Simulating dynamics and topological phases of cold fermionic gases

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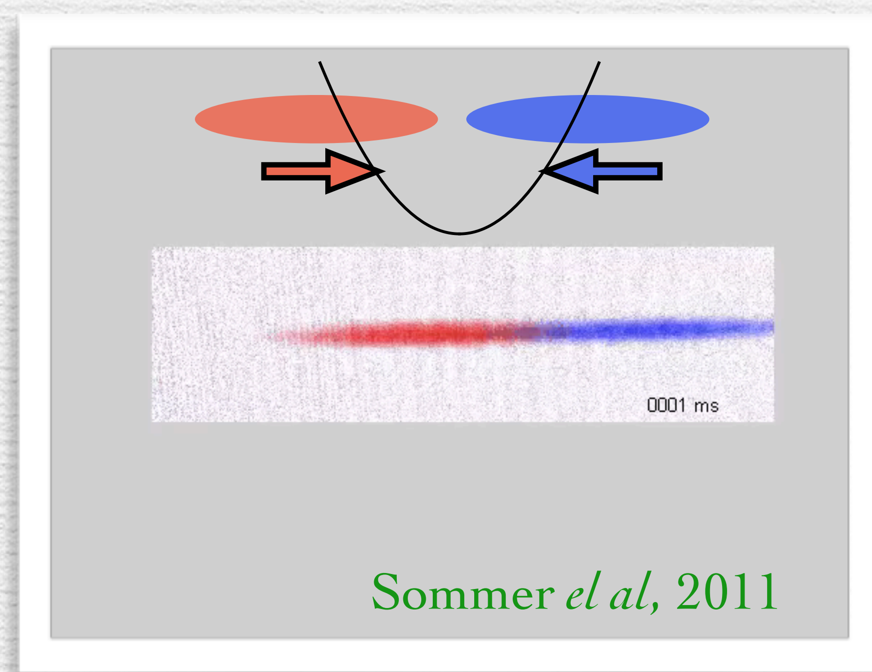
ICTP
Sebastiano Pilati

Institute of Physics CAS
Xi Dai

Numerical simulations for dynamics of fermionic atoms in high dimensions

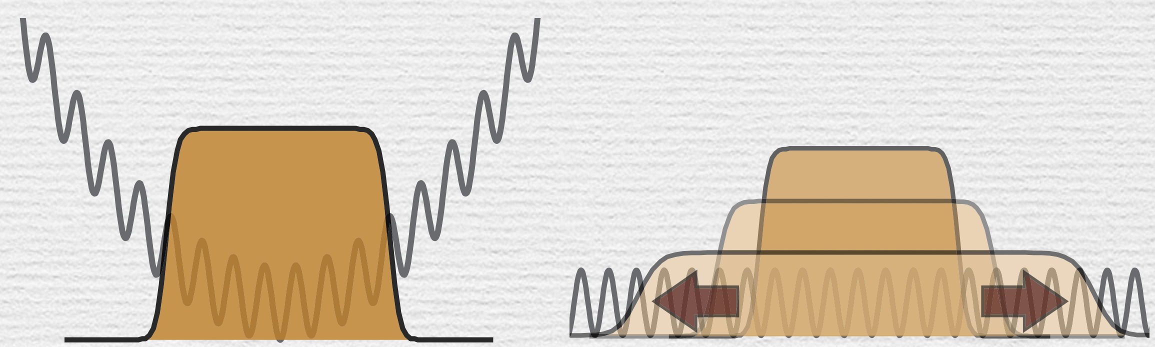
- It is difficult
- But, there were experiments…

Collision in a 3D trap



Sommer *el al,* 2011

Expansion in 2D optical lattice



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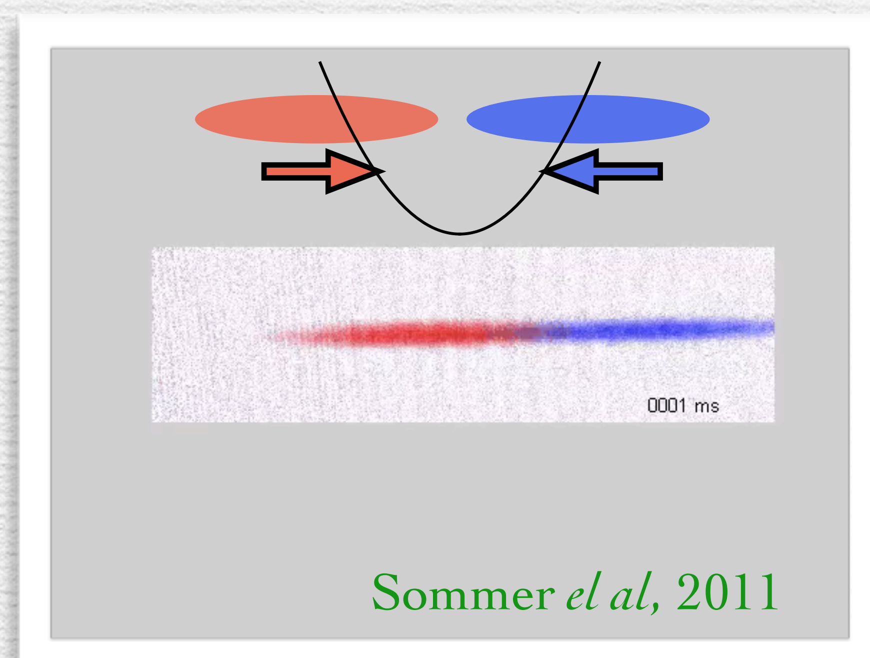

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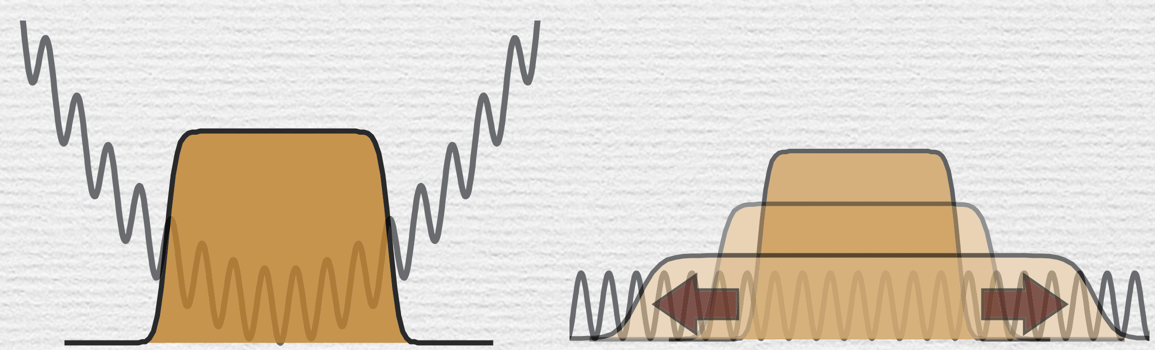

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Density functional theory

Hohenberg and Kohn 1964

$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$ → $\rho(\mathbf{r})$

$\mathbb{R}^{3N} \mapsto \mathbb{C}$ → $\mathbb{R}^3 \mapsto \mathbb{R}$

- Hohenberg-Kohn theorem: **All** properties of the system are **completely** determined by the ground state density.
- **Exact** ground state density and energy can be obtained by minimizing a **universal** density functional.

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- Hohenberg-Kohn theorem: **All** properties of the system are **completely** determined by the ground state density.
- **Exact** ground state density and energy can be obtained by minimizing a **universal** density functional.
- In practice, obtain many-particle density from an auxiliary noninteracting system

Kohn and Sham 1965

$$(-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}} + V_{\text{H}}[\rho] + \boxed{V_{\text{XC}}[\rho]})\psi_j = \varepsilon_j\psi_j$$

Time-dependent DFT

Runge and Gross, 1984

- Time-dependent density also plays a central role for non-equilibrium systems

- In practice, it is obtained from

$$i\frac{\partial}{\partial t}\psi_j(\mathbf{r},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\rm ext}(\mathbf{r},t) + V_{\rm H}(\mathbf{r},t) + V_{\rm xc}[\rho(\mathbf{r}',t')](\mathbf{r},t)\right]\psi_j(\mathbf{r},t)$$

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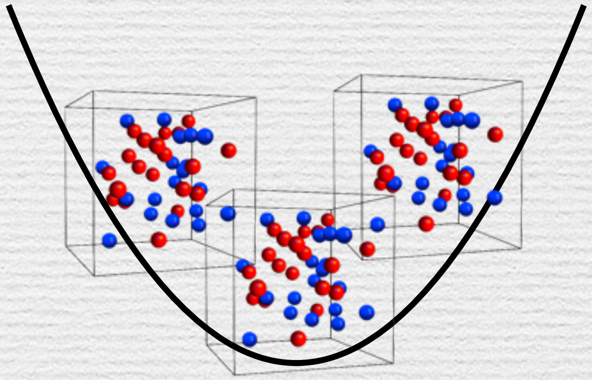
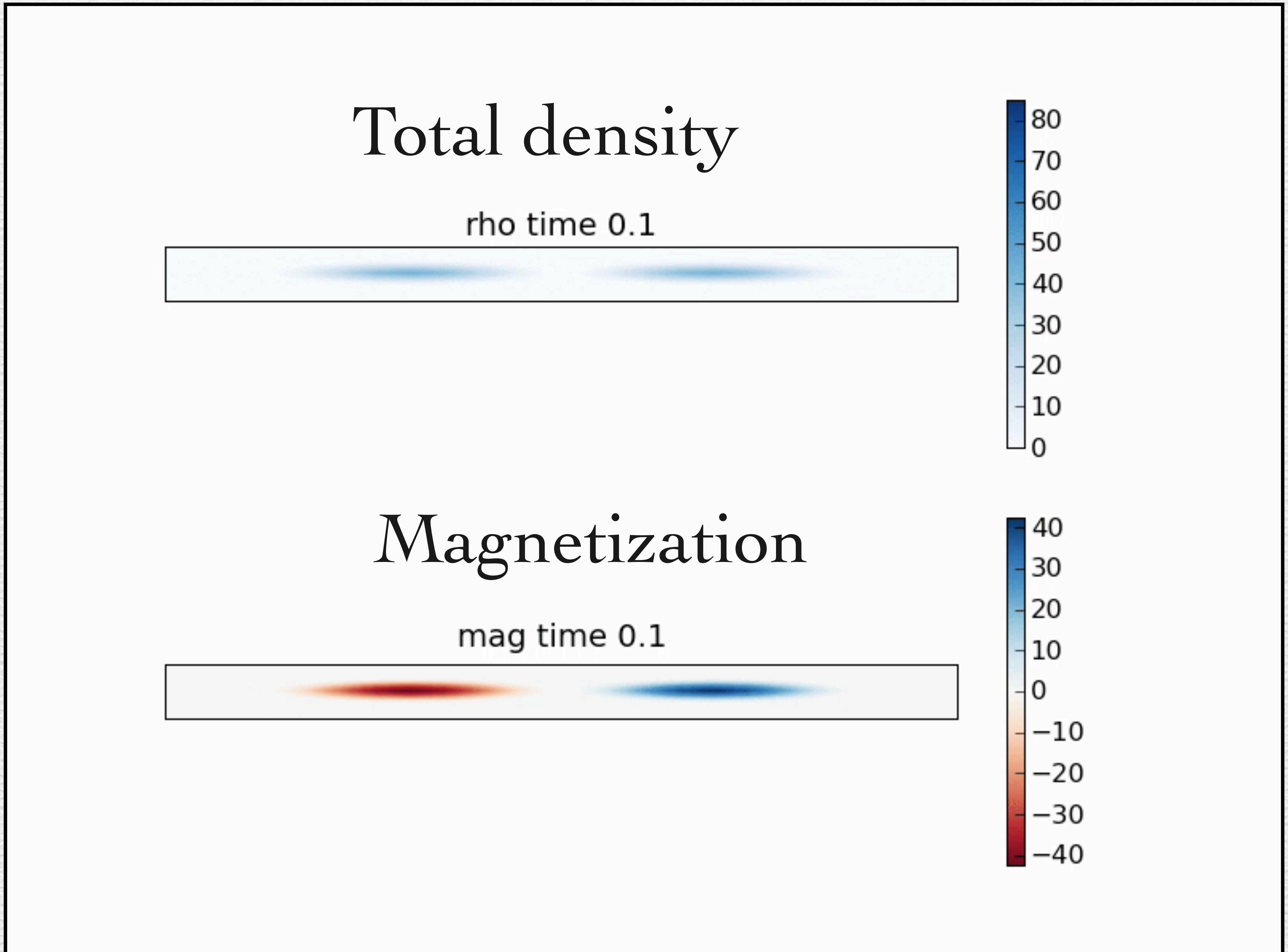
- We use the adiabatic local-density approximation



- Vxc from diffusion Monte-Carlo simulation of uniform atomic gases Pilati *et al* 2010, Ping Nang Ma *et al* 2012

