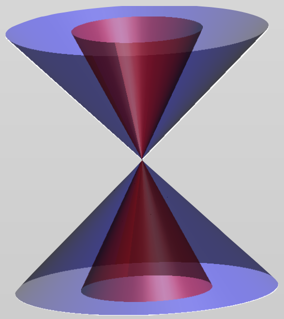
spinless fermions

LW, Troyer, PRL 2014

LW, Corboz, Troyer, NJP 2014

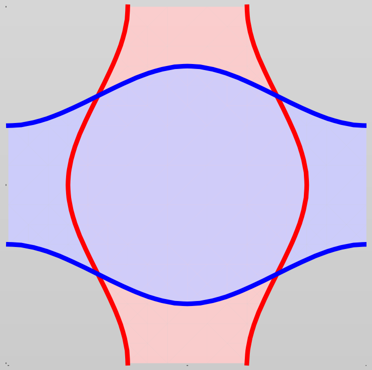
LW, Iazzi, Corboz, Troyer, PRB, 2015

LW, Liu and Troyer, arXiv 2016

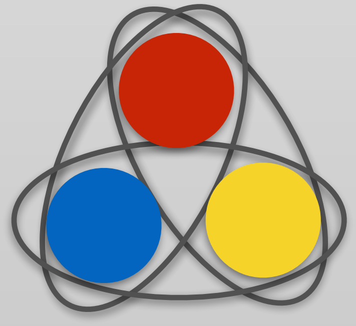


mass imbalance

Liu and LW, PRB 2015



spin nematicity



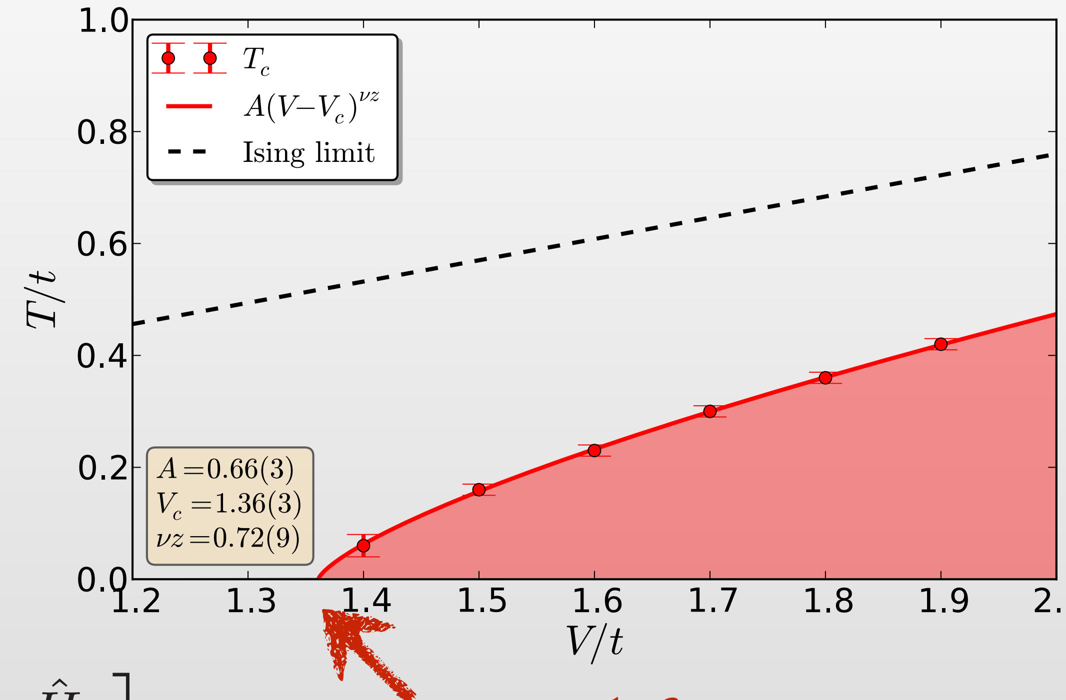
SU(3)

Solutions to the spinless t-V model

$$\hat{H}_0 = -t \sum_{\langle i,j \rangle} \left(\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i \right)$$

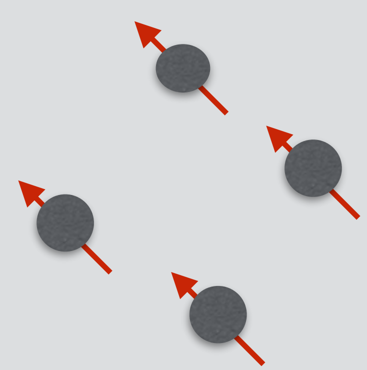
$$\hat{H}_1 = V \sum_{\langle i,j \rangle} \left(\hat{n}_i - \frac{1}{2} \right) \left(\hat{n}_j - \frac{1}{2} \right)$$

$$w(\mathcal{C}_k) \sim \mathrm{Tr}\left[(-1)^k e^{-(\beta-\tau_k)\hat{H}_0} \hat{H}_1 \ldots \hat{H}_1 e^{-\tau_1 \hat{H}_0} \right]$$



Novel fermionic quantum critical point

see also Li, Jiang, Yao 2015 Hesselmann and Wessel 2016



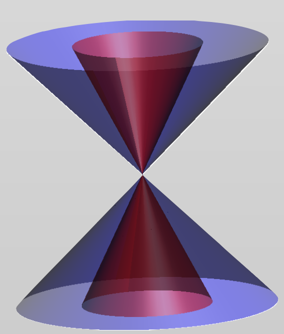
spinless fermions

LW, Troyer, PRL 2014

LW, Corboz, Troyer, NJP 2014

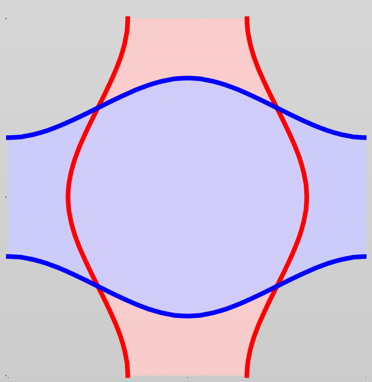
LW, Iazzi, Corboz, Troyer, PRB, 2015

LW, Liu and Troyer, PRB 2016

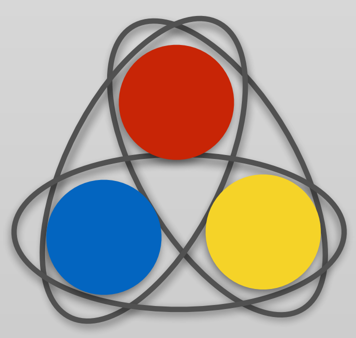


mass imbalance

Liu and LW, PRB 2015



spin nematicity



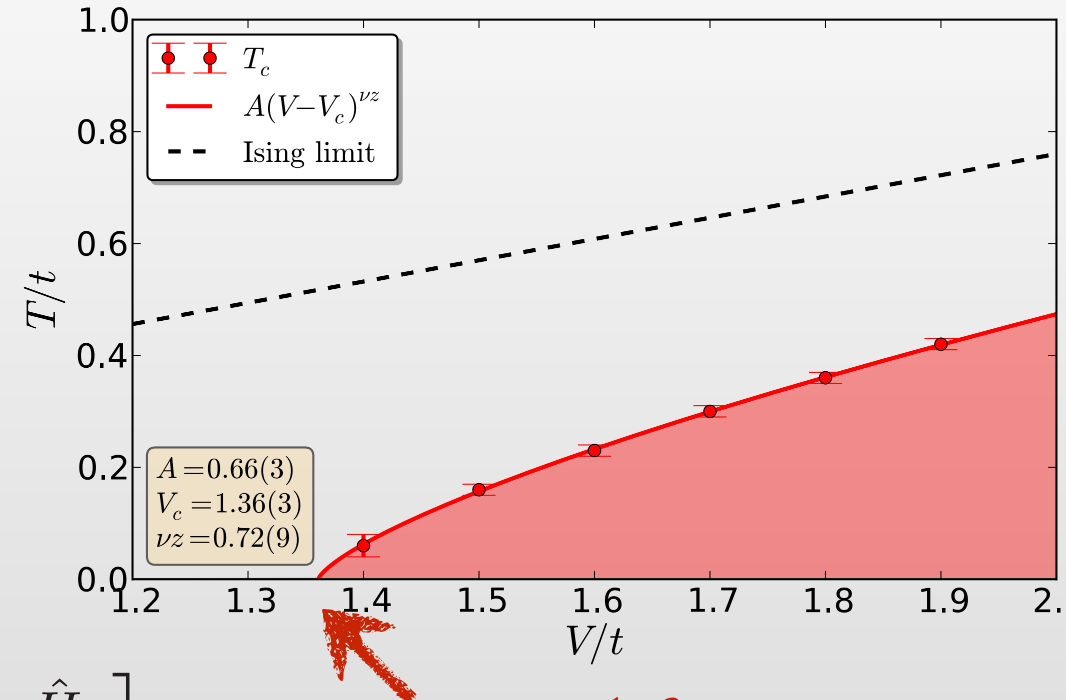
SU(3)

Solutions to the spinless t-V model

$$\hat{H}_0 = -t \sum_{\langle i,j \rangle} \left(\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i \right)$$

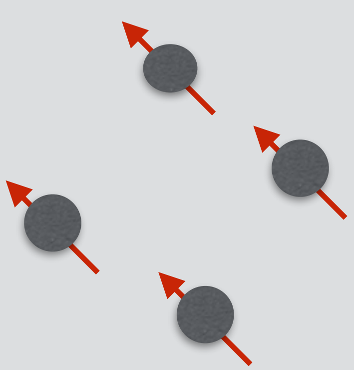
$$\hat{H}_1 = \frac{V}{4} \sum_{\langle i,j \rangle} e^{i\pi(\hat{n}_i + \hat{n}_j)}$$

$$w(\mathcal{C}_k) \sim \mathrm{Tr}\left[(-1)^k e^{-(\beta-\tau_k)\hat{H}_0} \hat{H}_1 \ldots \hat{H}_1 e^{-\tau_1 \hat{H}_0} \right]$$



Novel fermionic quantum critical point

see also Li, Jiang, Yao 2015 Hesselmann and Wessel 2016



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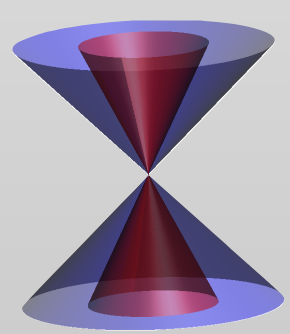

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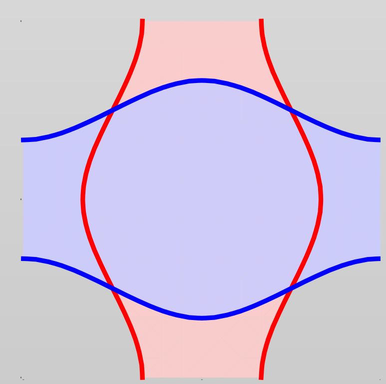

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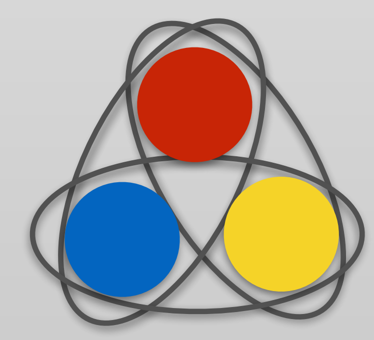