Simulation of cloud collisions



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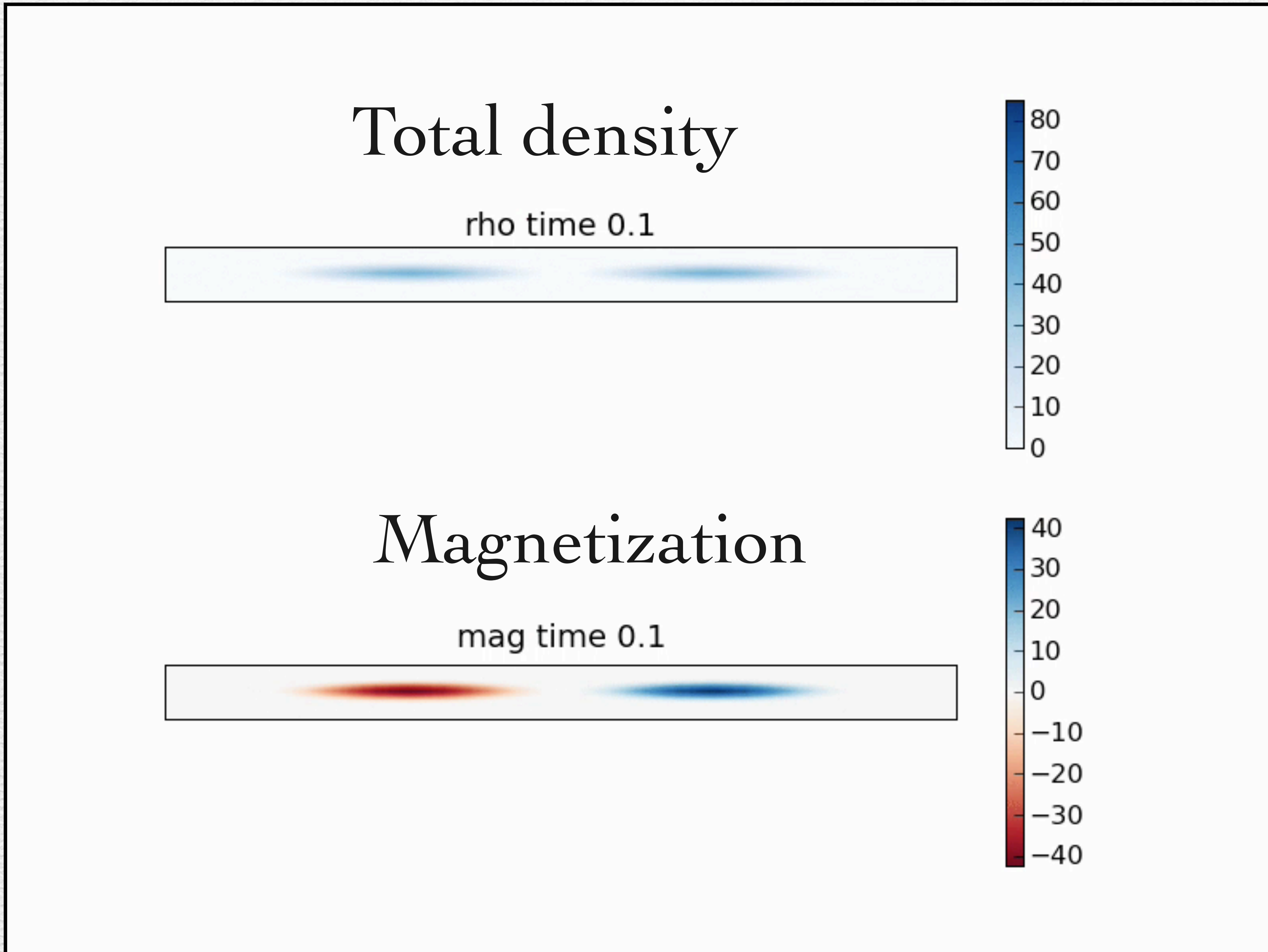

Deep optical lattices: Time-dependent Gutzwiller method

Schiro and Fabrizio, 2010

$$H = -\sum_{i,j,\sigma} t_{ij}\hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma} + \sum_{i,\sigma} v_i^{\text{ext}}\hat{n}_{i\sigma} + U\sum_i \hat{n}_{i\uparrow}\hat{n}_{i\downarrow}$$

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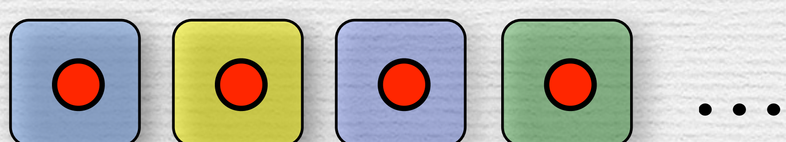

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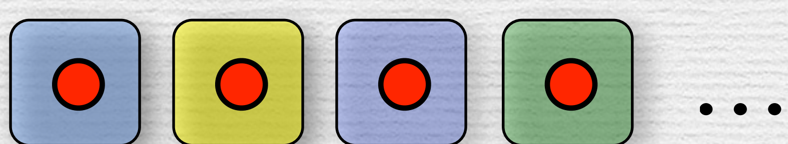

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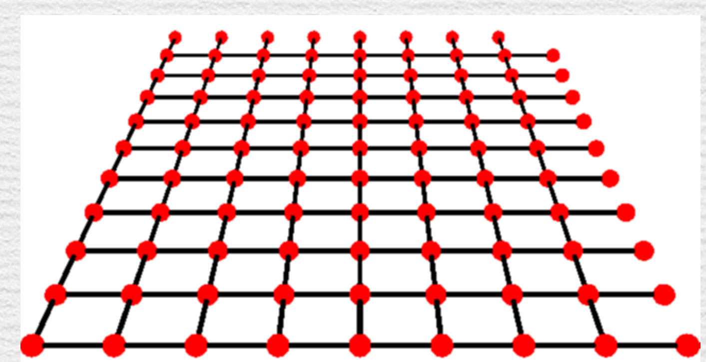

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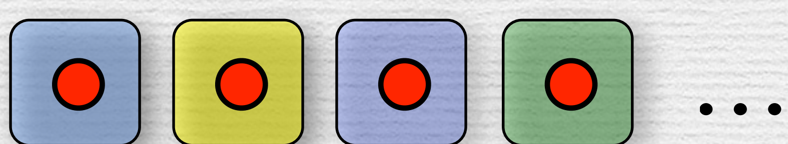

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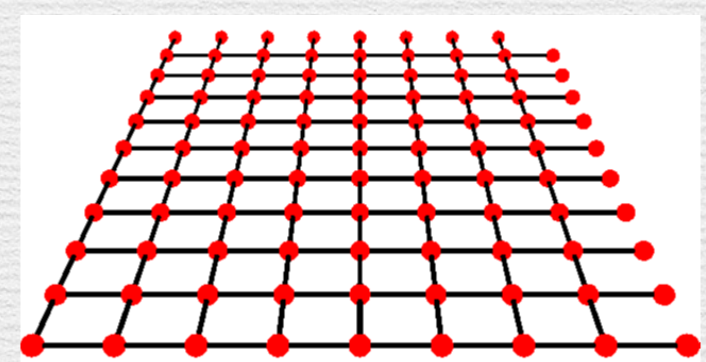

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Simulation of cloud expansion

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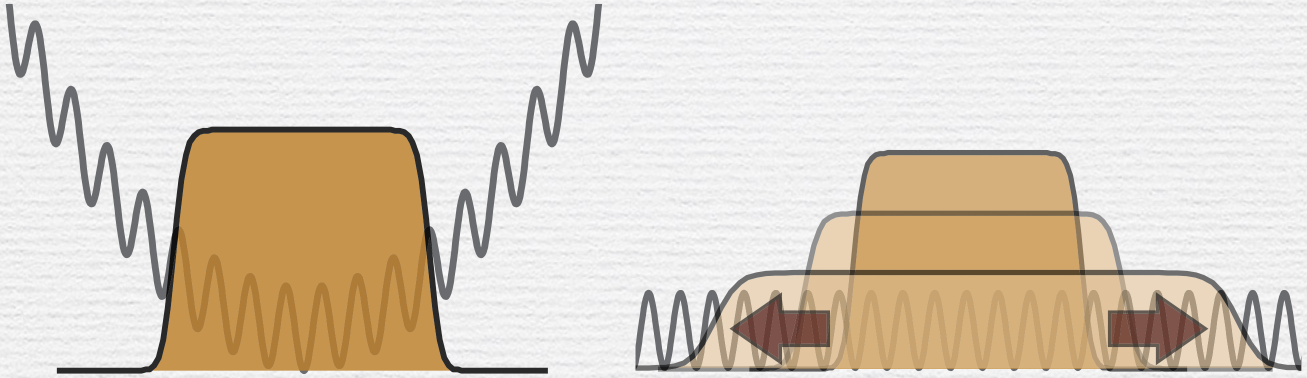

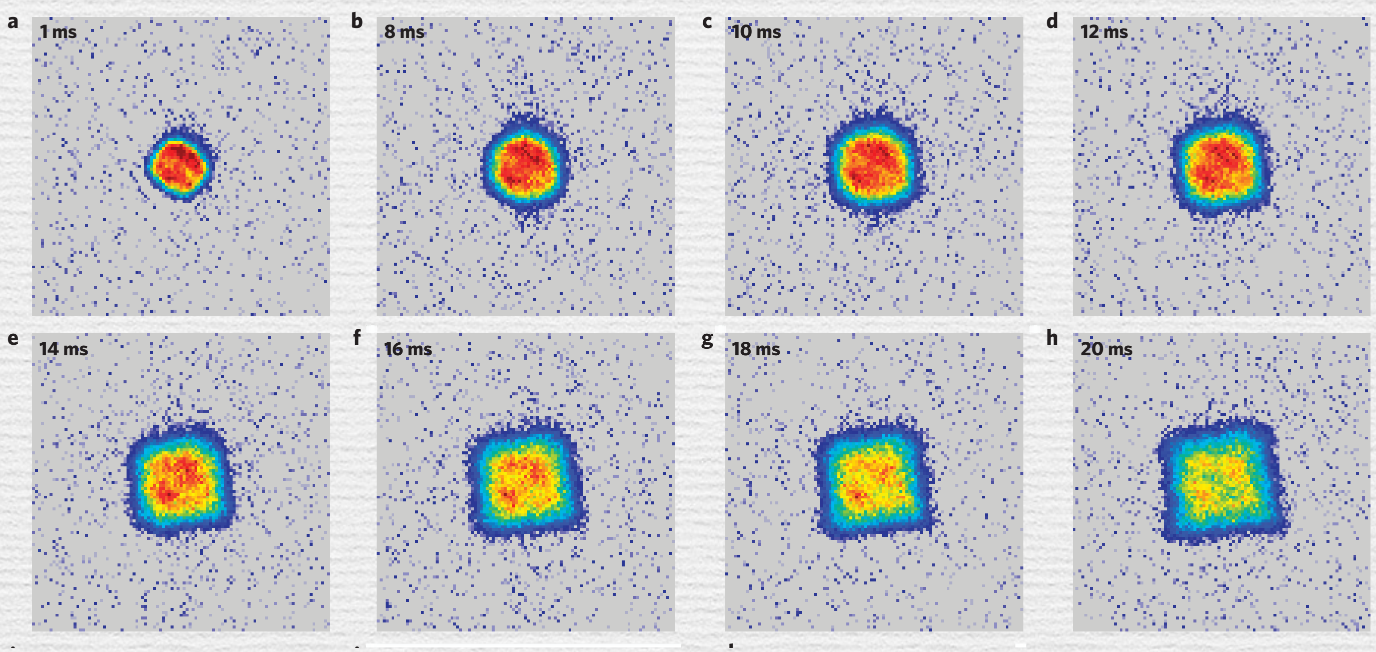

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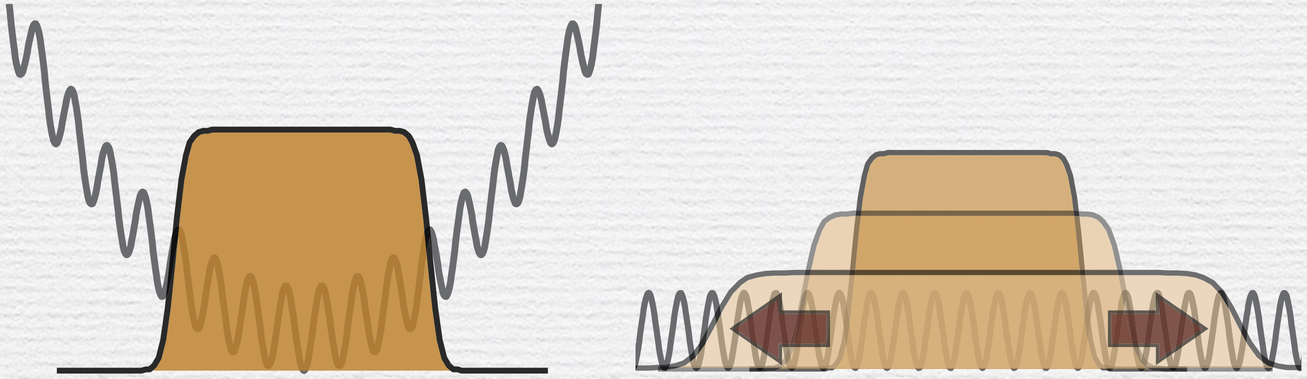