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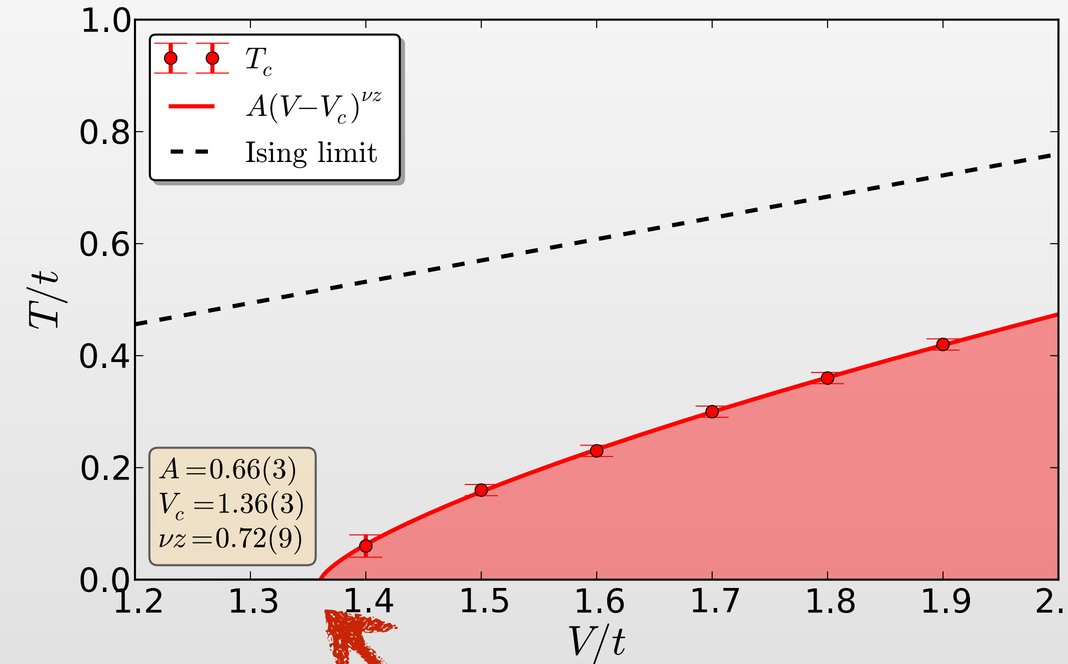

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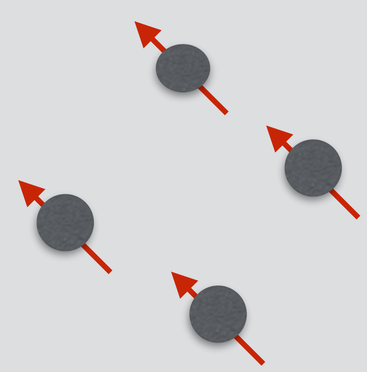

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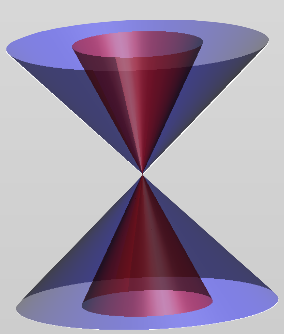

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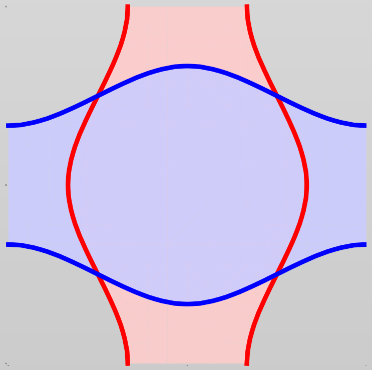

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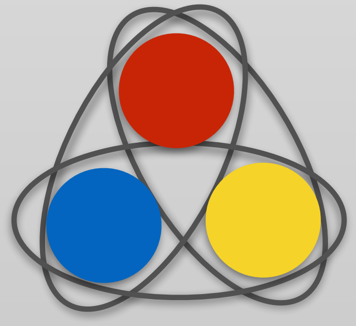

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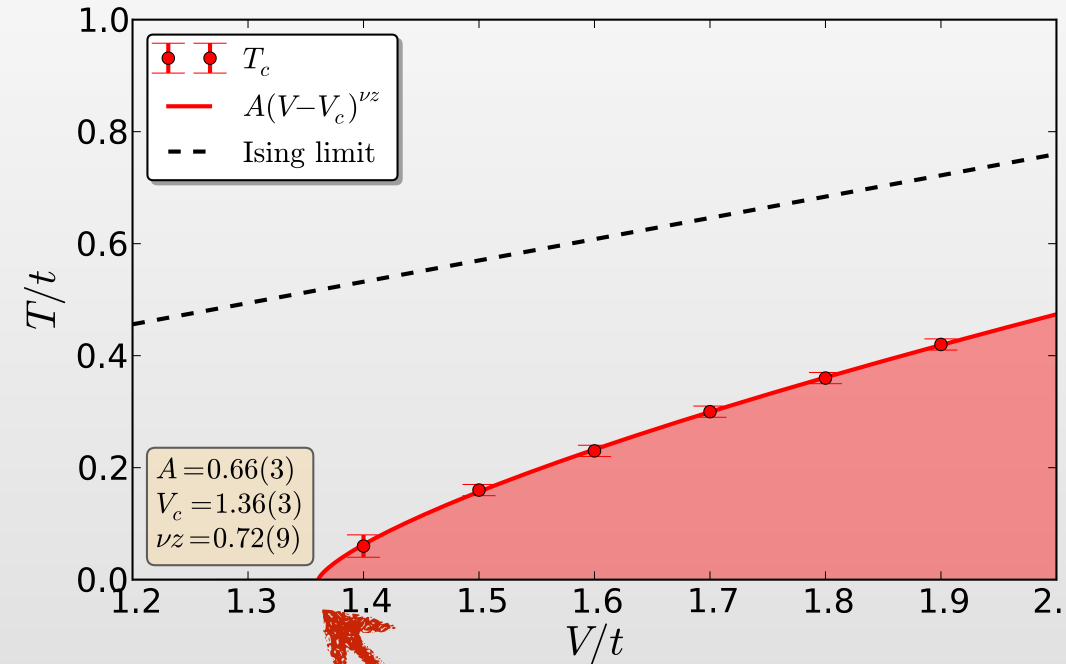

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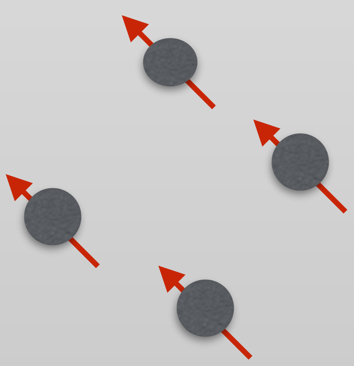

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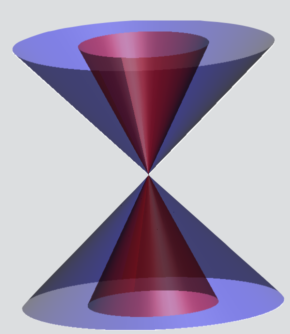

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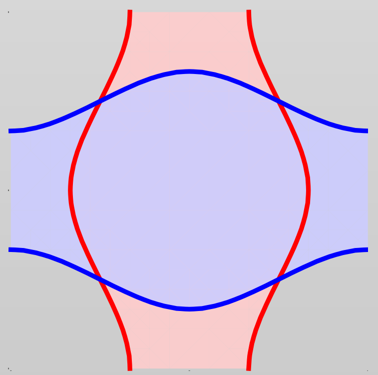

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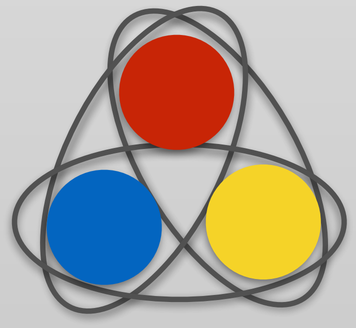
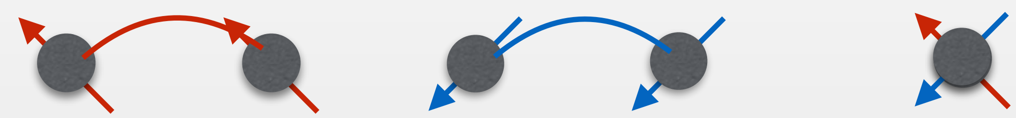


SU(3)

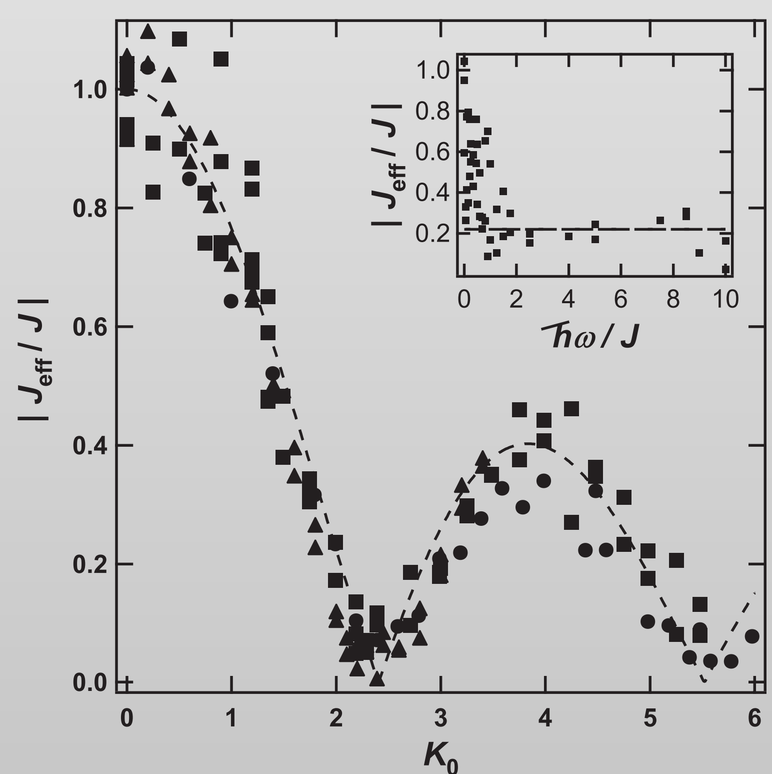
Asymmetric Hubbard model

$t_{\uparrow} \neq t_{\downarrow}$ $\quad U$

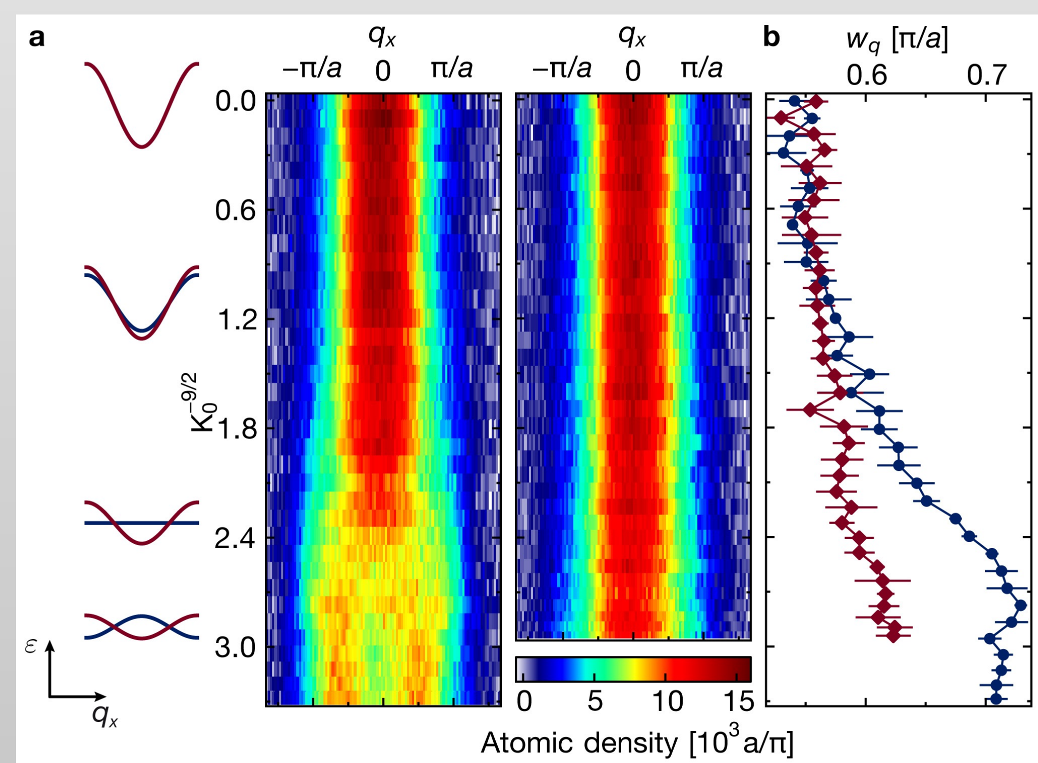


- Realization: mixture of ultracold fermions (e.g. ^{6}Li and ^{40}K)
- Now, continuously tunable by spin-dependent modulations

$$t_{\downarrow}/t_{\uparrow} \in (-\infty, \infty)$$

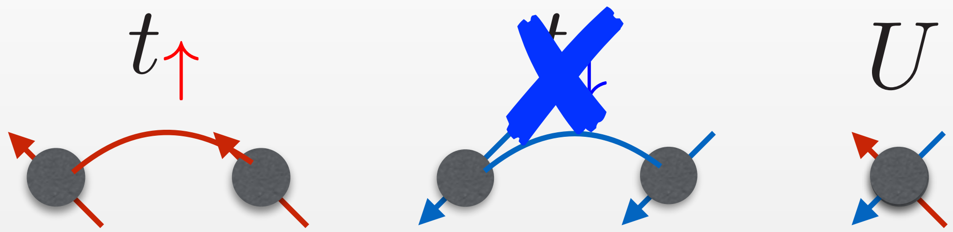


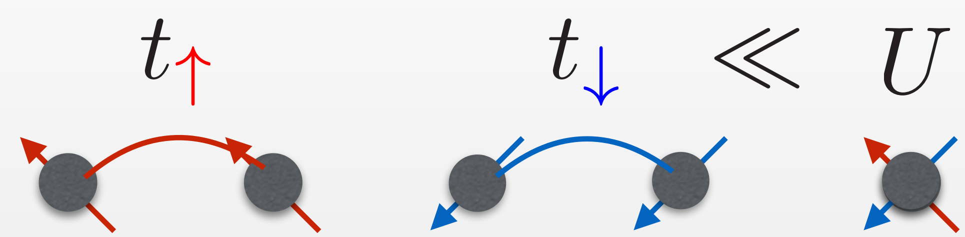
Lignier et al, PRL 2007 and many others



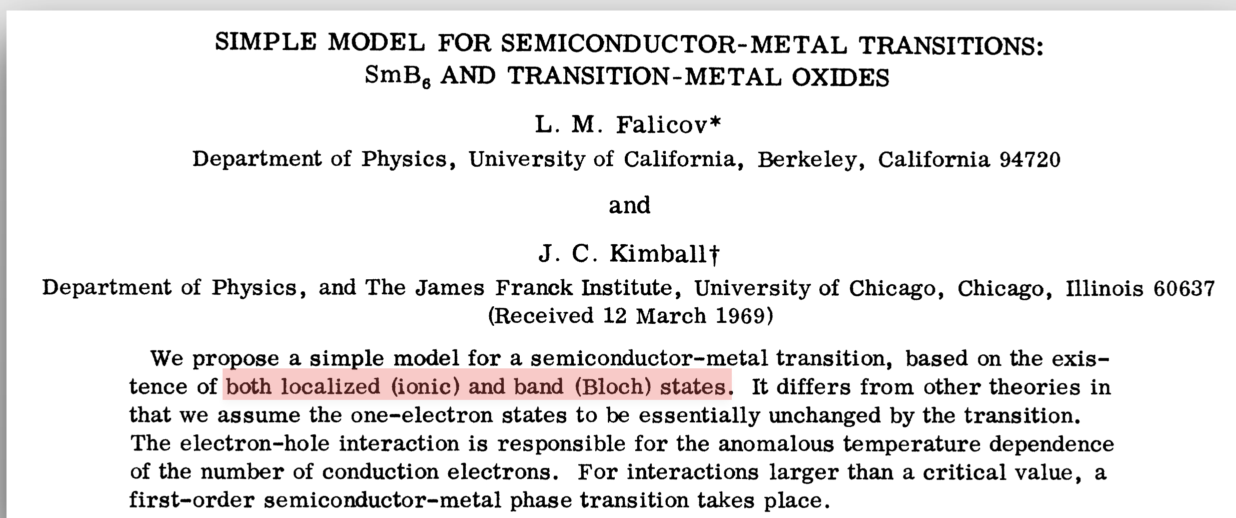
Jotzu et al, PRL 2015

Two limiting cases





Falicov-Kamball Limit



SIMPLE MODEL FOR SEMICONDUCTOR-METAL TRANSITIONS: SmB_6 AND TRANSITION-METAL OXIDES

L. M. Falicov*
Department of Physics, University of California, Berkeley, California 94720

and

J. C. Kimball†
Department of Physics, and The James Franck Institute, University of Chicago, Chicago, Illinois 60637

(Received 12 March 1969)

We propose a simple model for a semiconductor-metal transition, based on the existence of both localized (ionic) and band (Bloch) states. It differs from other theories in that we assume the one-electron states to be essentially unchanged by the transition. The electron-hole interaction is responsible for the anomalous temperature dependence of the number of conduction electrons. For interactions larger than a critical value, a first-order semiconductor-metal phase transition takes place.

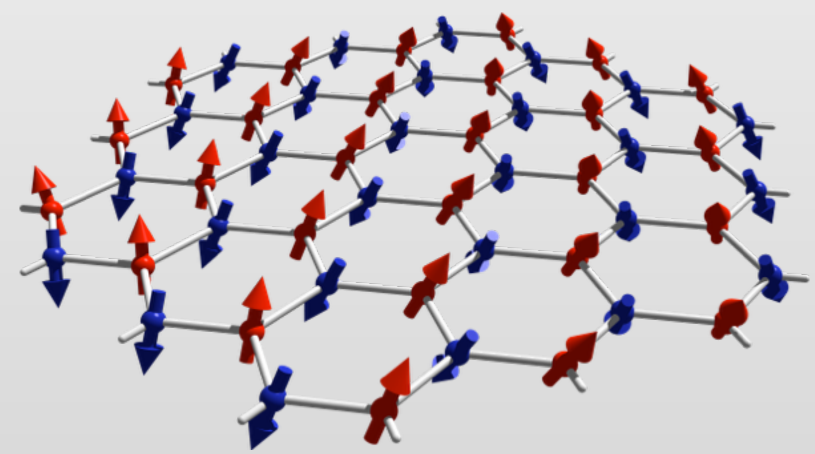
Long-range spin order on bipartite lattices with infinitesimal repulsion

Kennedy and Lieb 1986

"Fruit fly" of DMFT

Freericks and Zlatić, RMP, 2003

Strong Coupling Limit

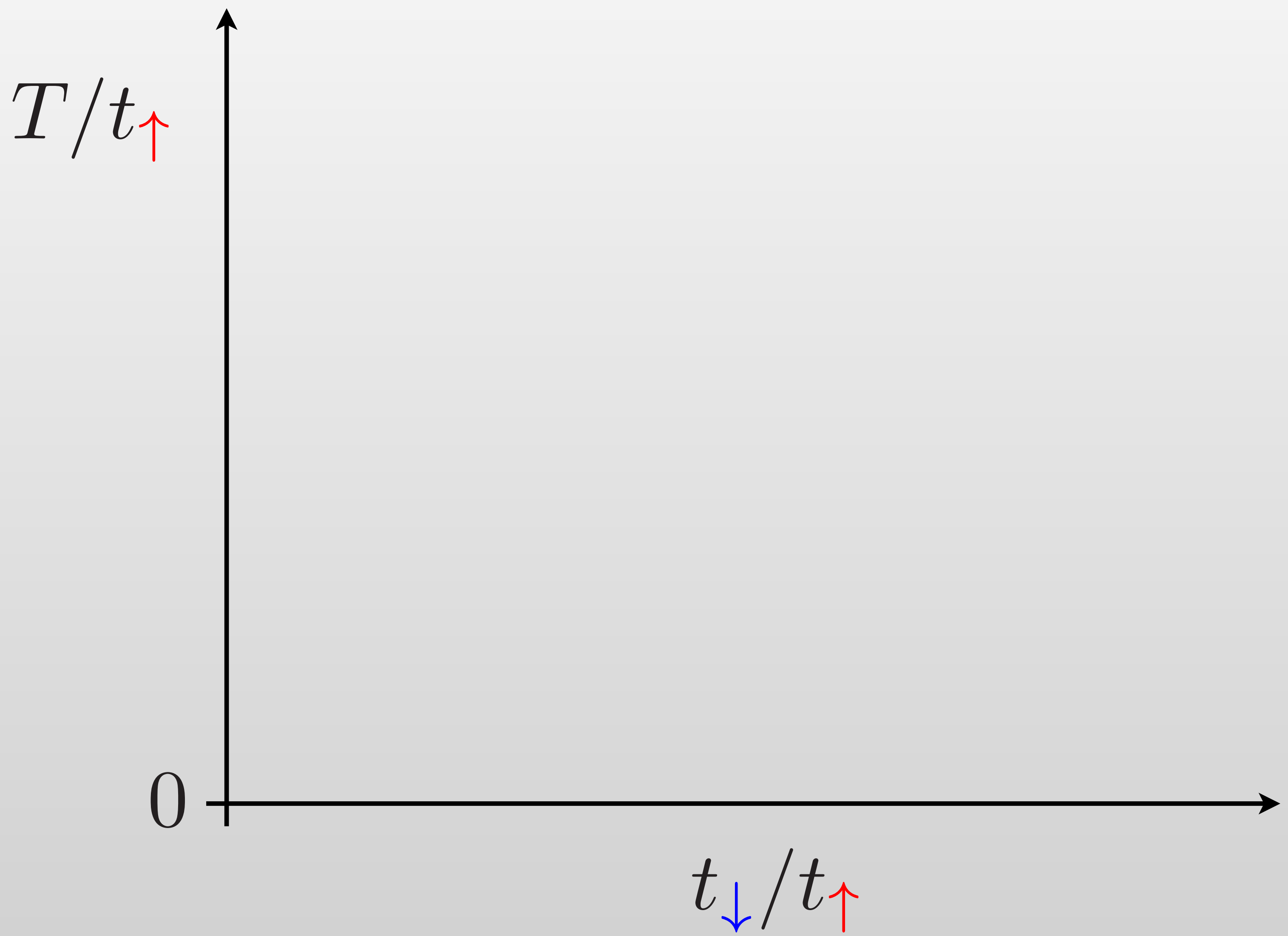


$$J_{xy}\left(\hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y\right) + J_z \hat{S}_i^z \hat{S}_j^z$$

$$\frac{4t_\uparrow t_\downarrow}{U} \leq \frac{2(t_\uparrow^2 + t_\downarrow^2)}{U}$$

XXZ model with Ising anisotropy

Physics on a square lattice



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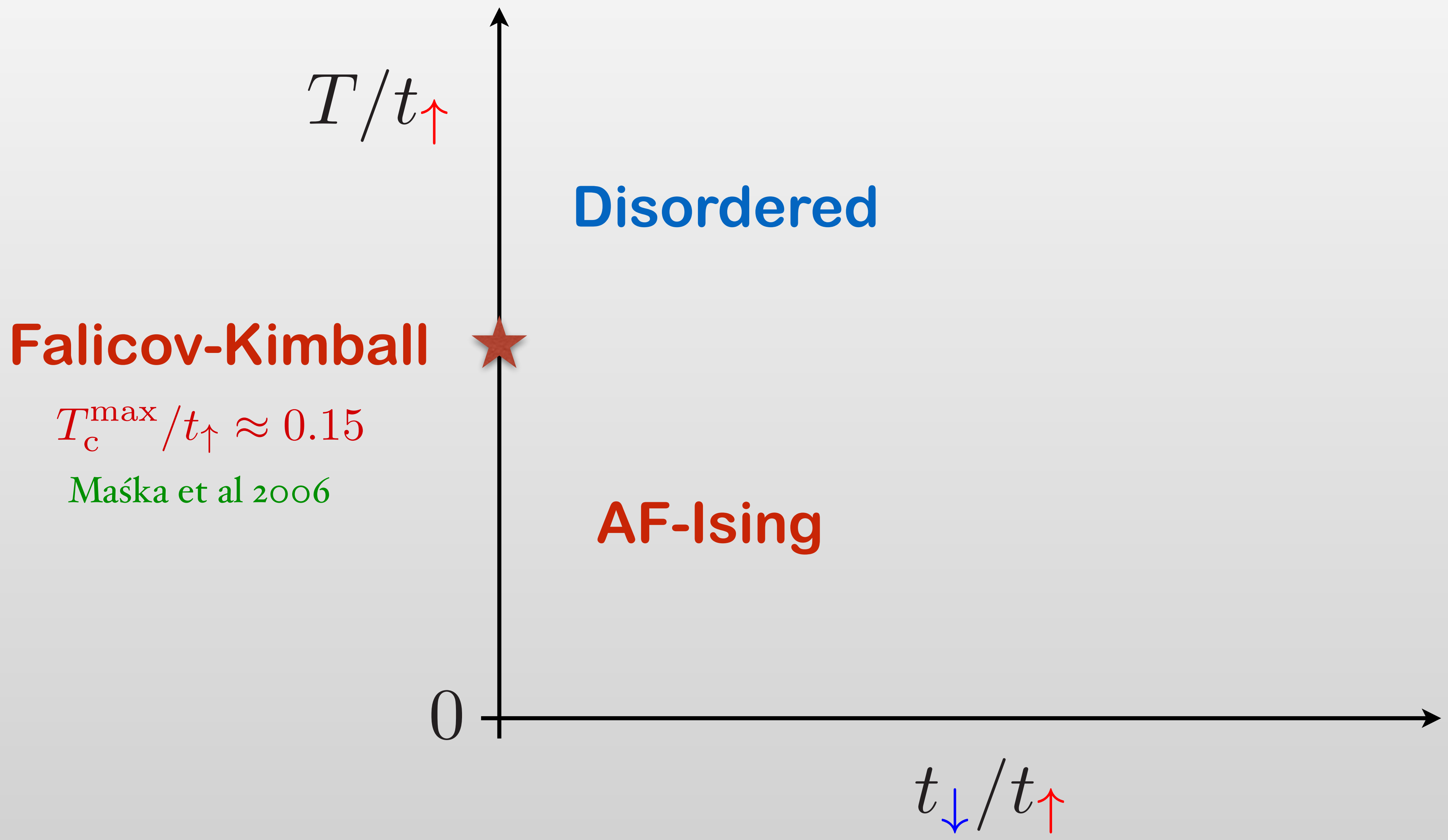