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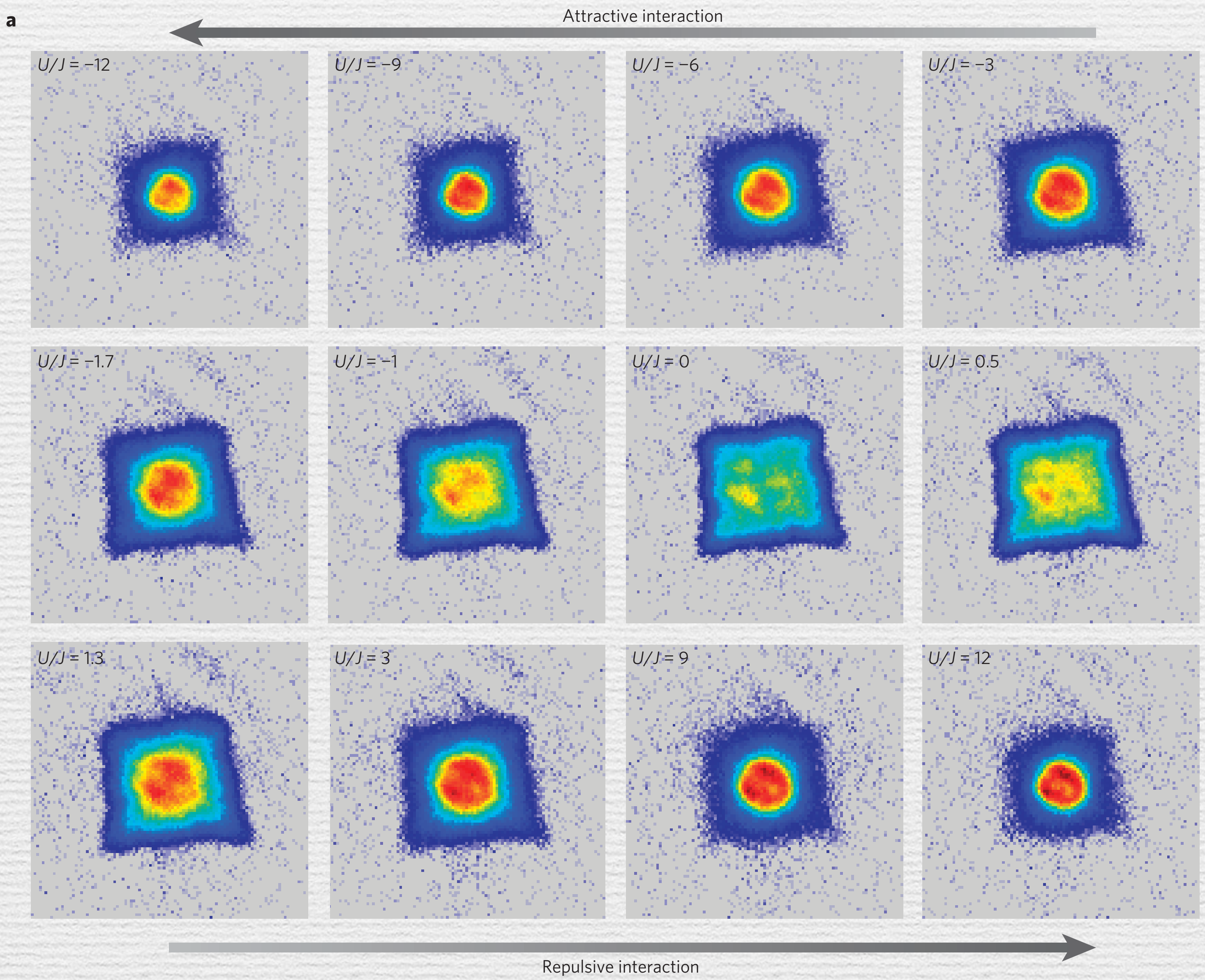

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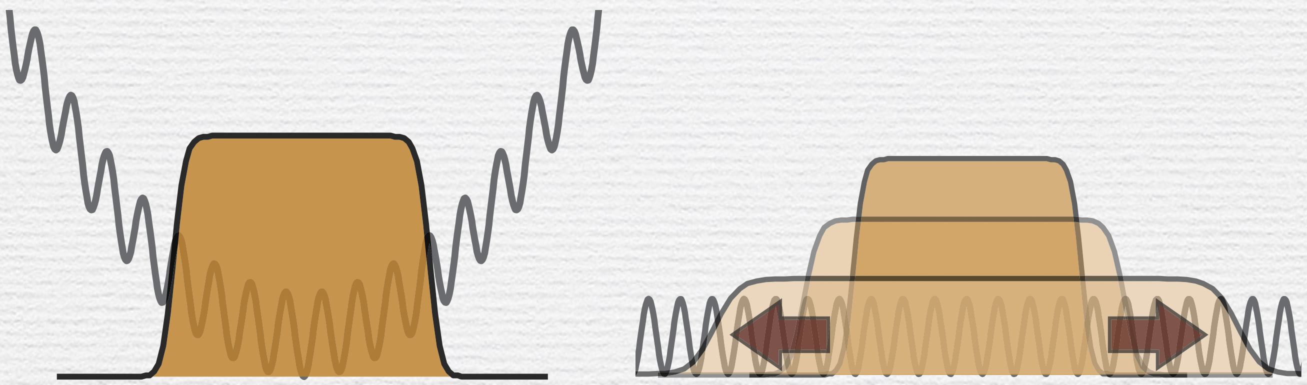