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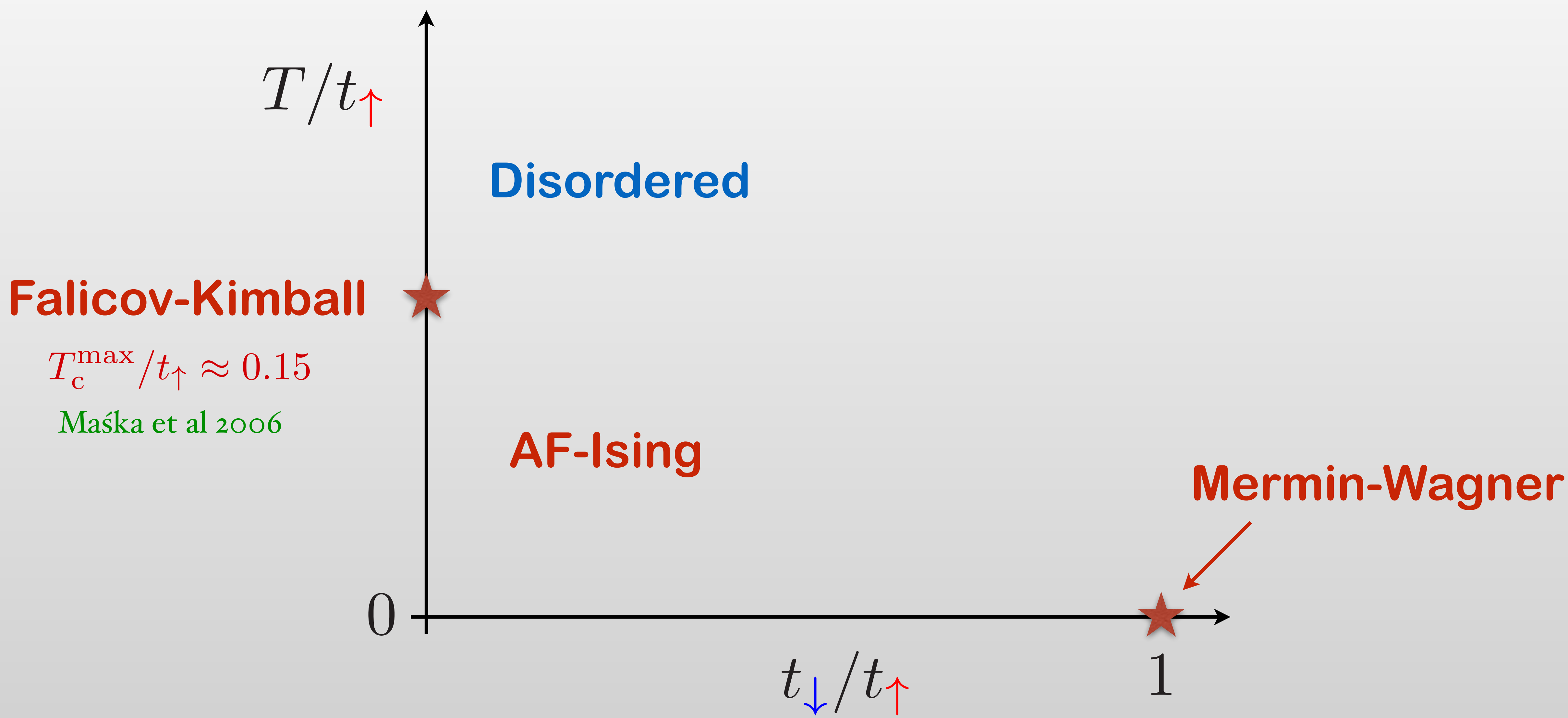

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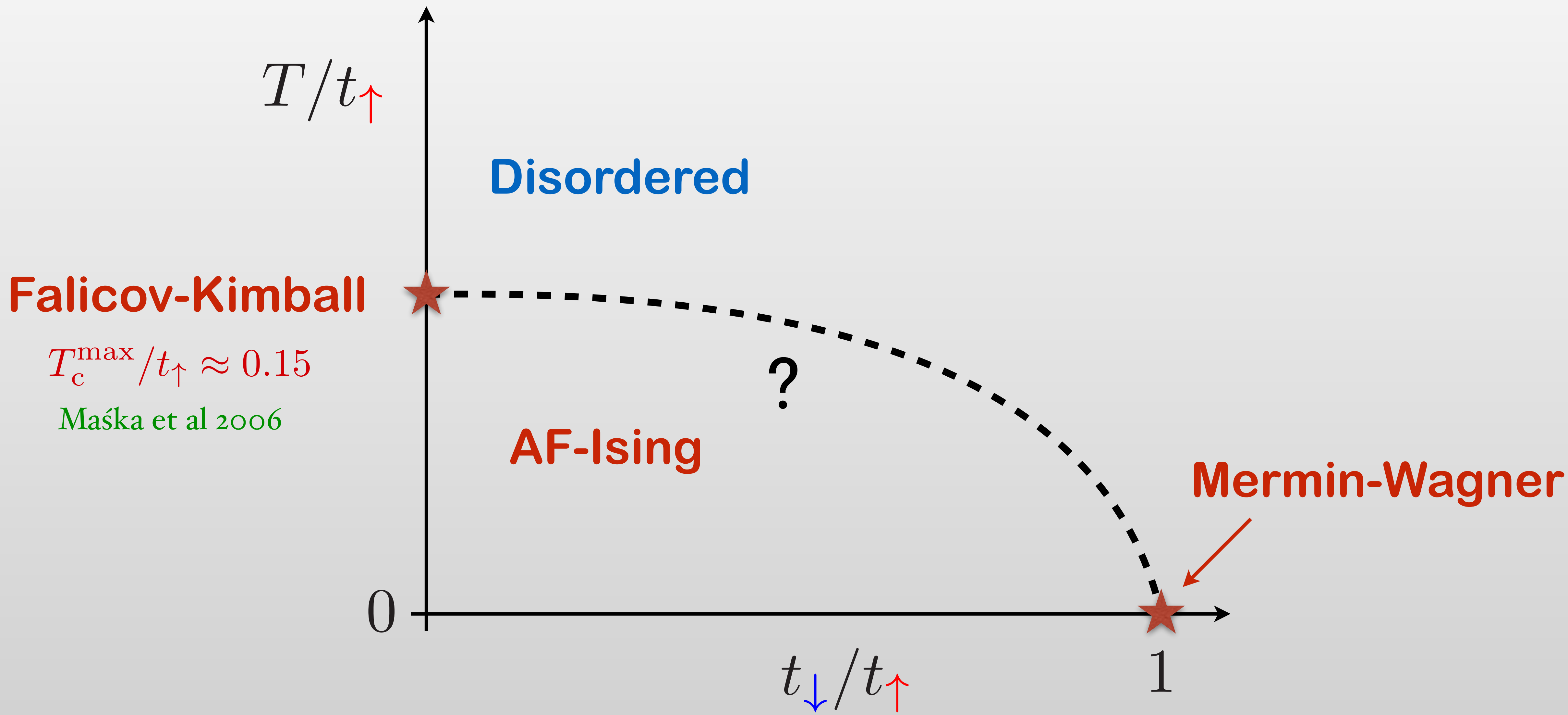

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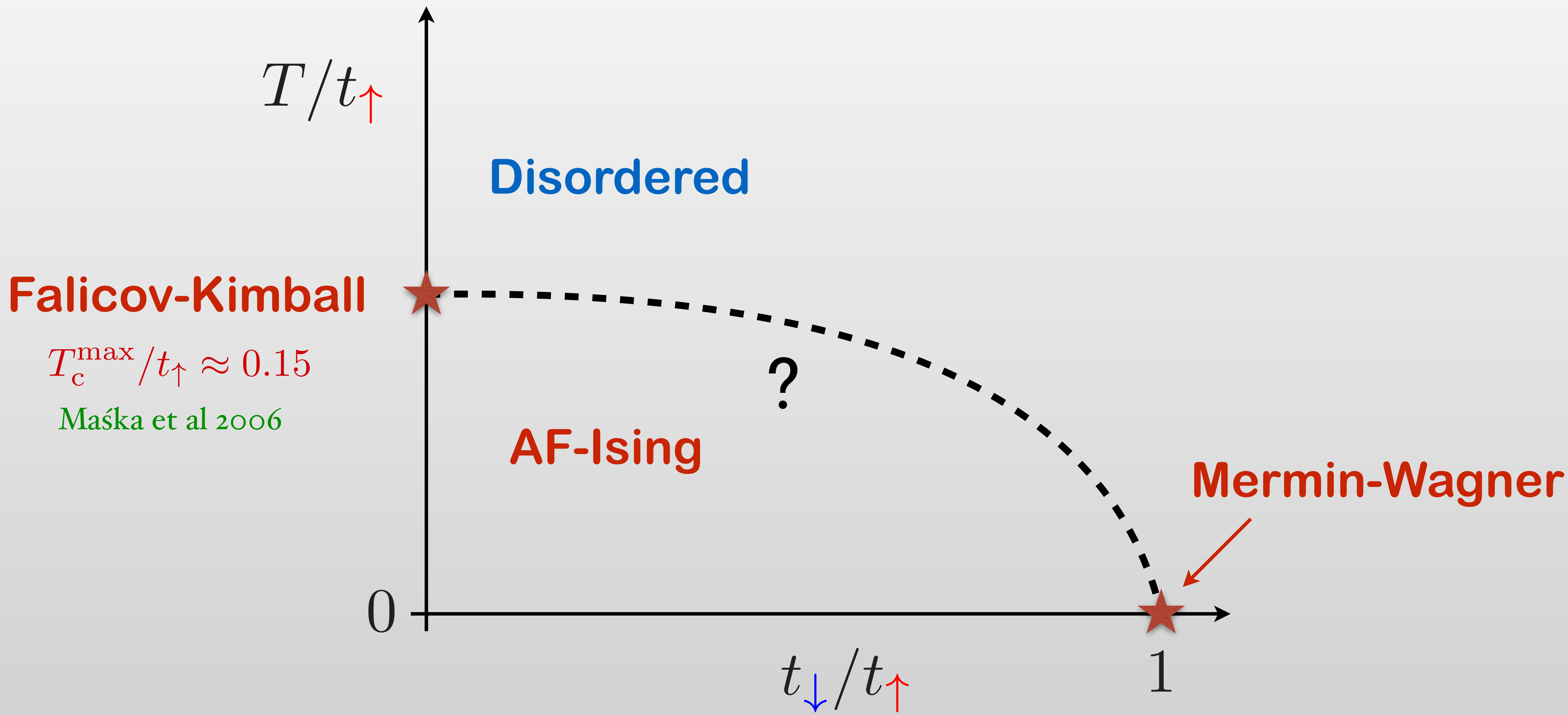
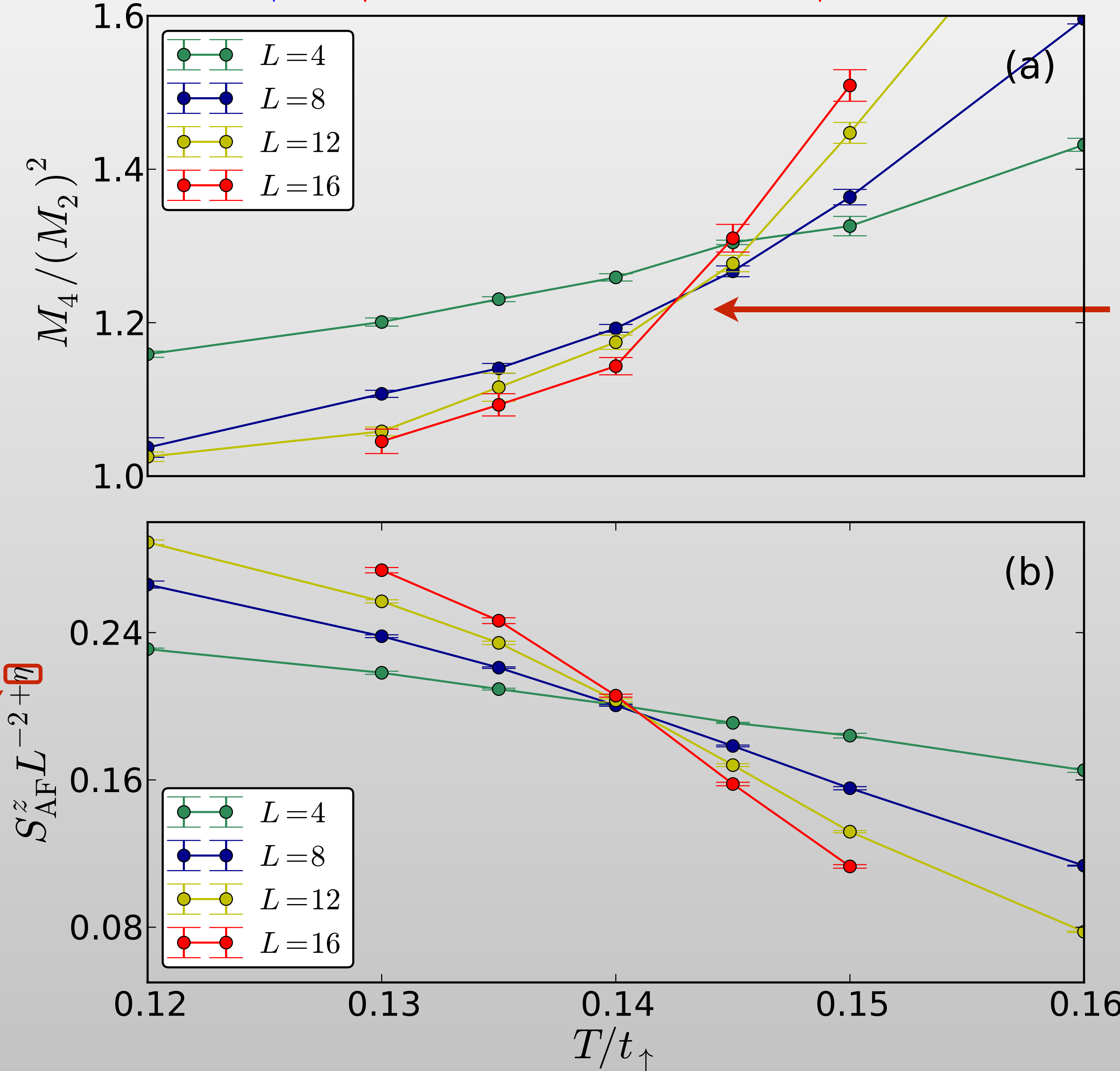