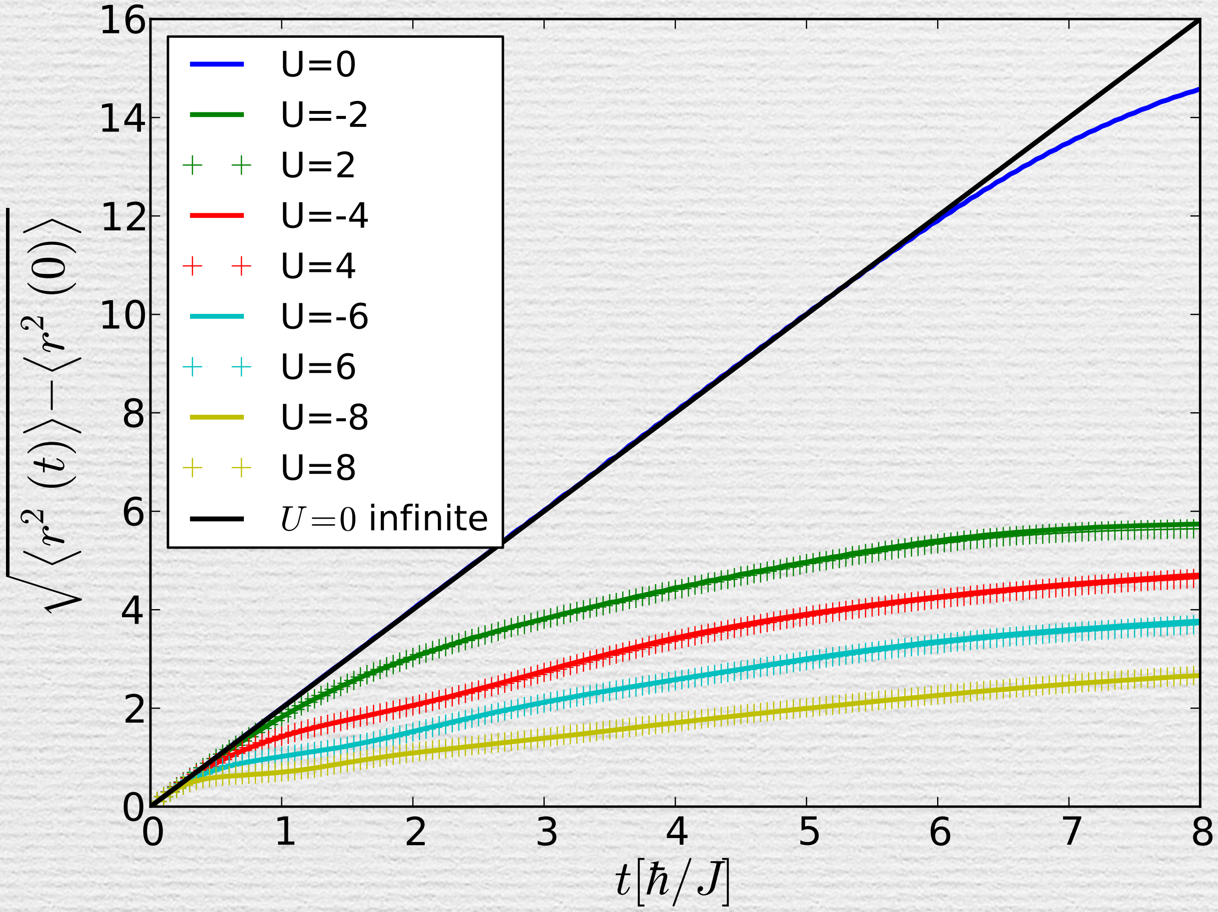

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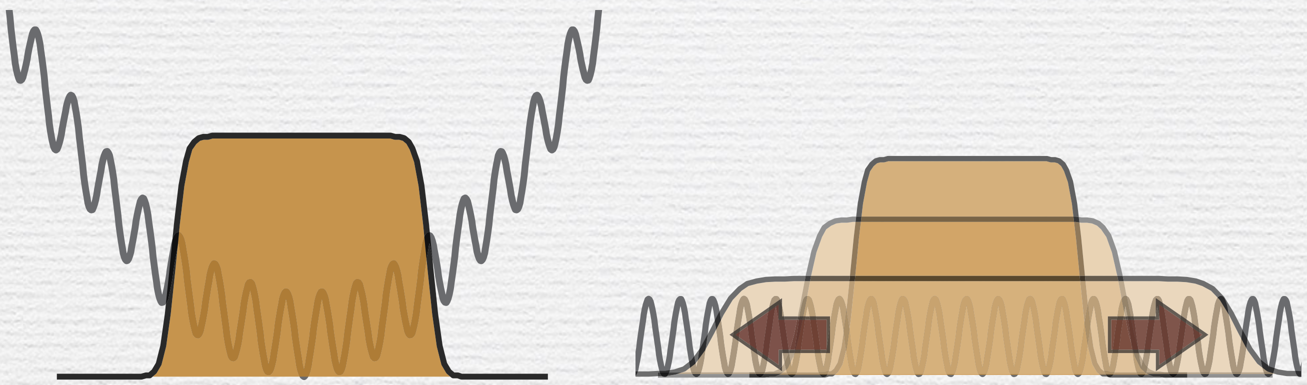
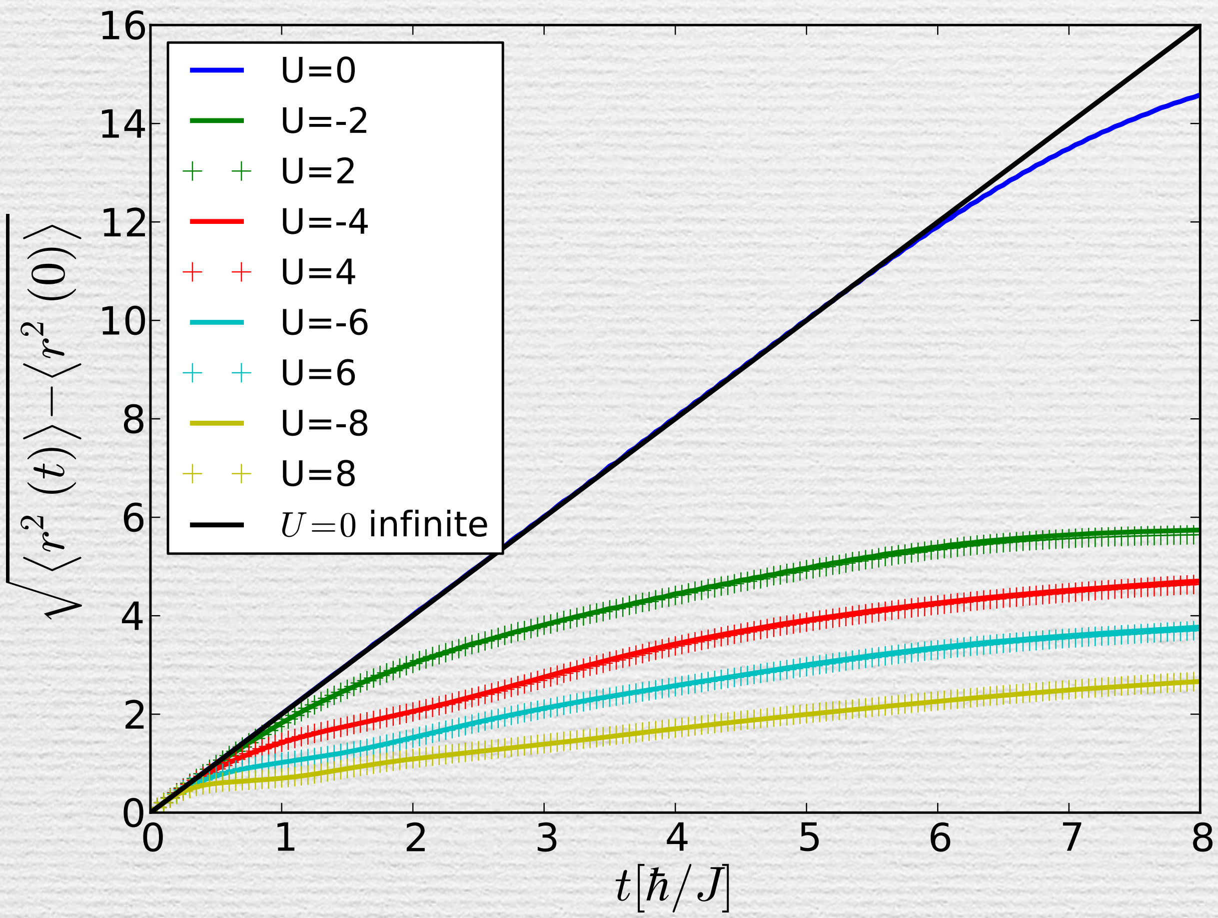
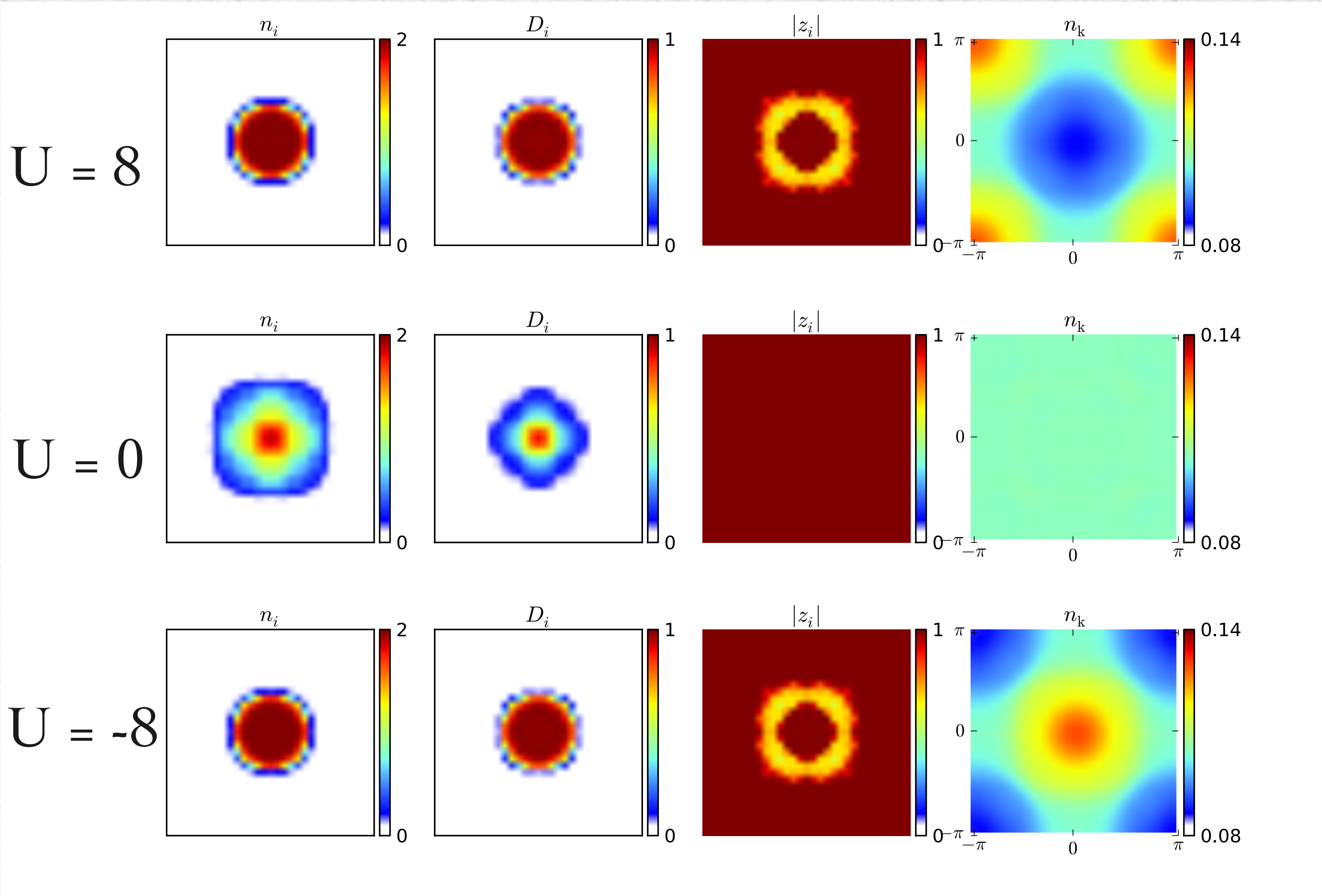
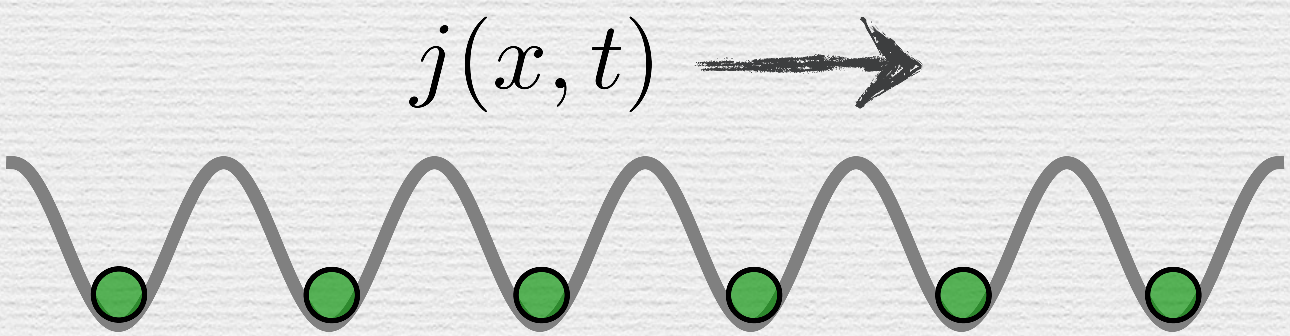






Topological charge pumping of cold atoms



Pumps

A **pump** is a device that moves fluids, or sometimes slurries, by mechanical action.

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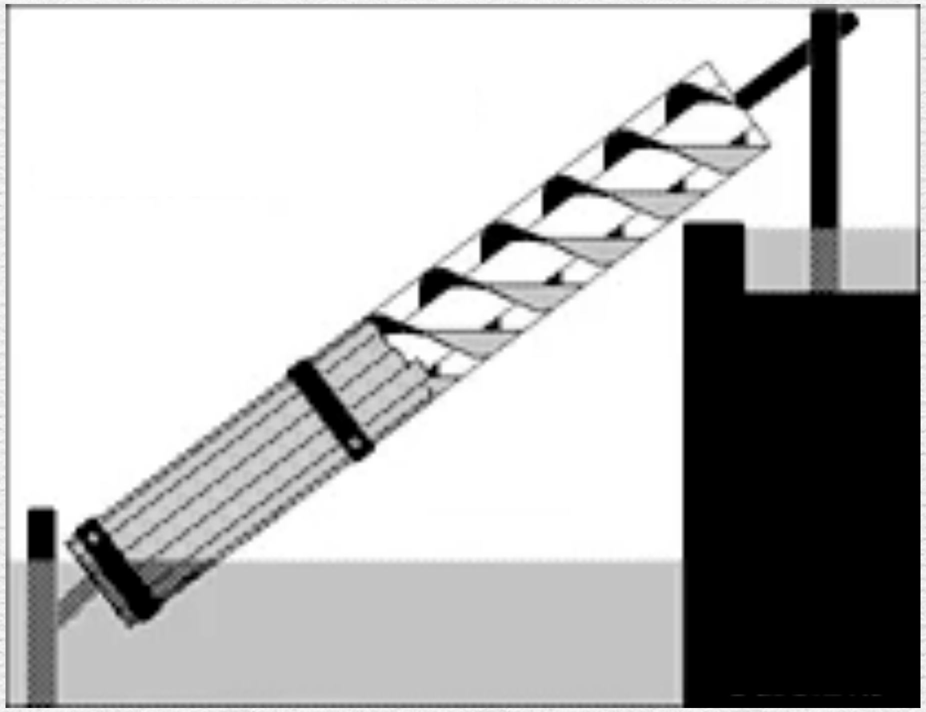

Archimedes' screw ~250 BC

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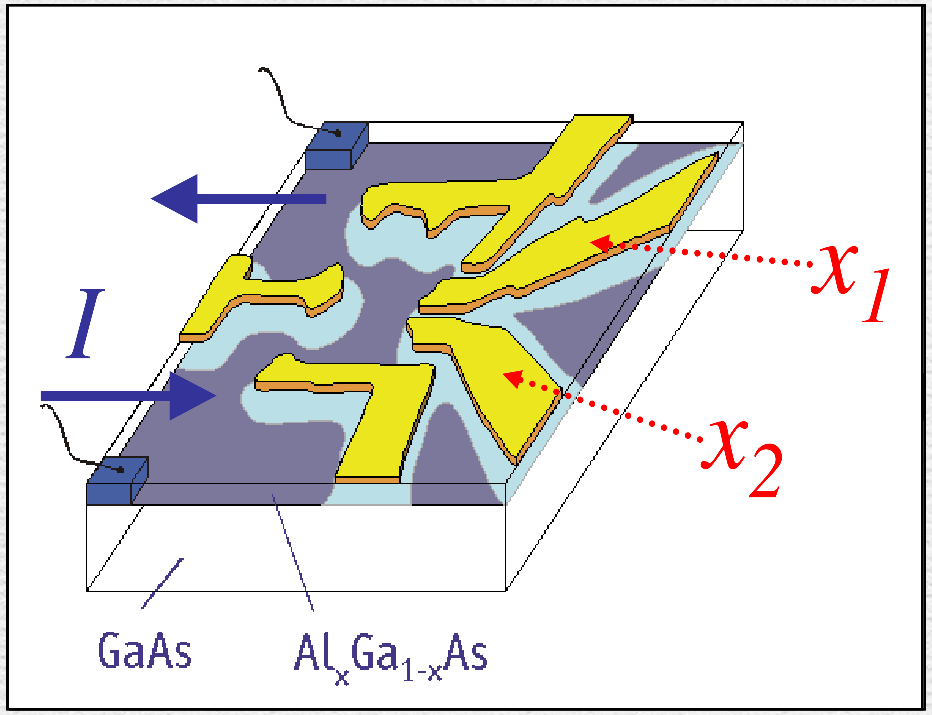
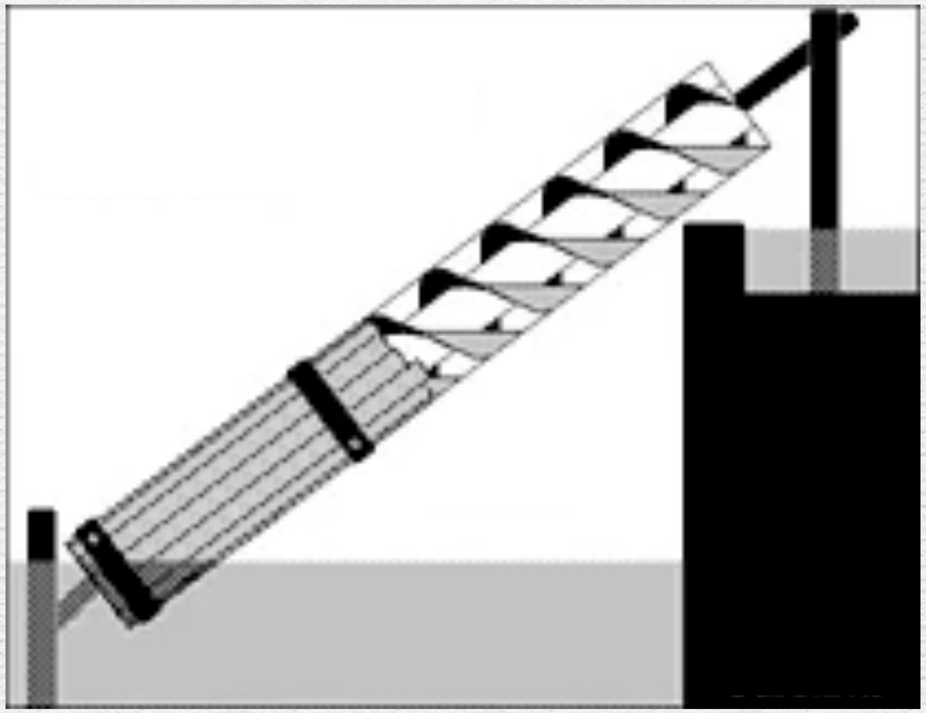


Switkes *et al* 1999

Buttiker, Brouwer, Zhou, Spivak, Altshuler ...



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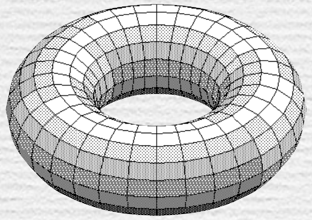
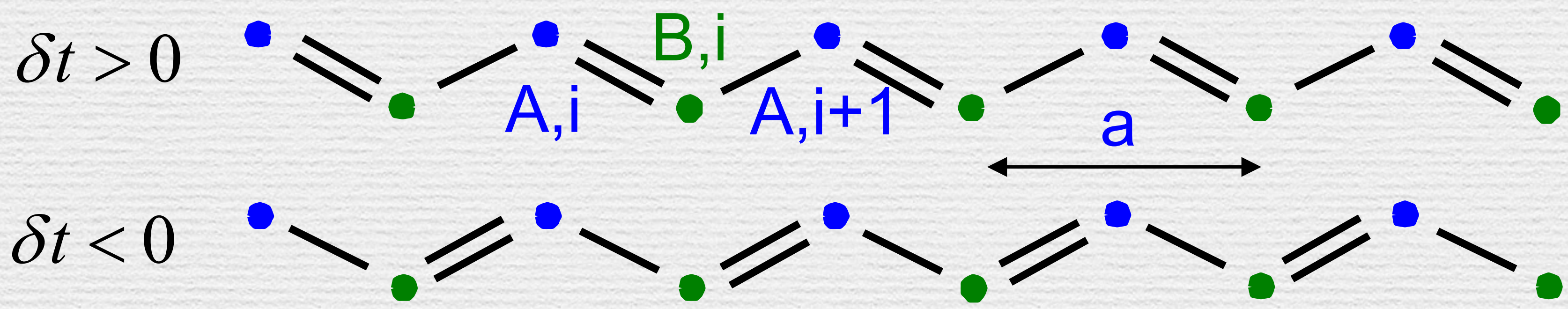


A **topological pump** transfers quantized charge in each pumping cycle.