- Finite T_c in 2D because of broken discrete Z_2 symmetry
- Advantage in 3D ? Sotnikov et al, PRL 2012

Locate the critical temperature

Liu and LW, PRB 2015

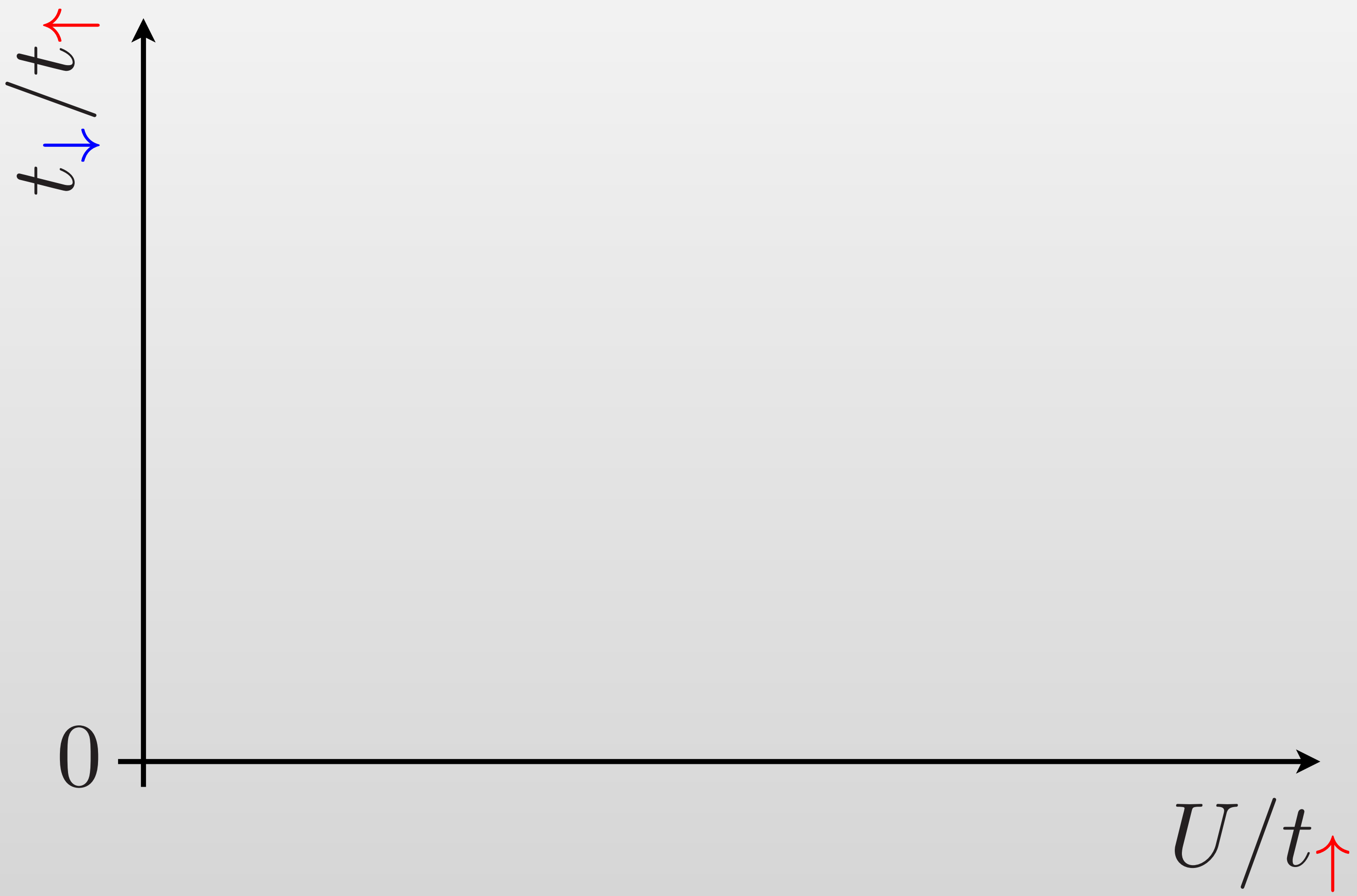
$t_{\downarrow}/t_{\uparrow} = 0.15 \quad U/t_{\uparrow} = 4.0$



Universal crossing value of 2D Ising

2D Ising value
η=0.25

Physics on a honeycomb lattice



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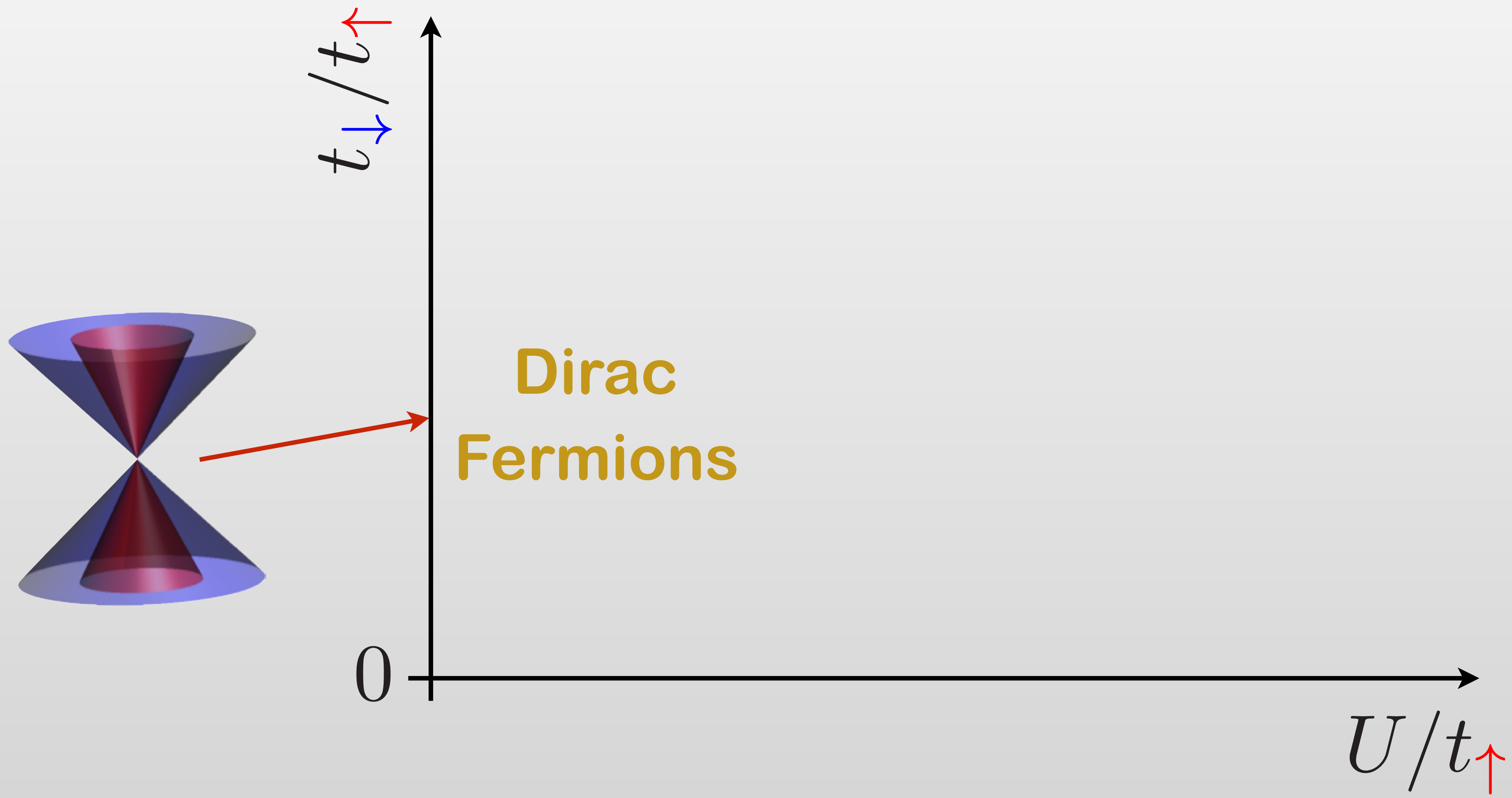

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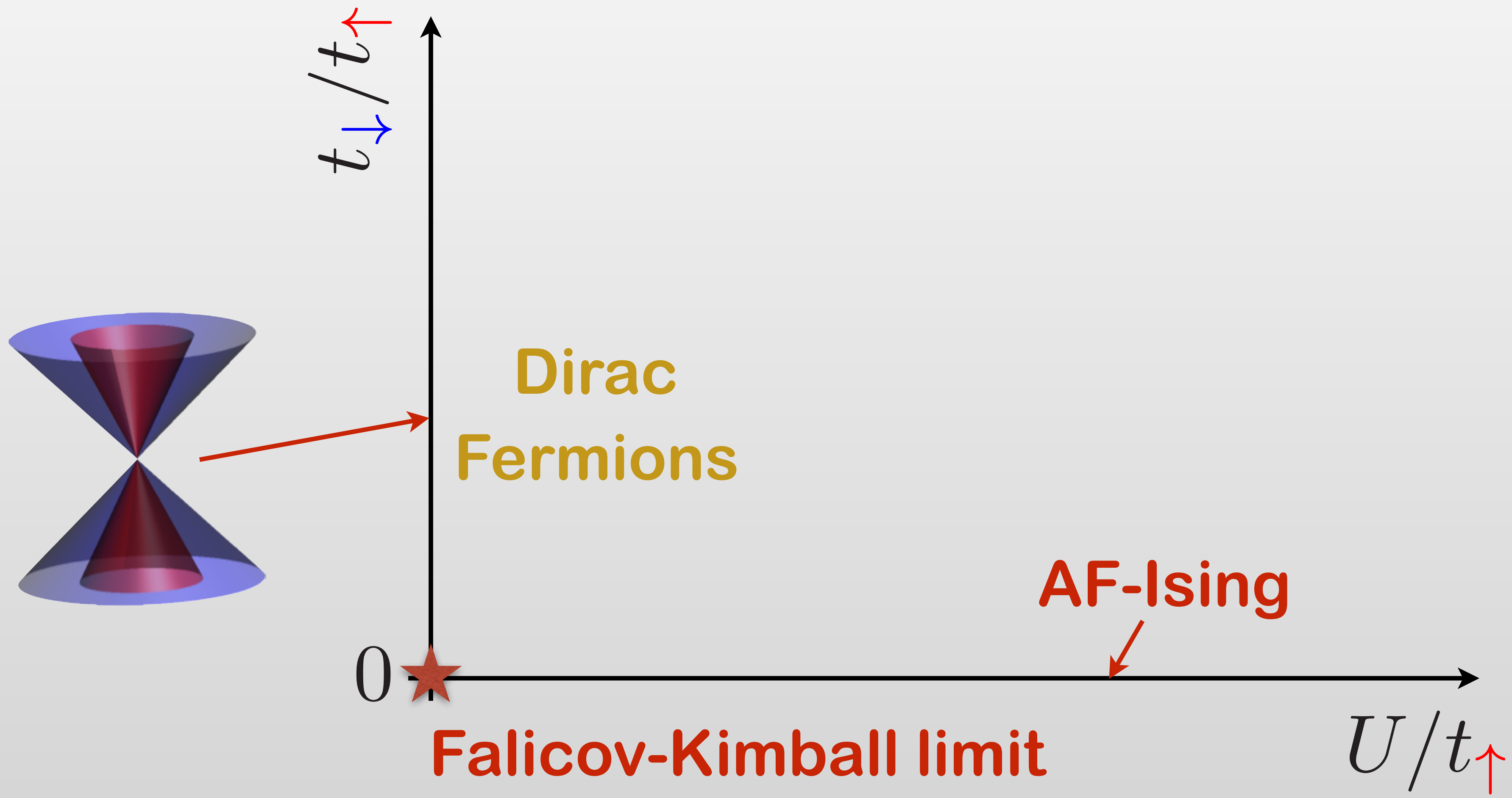

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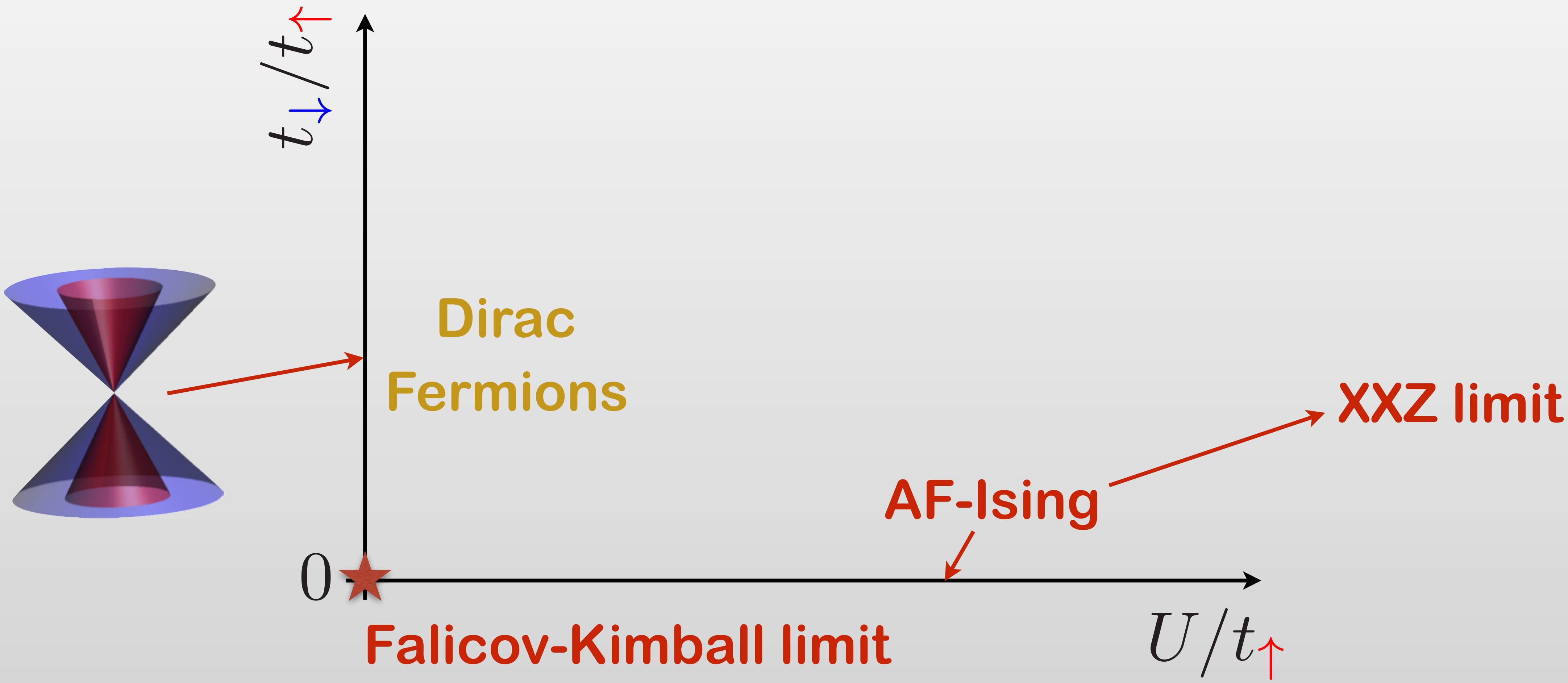

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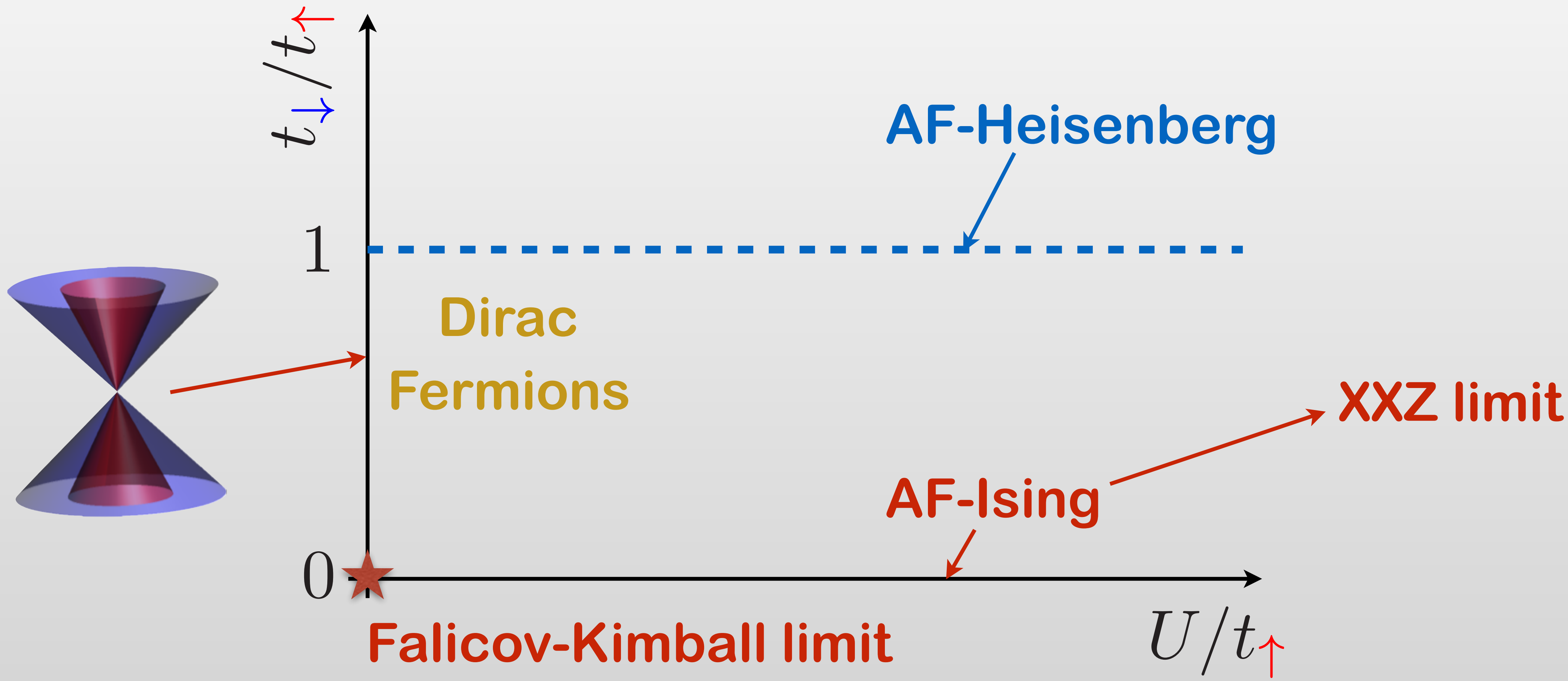

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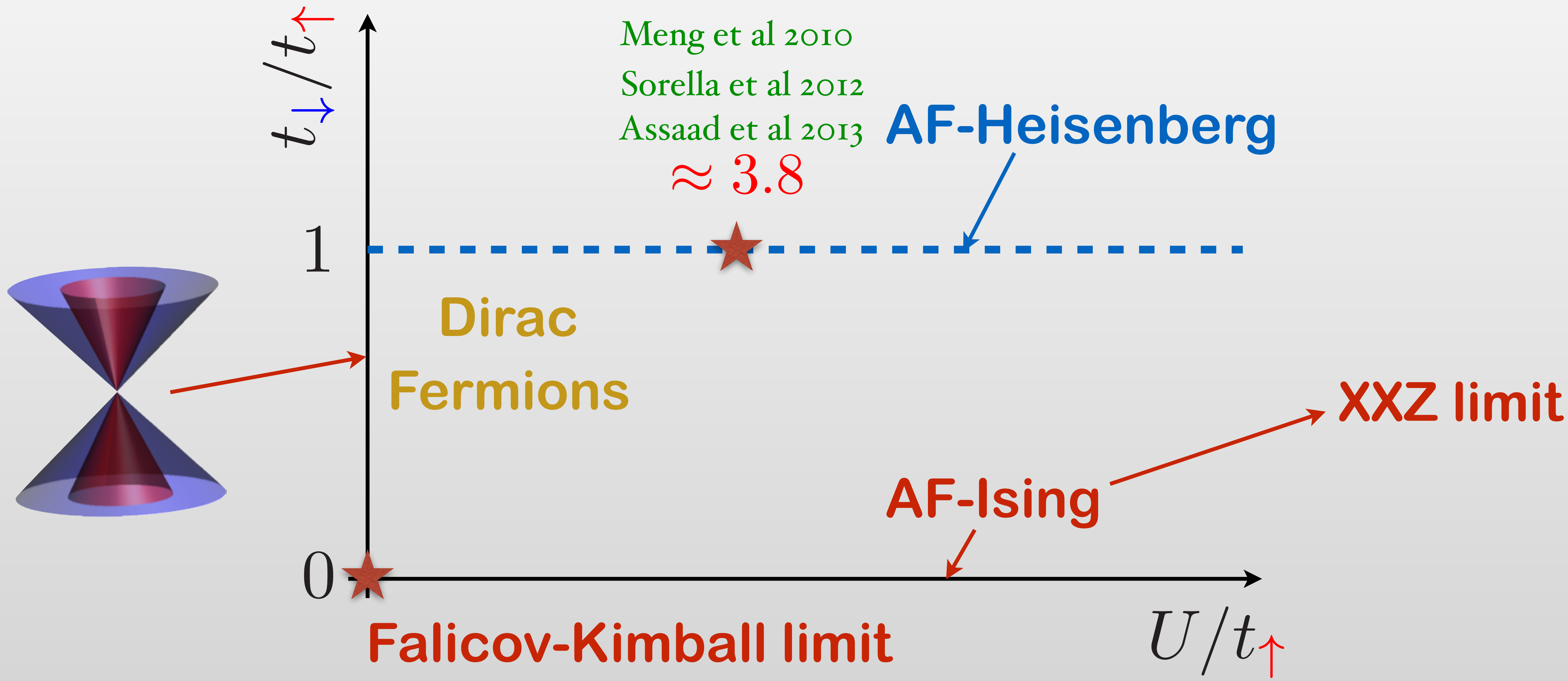