Thouless, Niu, 1980s

$$H = \sum_i (t+\delta t) c^\dagger_{Ai} c_{Bi} + (t-\delta t) c^\dagger_{Ai+1} c_{Bi} + H.c.$$

Su, Schrieffer, Heeger, 1979

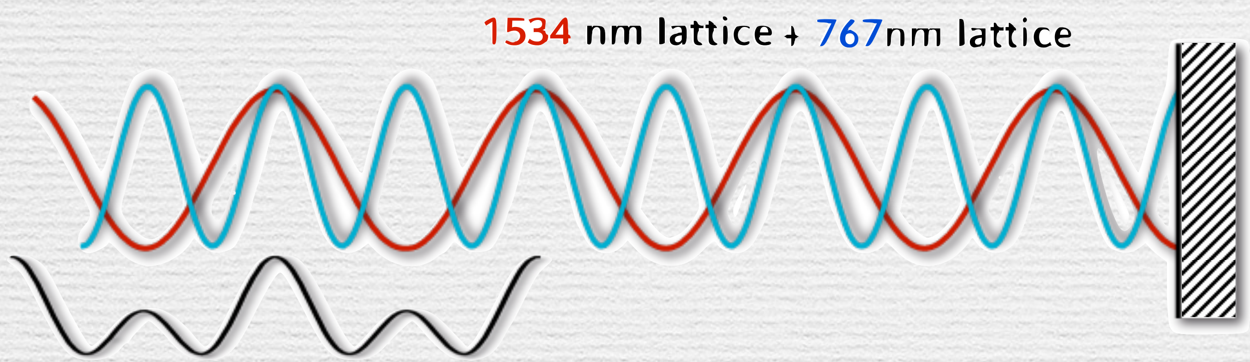


- Current flows in an insulating state
- No dissipation!
- Dynamical analog of quantum Hall effect

Experimental progresses

Optical Superlattice

Fölling *et al*, Atala *et al*

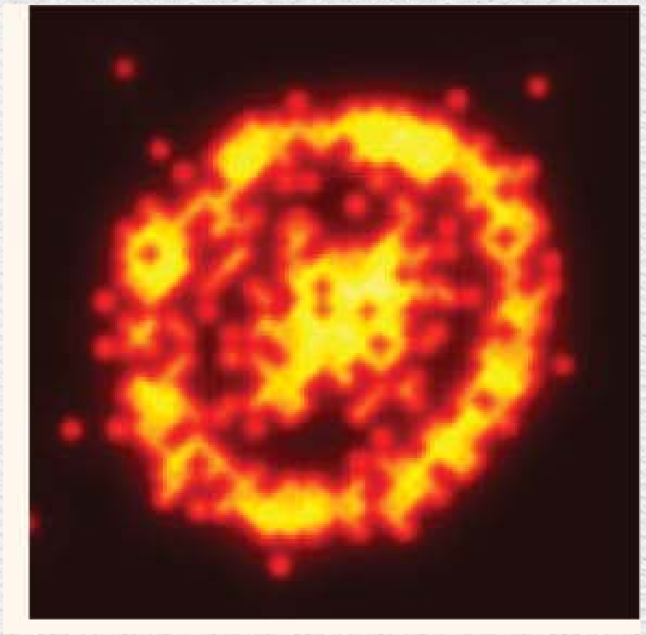


$$V_{\rm OL}(x) = V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \varphi\right)$$

Full (independent) dynamical control over V_1, V_2 and $\boldsymbol{\varphi}$

in-situ imaging

Gemelke, *et al*, Sherson *et al*, Bakr *et al*



1D pumping lattices

$$V_{\mathrm{OL}}(x,t) = V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \frac{\pi t}{T}\right)$$

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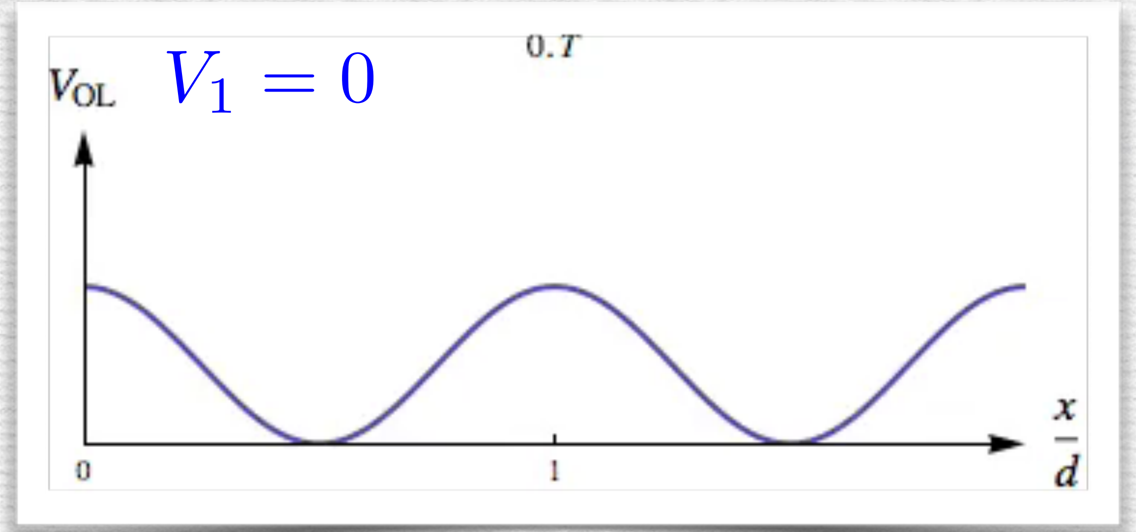

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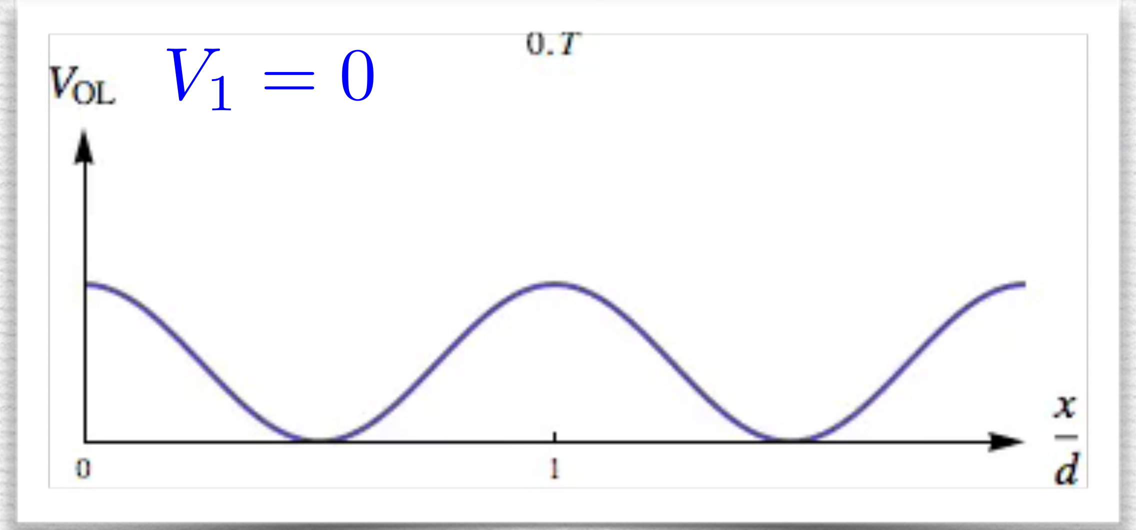

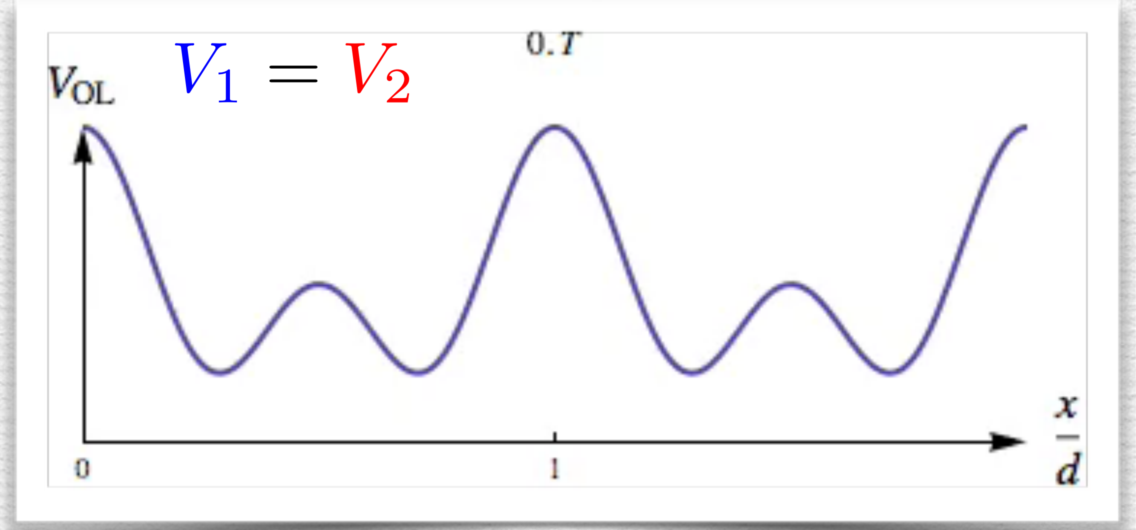

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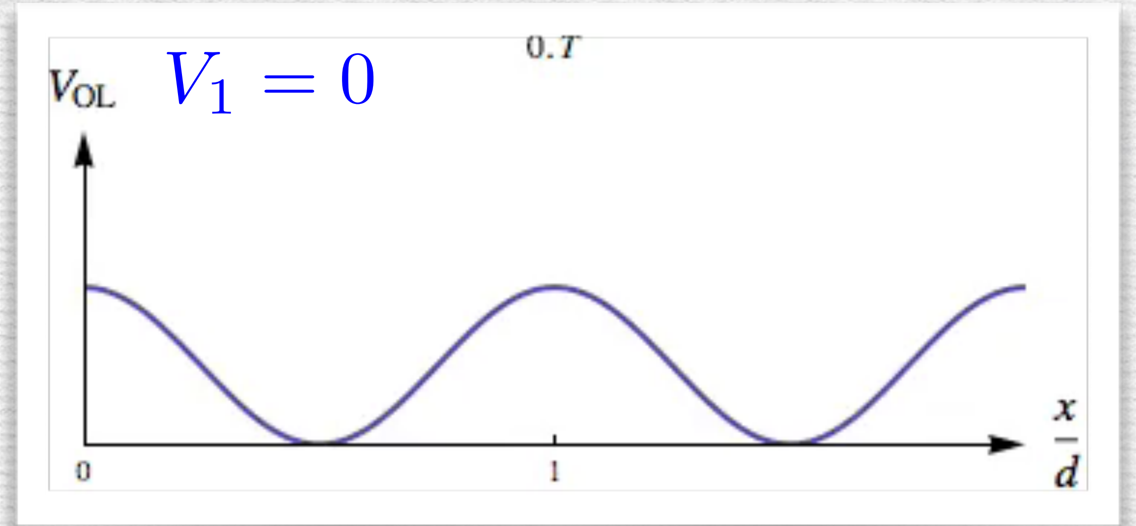