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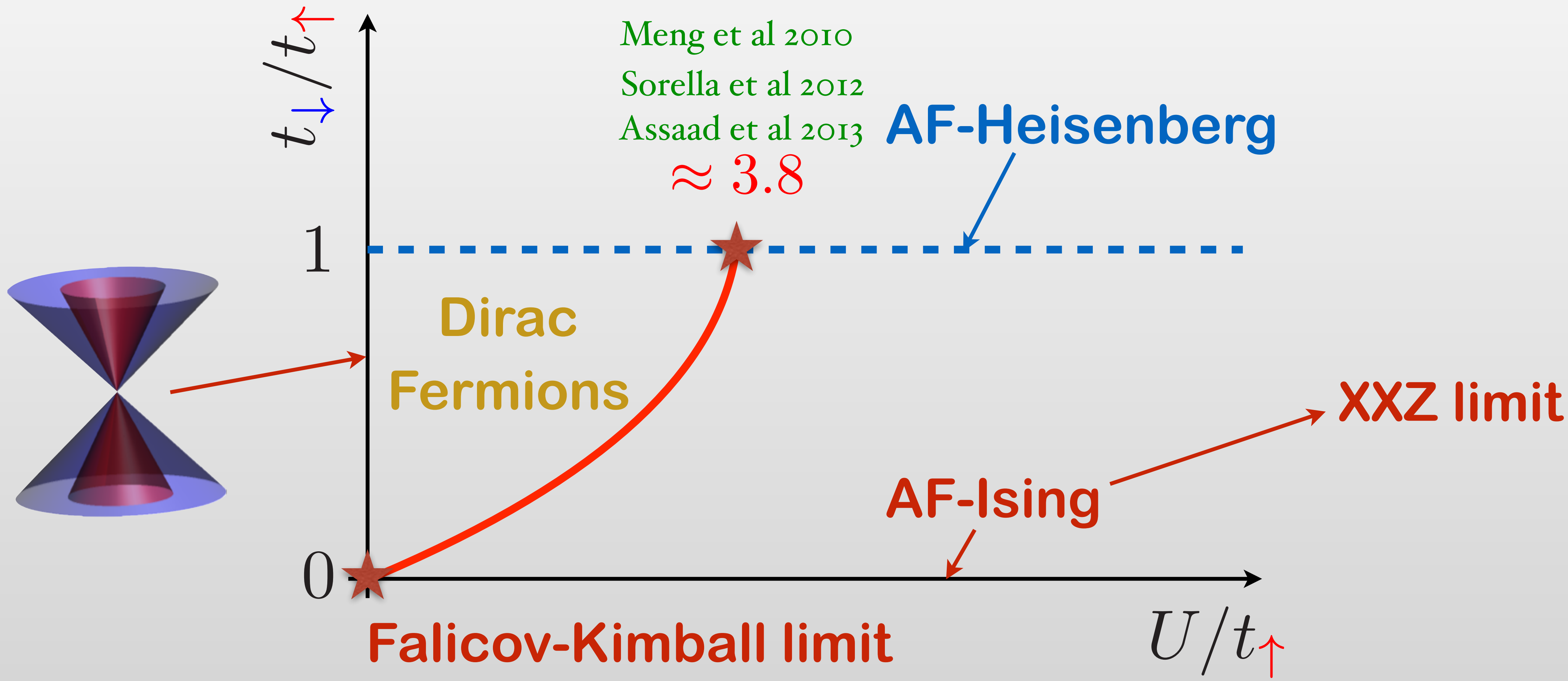
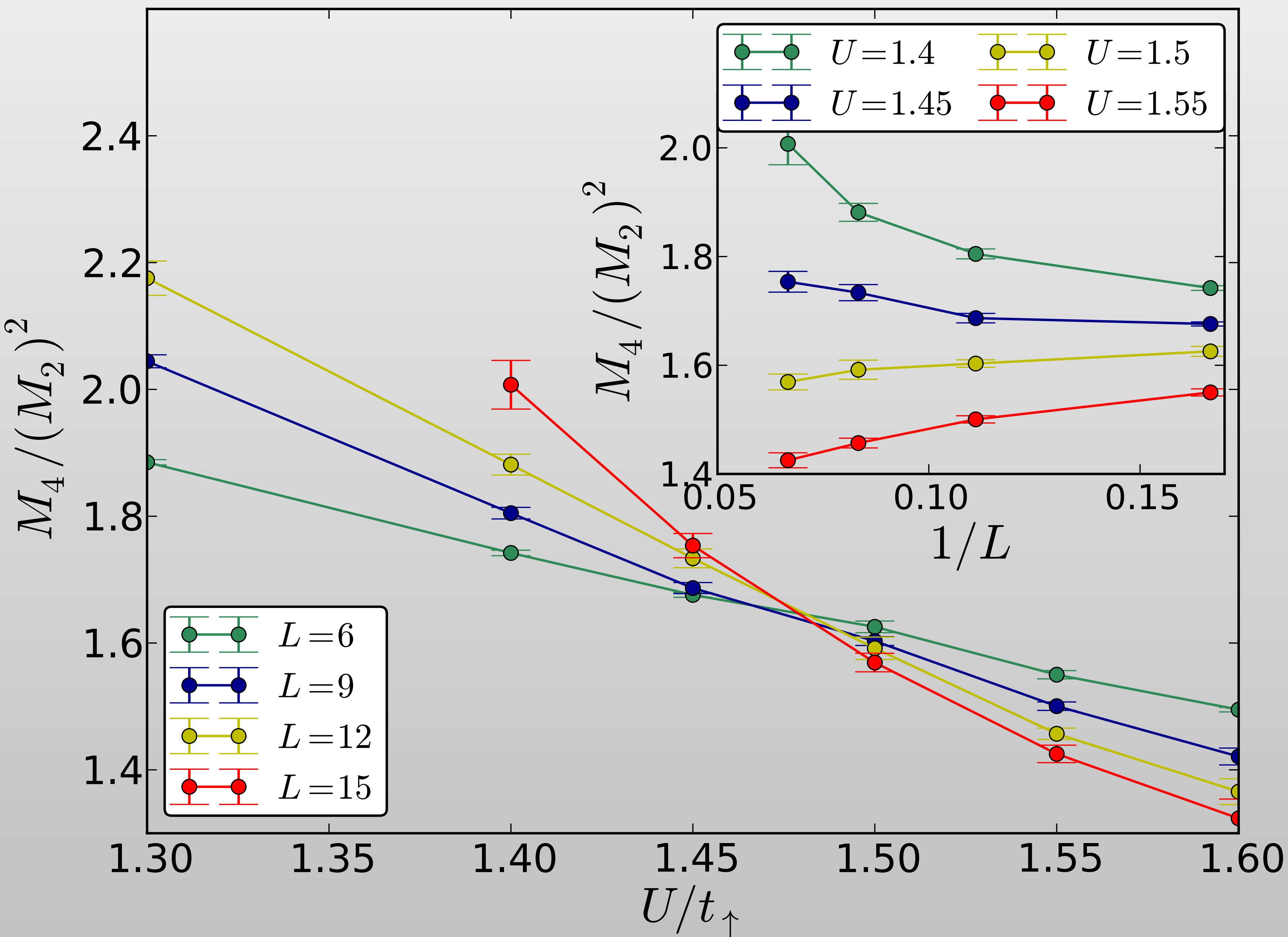


- How to connect the phase boundary ?
- What is the universality class ?

Binder ratio

$t_\downarrow/t_\uparrow = 0.15$

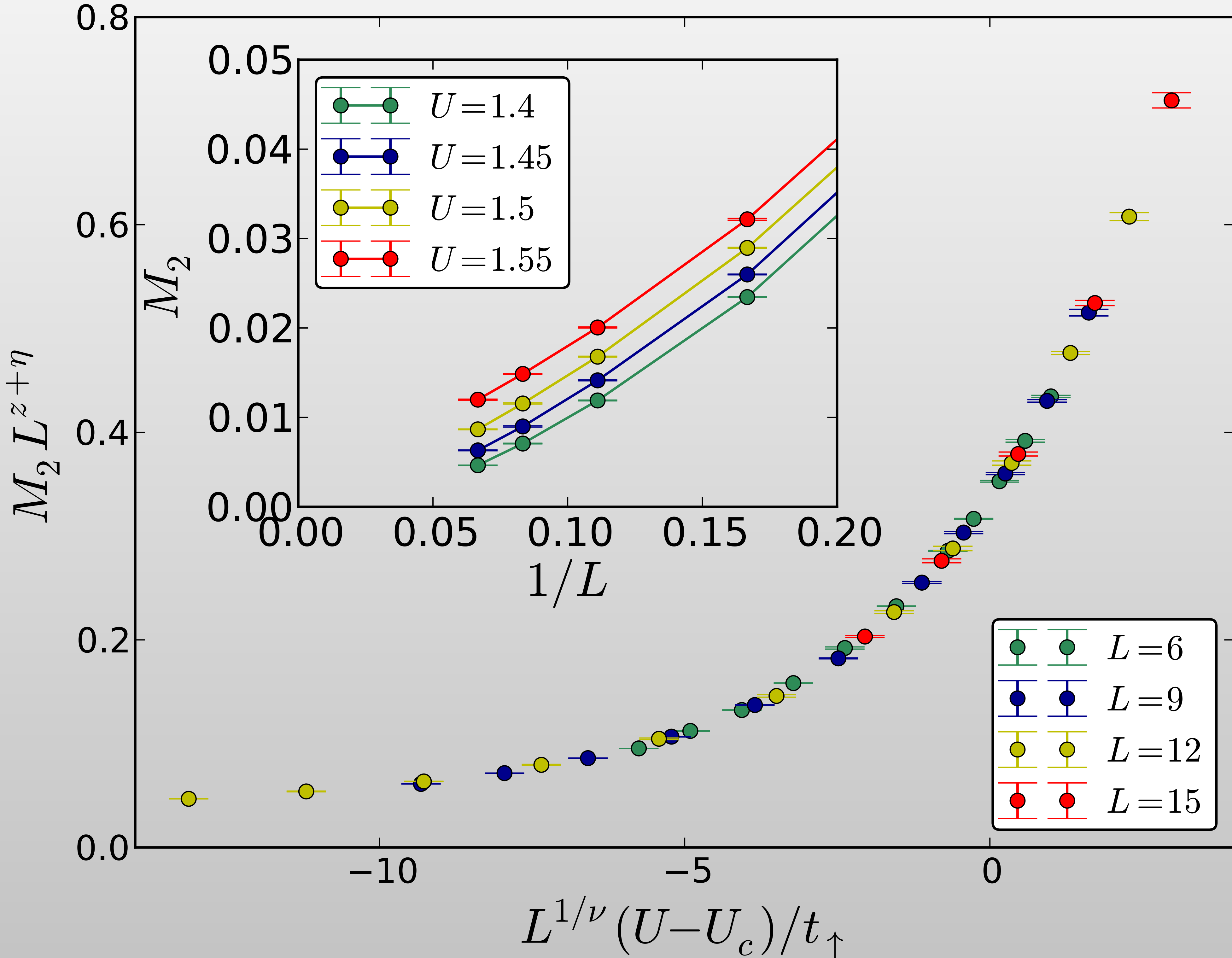
$$M_2 = \left\langle \left(\frac{1}{N} \sum_{\mathbf{r}} e^{i\mathbf{Q}\cdot\mathbf{r}} \frac{\hat{n}_{\mathbf{r}\uparrow} - \hat{n}_{\mathbf{r}\downarrow}}{2} \right)^2 \right\rangle \qquad M_4 = \left\langle \left(\frac{1}{N} \sum_{\mathbf{r}} e^{i\mathbf{Q}\cdot\mathbf{r}} \frac{\hat{n}_{\mathbf{r}\uparrow} - \hat{n}_{\mathbf{r}\downarrow}}{2} \right)^4 \right\rangle$$