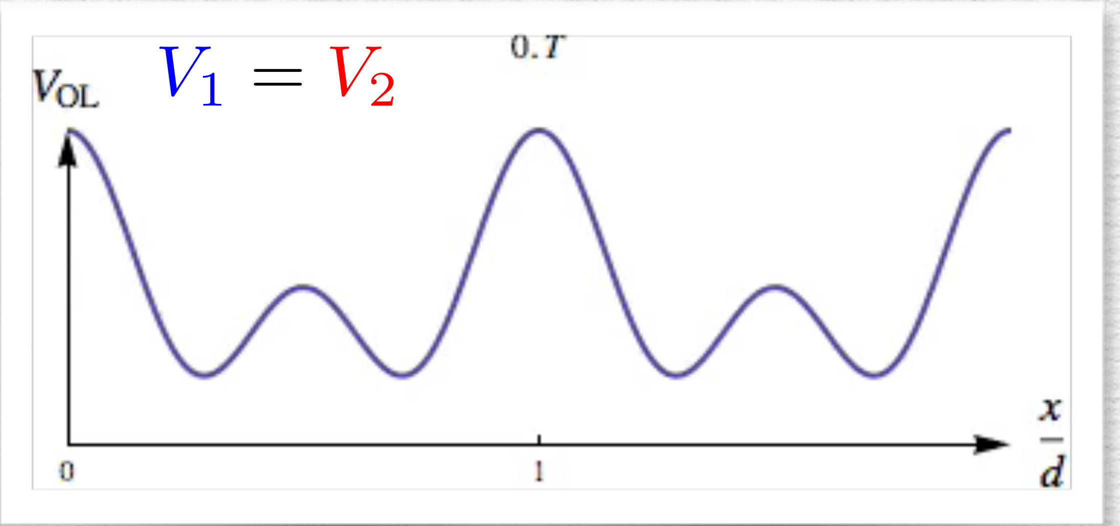

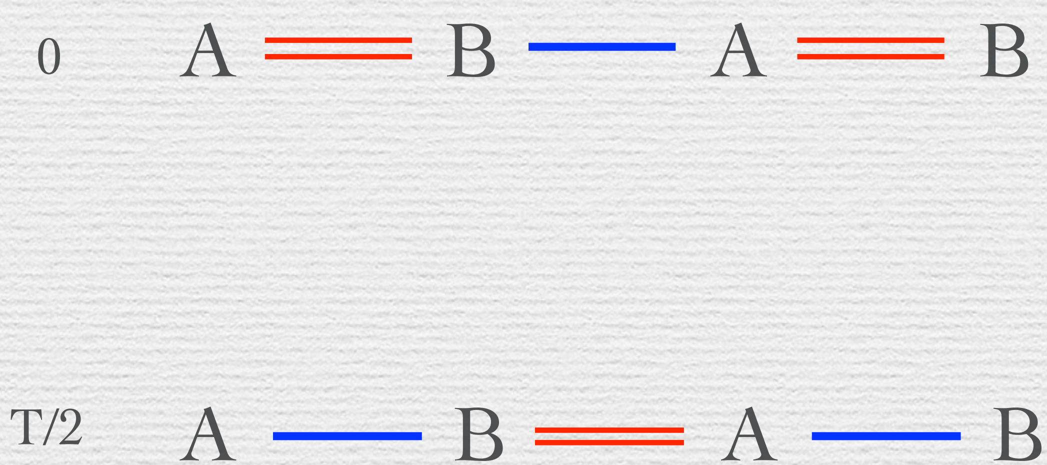

Su, Schrieffer, Heeger, 1979

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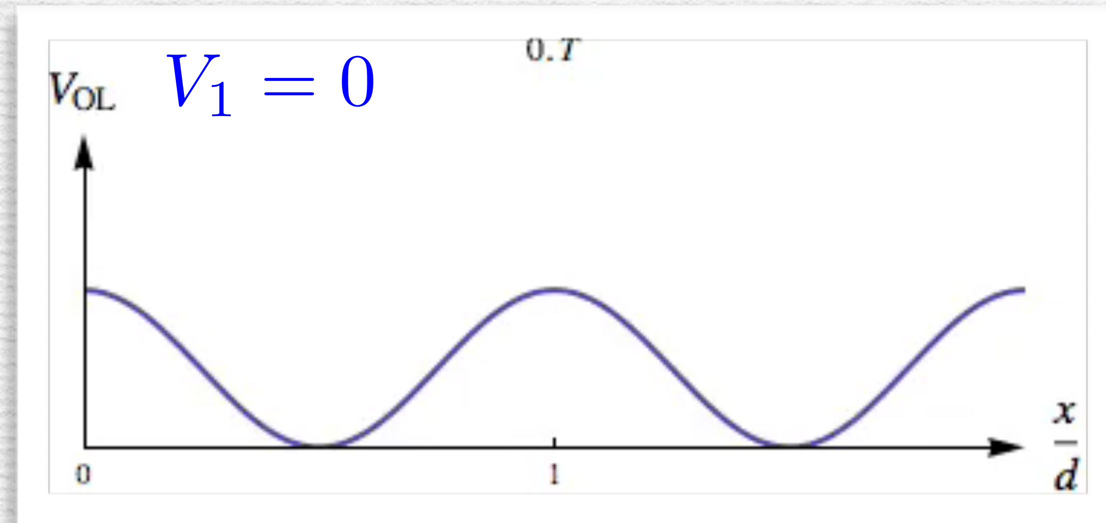
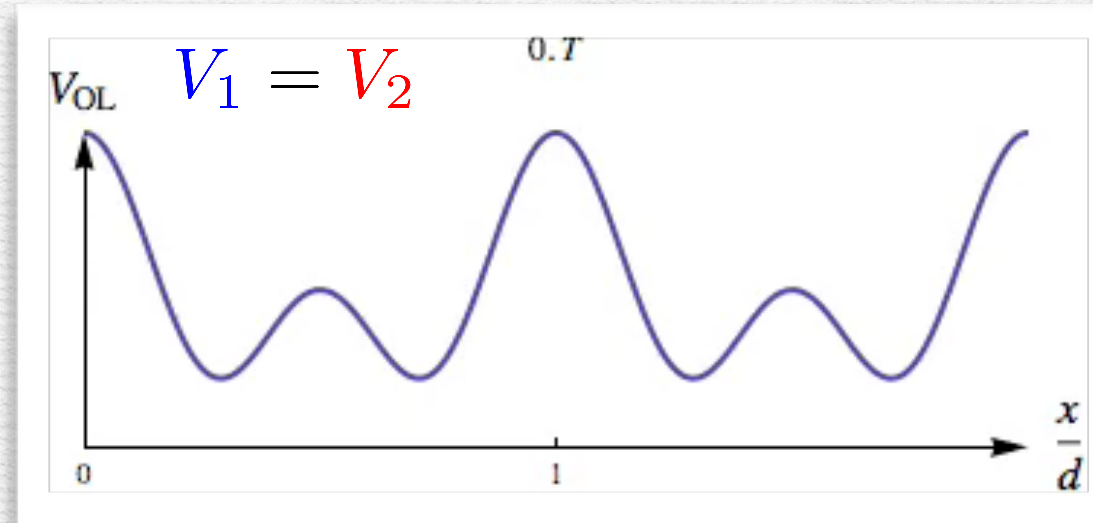
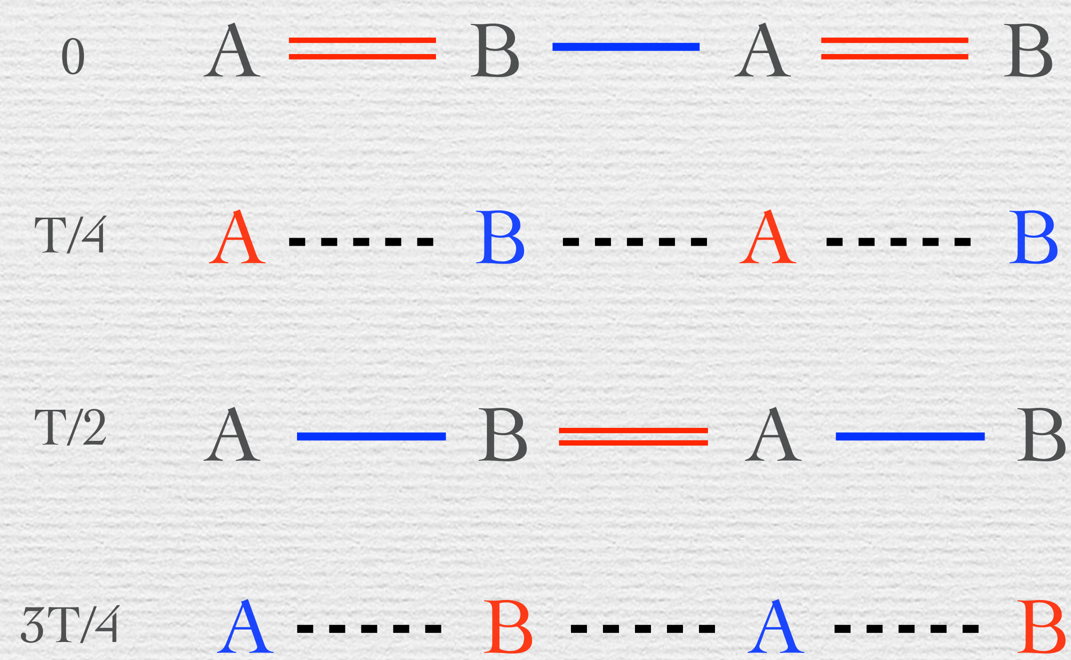


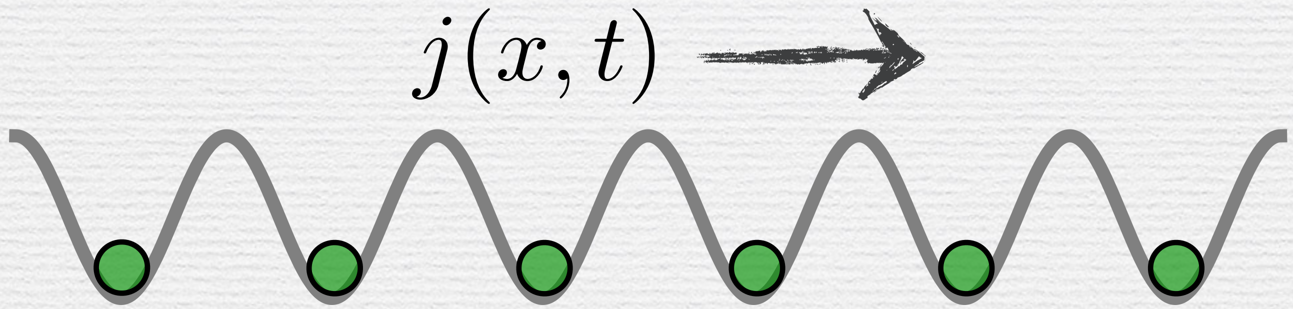




Su, Schrieffer, Heeger, 1979

Rice, Mele, 1982

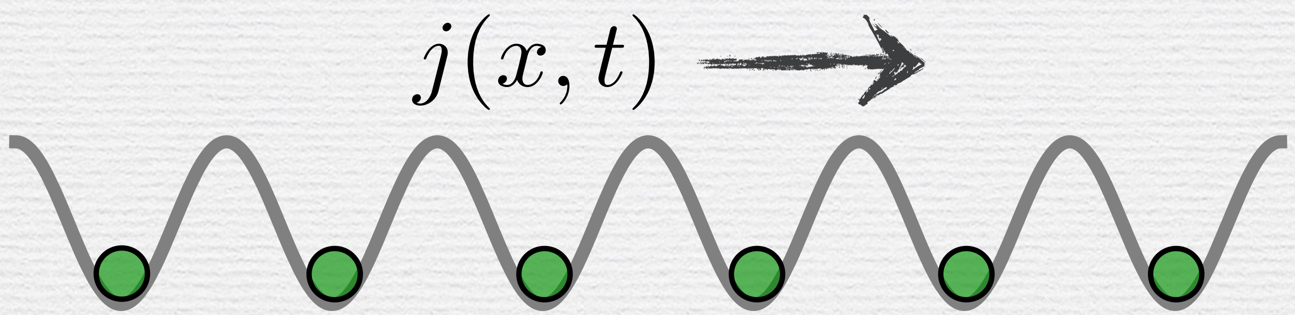
Quantum dynamics

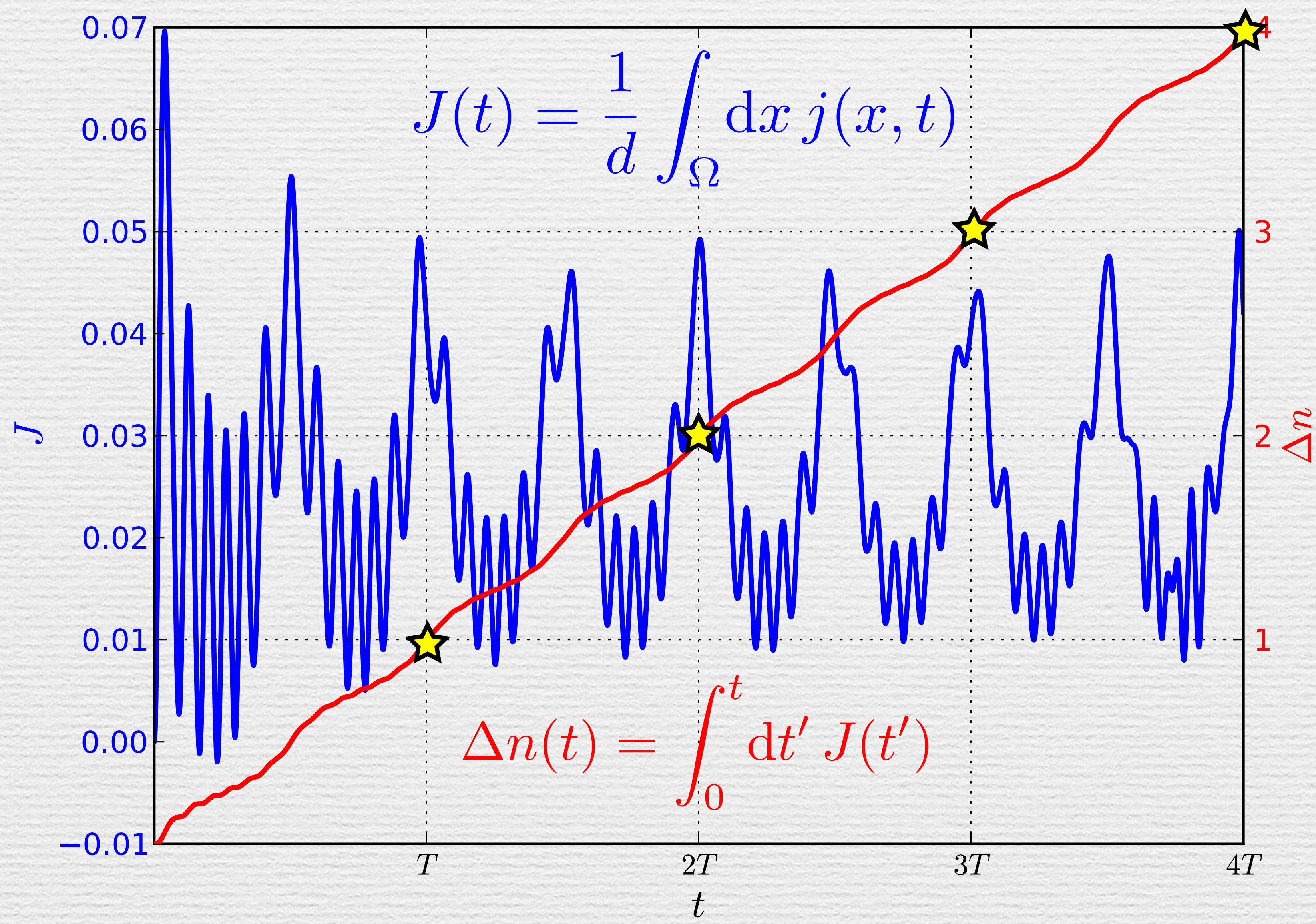


$$H(x,t) = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{OL}}(x,t)$$

$$i\frac{\partial}{\partial t}|\Psi\rangle = H(x,t)|\Psi\rangle$$

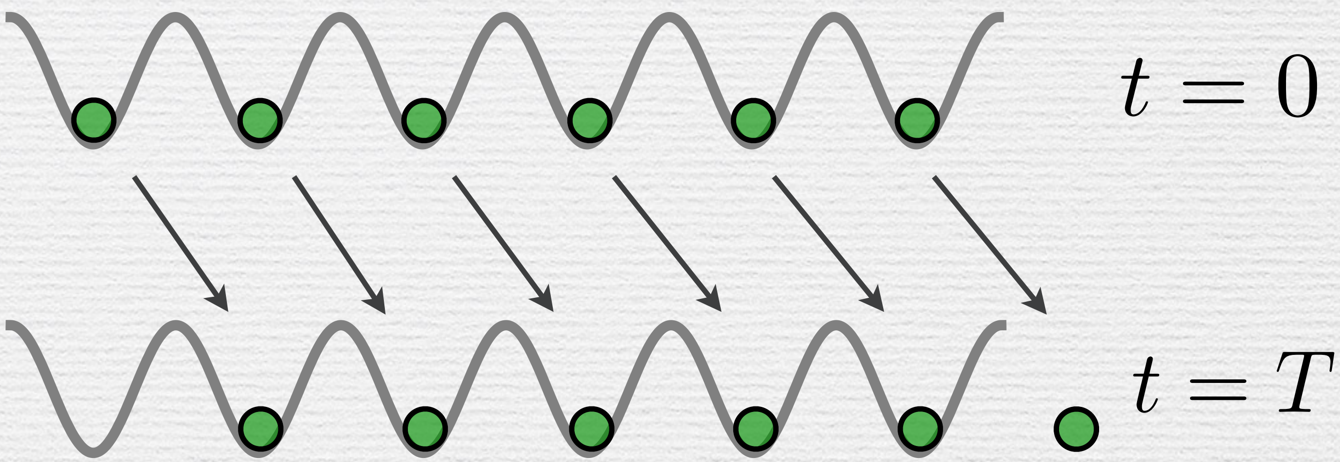
Quantum dynamics

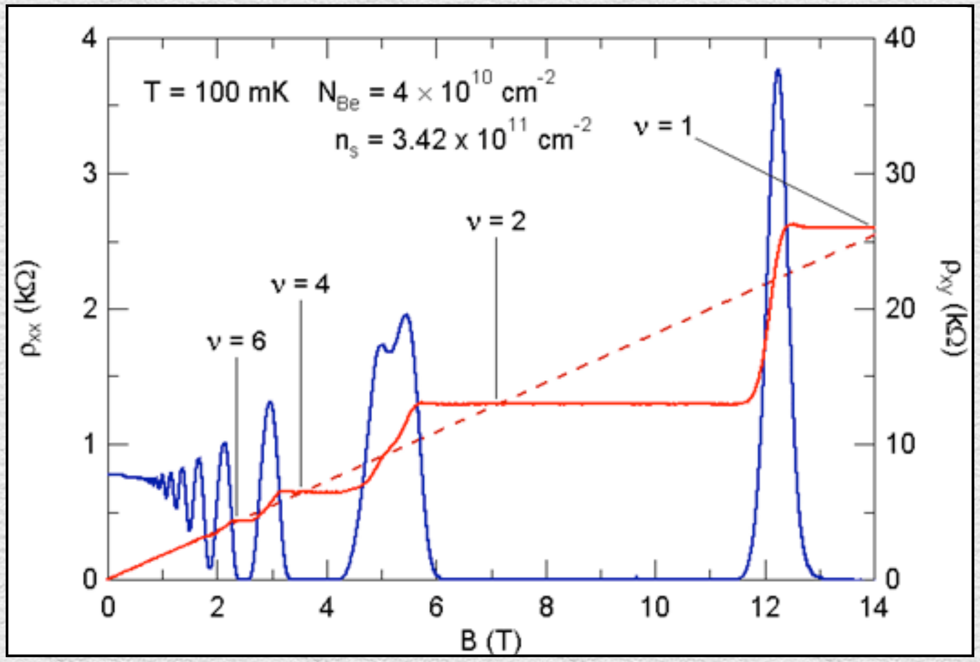




1D pump and 2D QHE

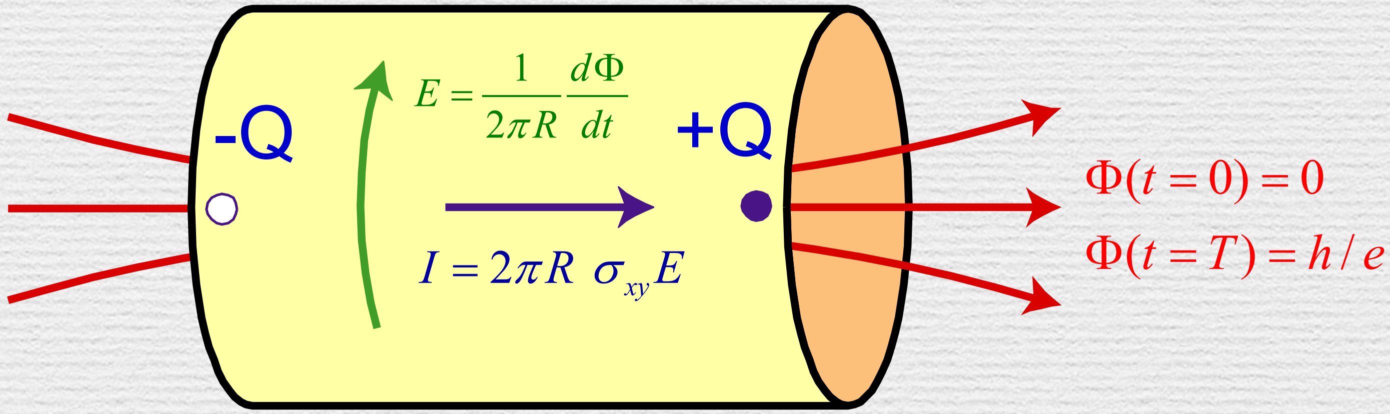
$$H(k_x, t) = H(k_x, t+T)$$





Von Klitzing *et al*, 1980

Adiabatically thread a quantum of magnetic flux through cylinder.



Laughlin, 1981

Gap & Chern number

$V_1 = 0 \quad V_2 = 4E_R$

$$H(x,t) = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{OL}}(x,t)$$

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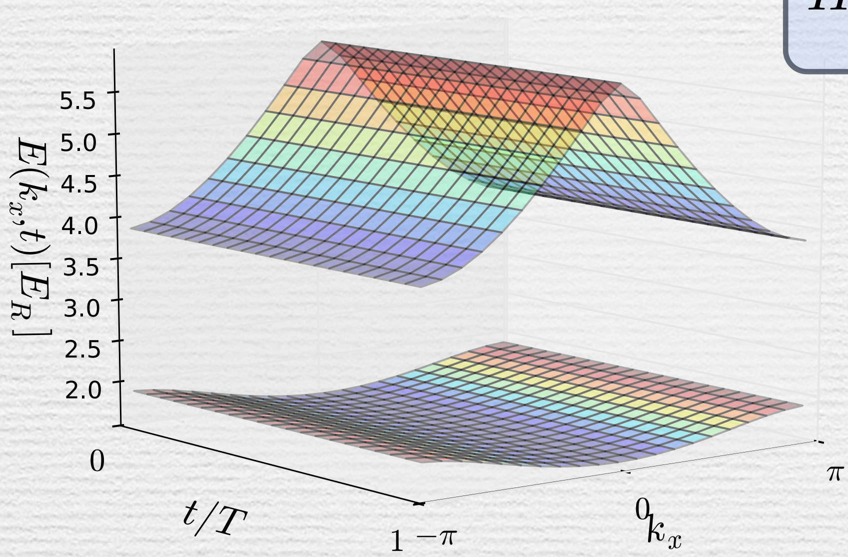

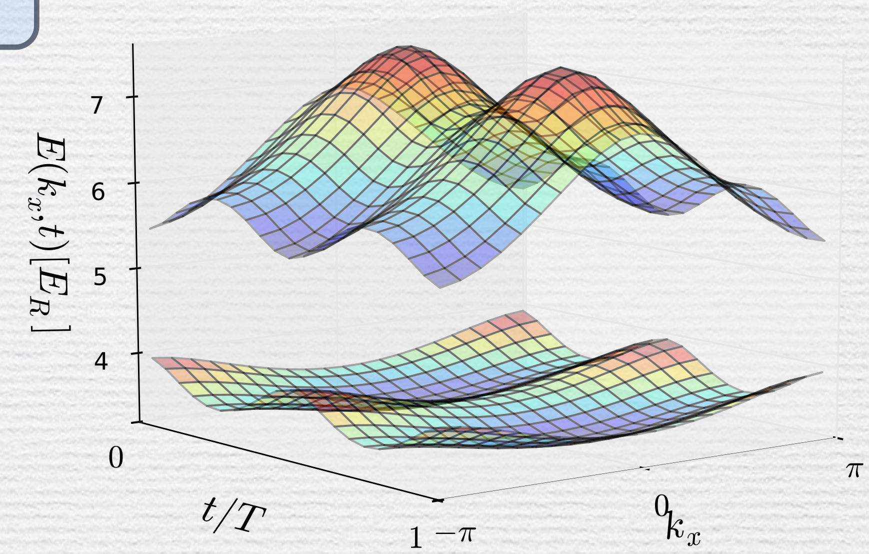

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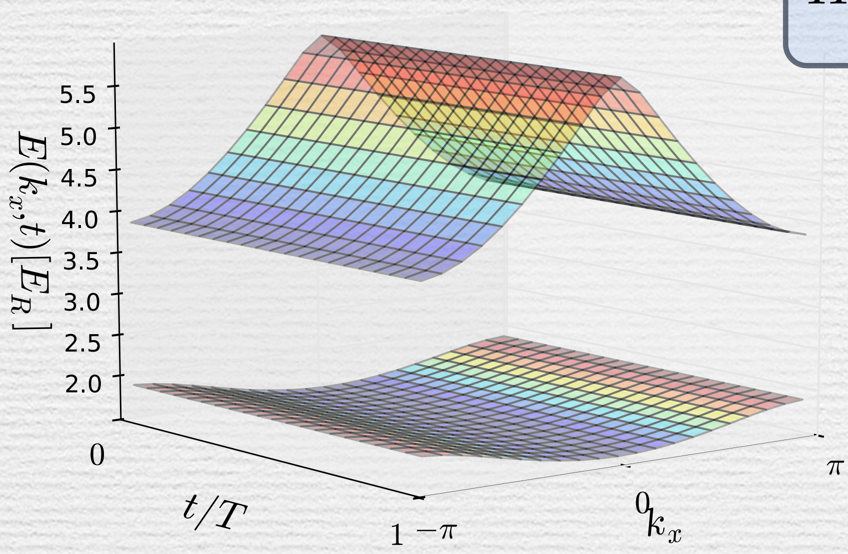

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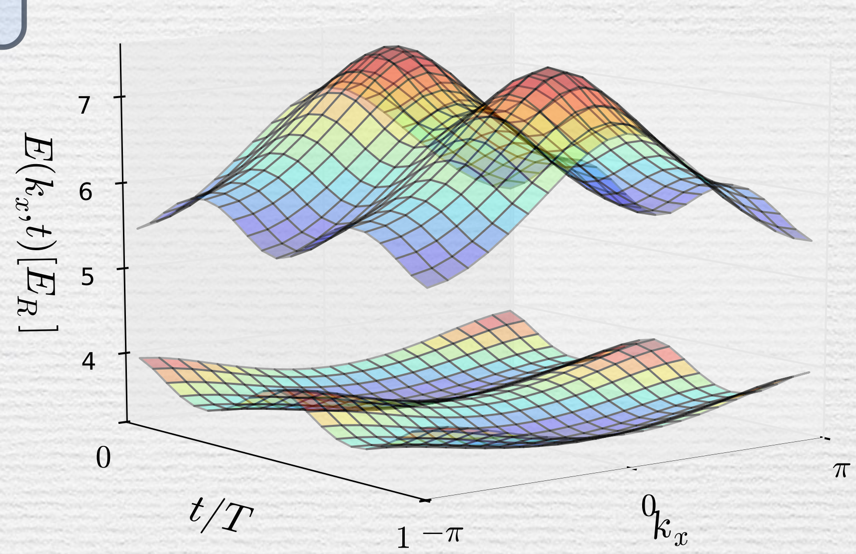