Liu and Wang, PRB 2015

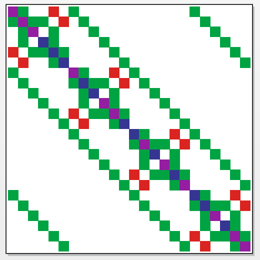
Scaling analysis

$$\nu = 0.84(4)$$
$$z + \eta = 1.395(7)$$



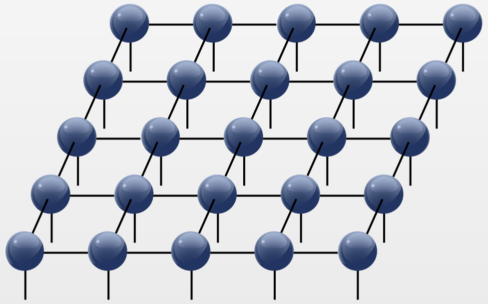
Liu and Wang, PRB 2015

Summary

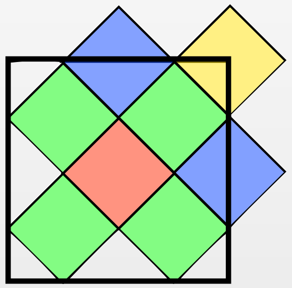


exact diagonalization

quantum Monte Carlo

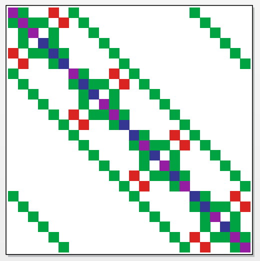


tensor network states



dynamical mean field theories

Summary



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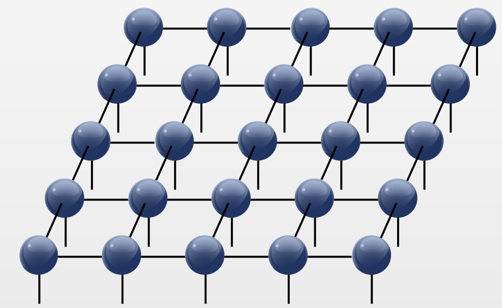

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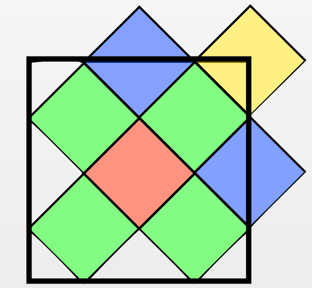
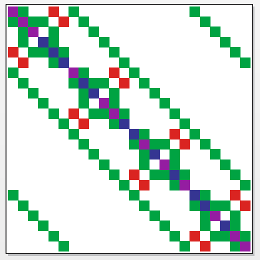


dynamical mean field theories

Algorithmic improvement in past 20 years outperformed Moore's law

Summary



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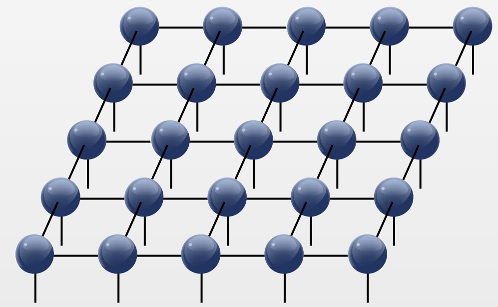

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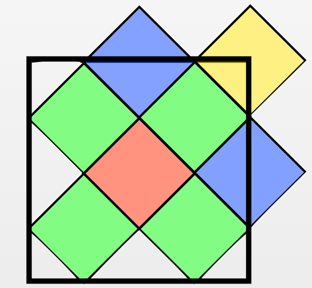


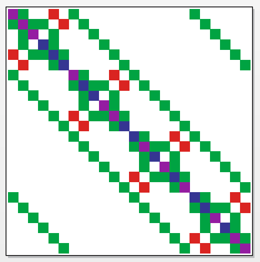
dynamical mean field theories

Modern algorithm

is faster than

Traditional algorithm

Summary



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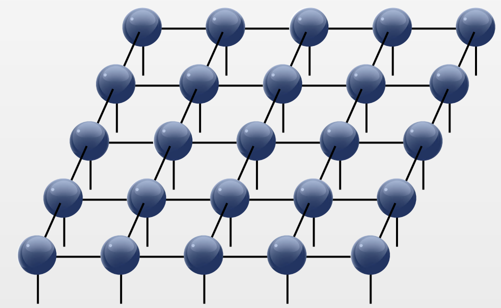

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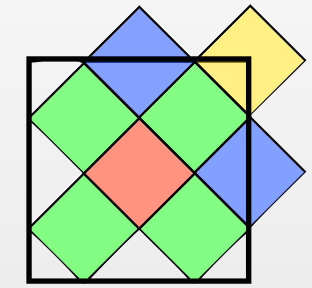
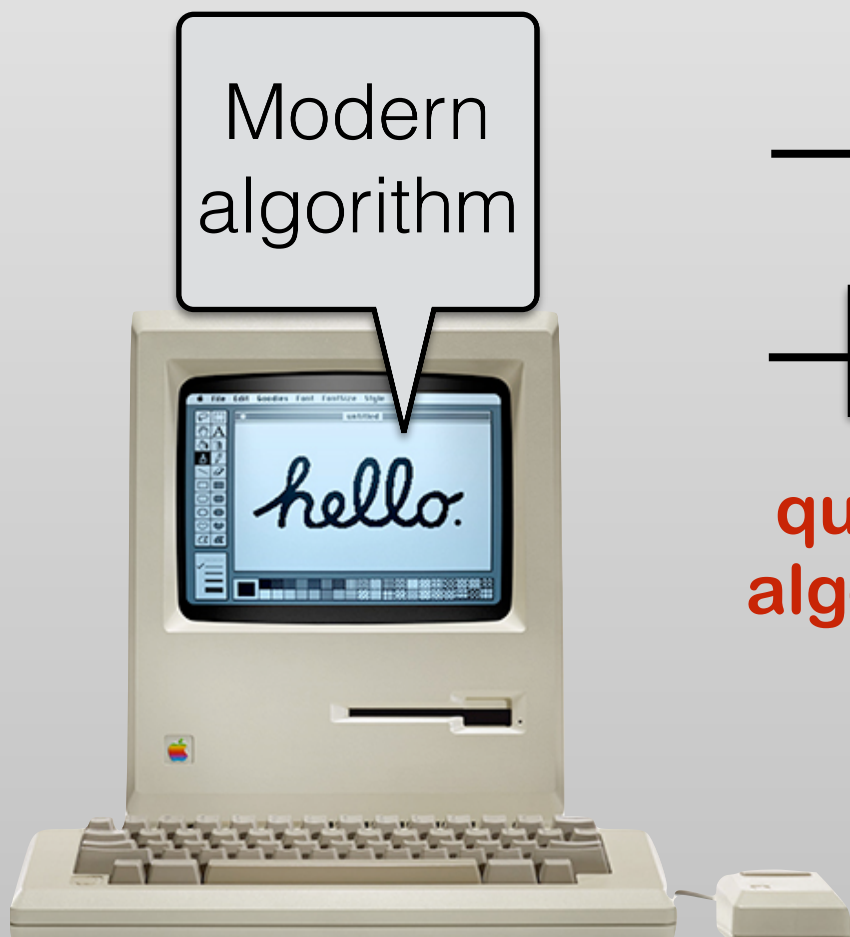
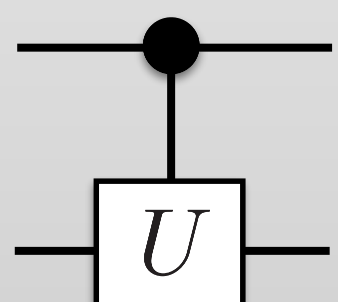
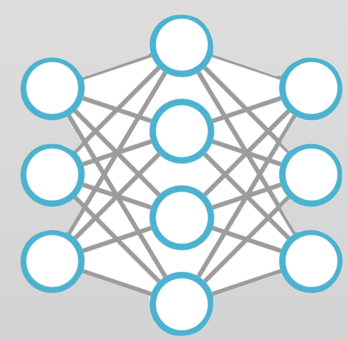


dynamical mean field theories

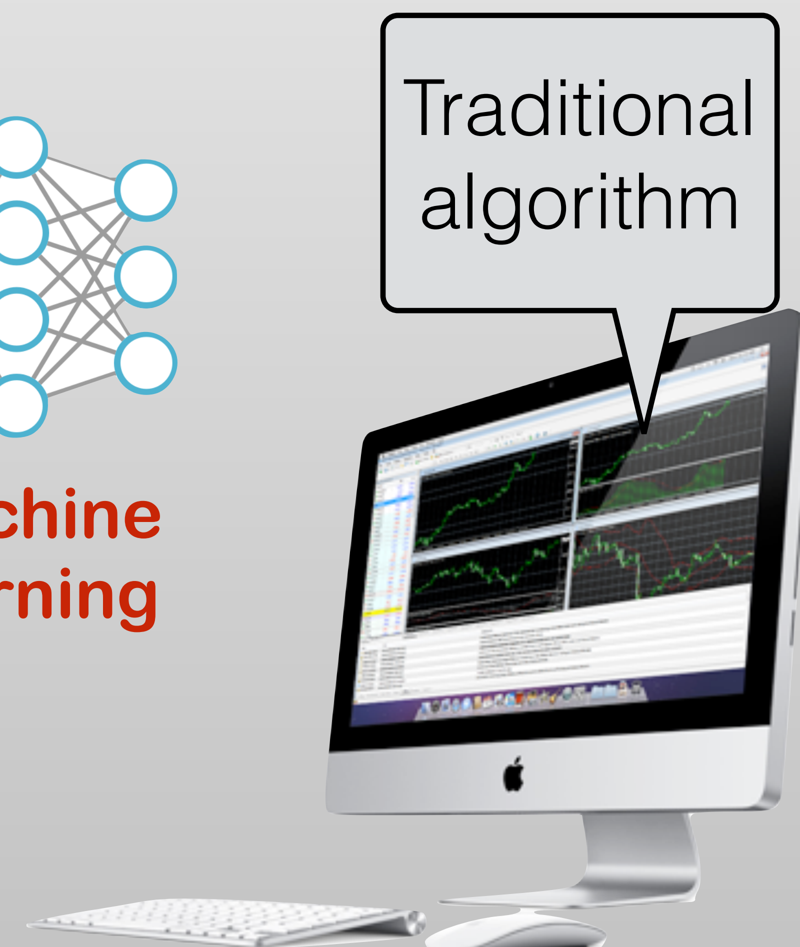




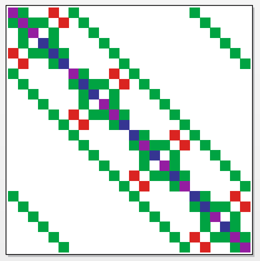
quantum algorithms



machine learning



Summary



exact
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quantum
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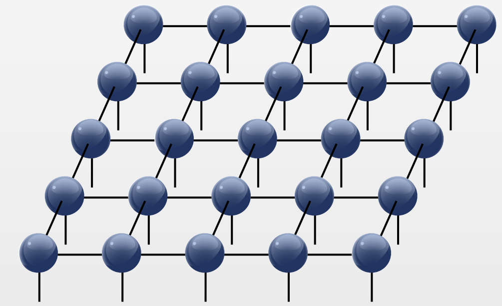
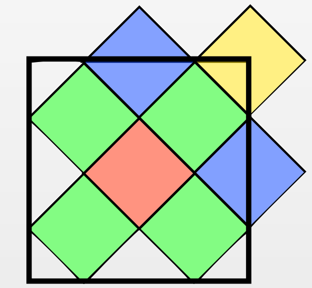


tensor network
states



dynamical mean
field theories

Thanks to my collaborators!

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广告

欢迎本科生毕业设计，博士生，博士后

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