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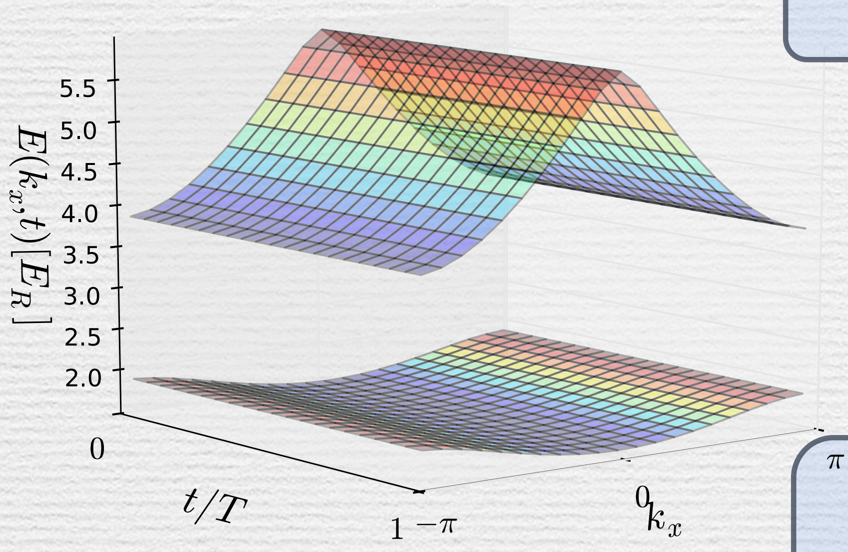

$$H(x,t) = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{OL}}(x,t)$$

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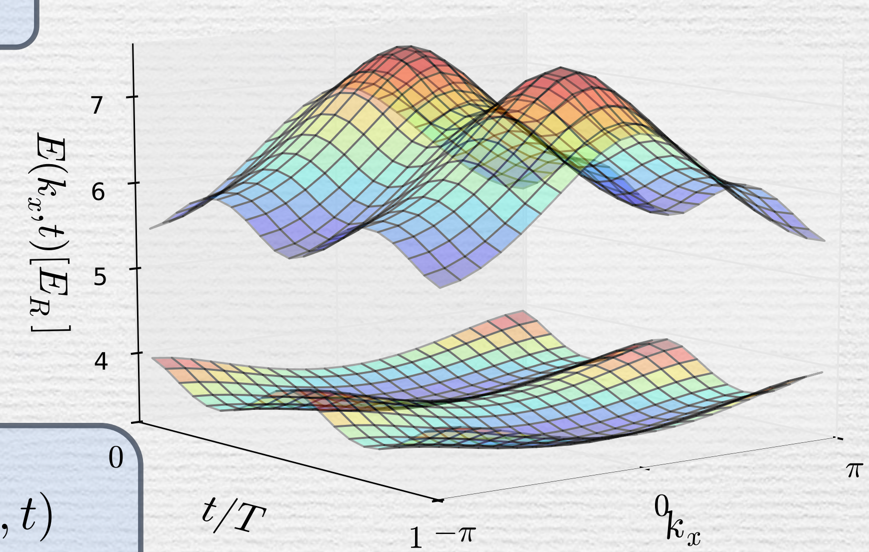

$$\Delta n = \frac{1}{2\pi}\int_0^T \mathrm{d}t \int_0^{2\pi} \mathrm{d}k_x\, \mathcal{F}(k_x,t)$$

$$\mathcal{F}(k_x,t) = \partial_t A_{k_x} - \partial_{k_x} A_t$$

$$A_{t(k_x)} = -i\langle\psi_{k_x}(t)|\partial_{t(k_x)}|\psi_{k_x}(t)\rangle$$

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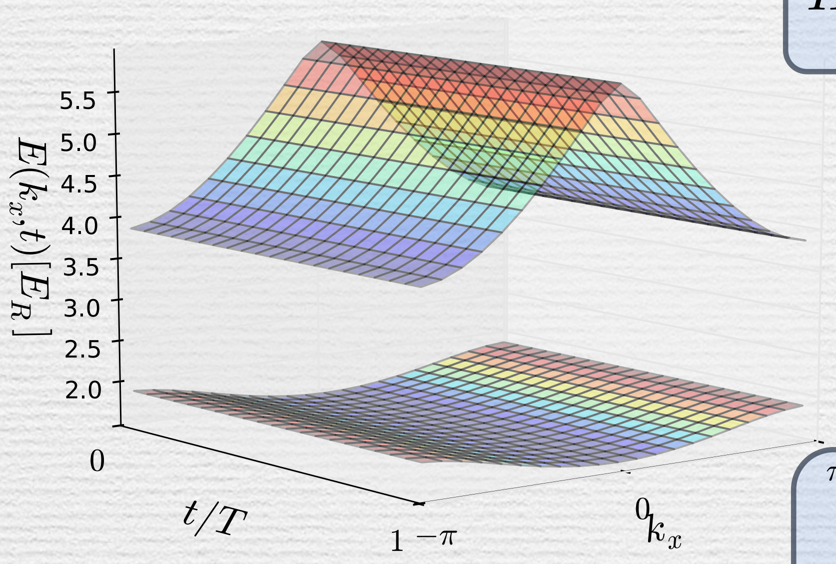

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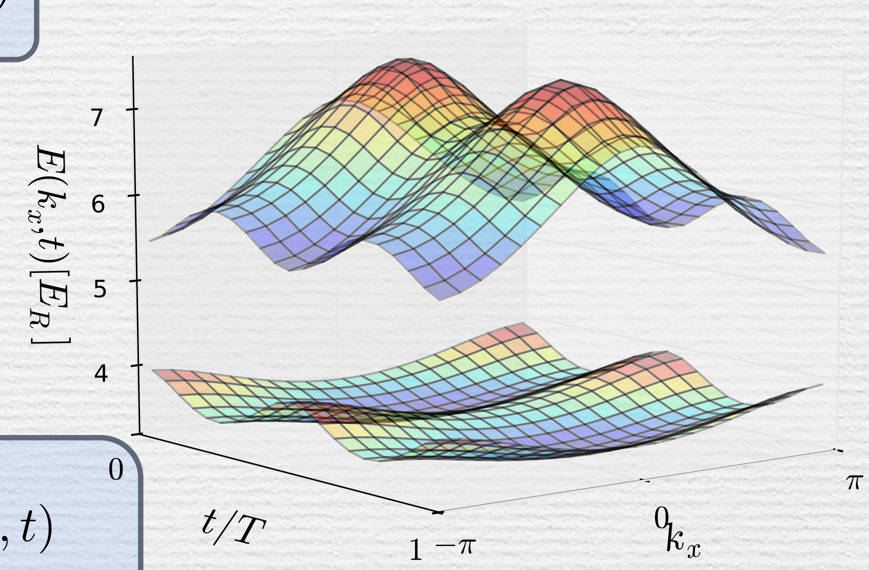

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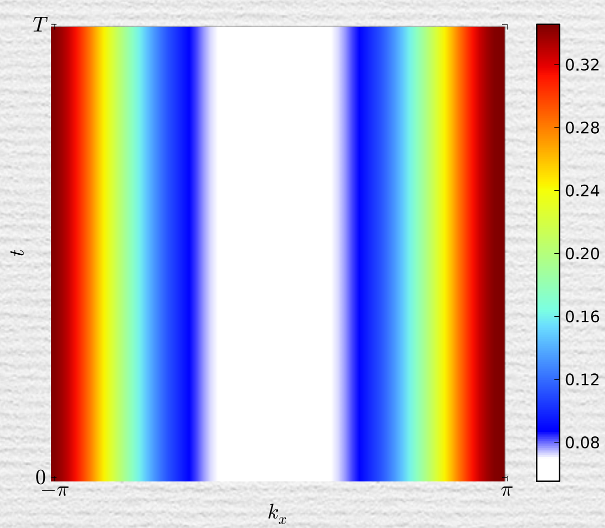

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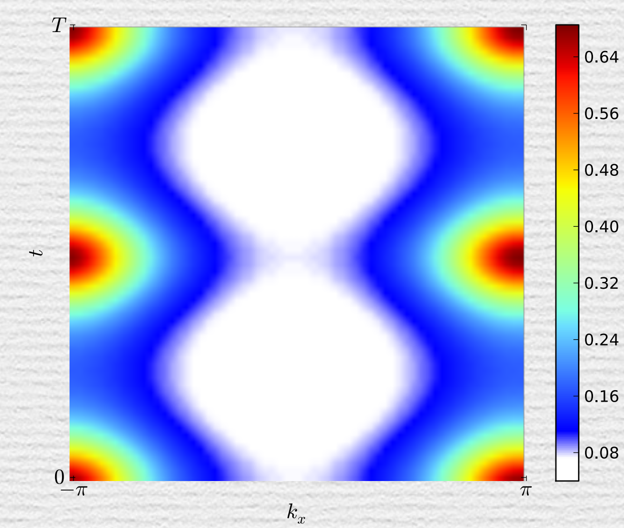

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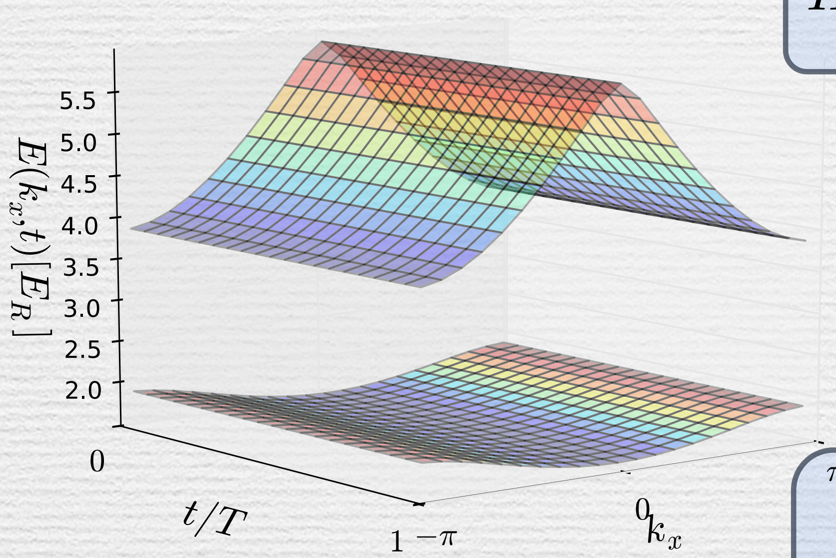

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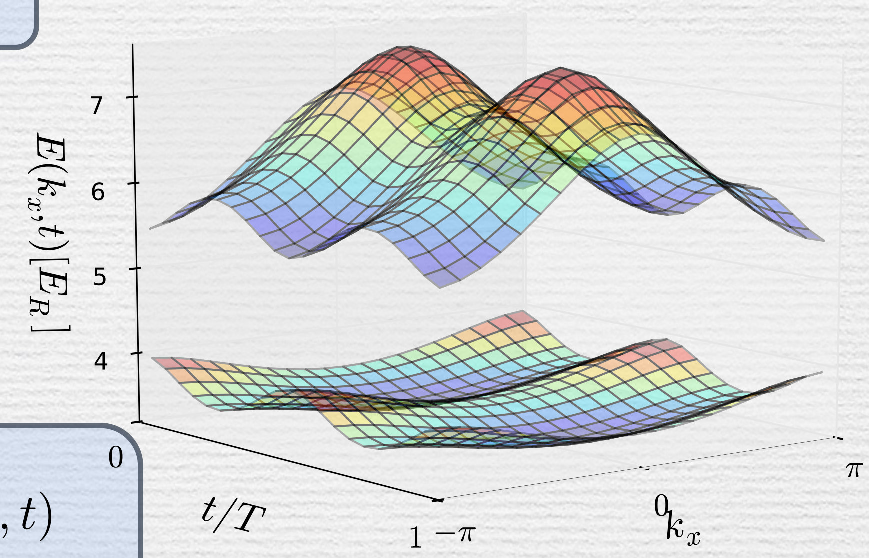

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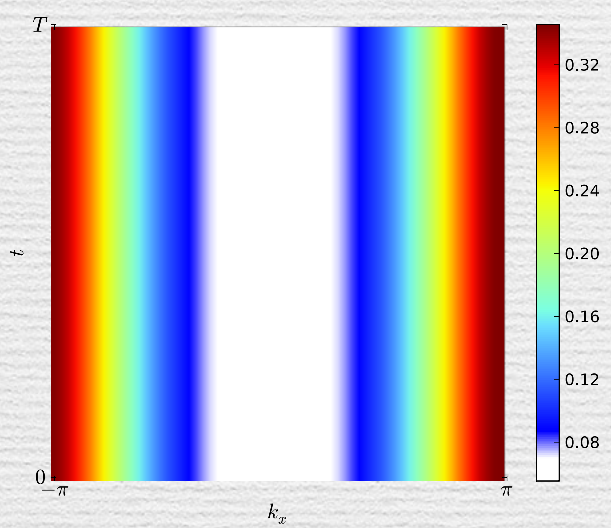

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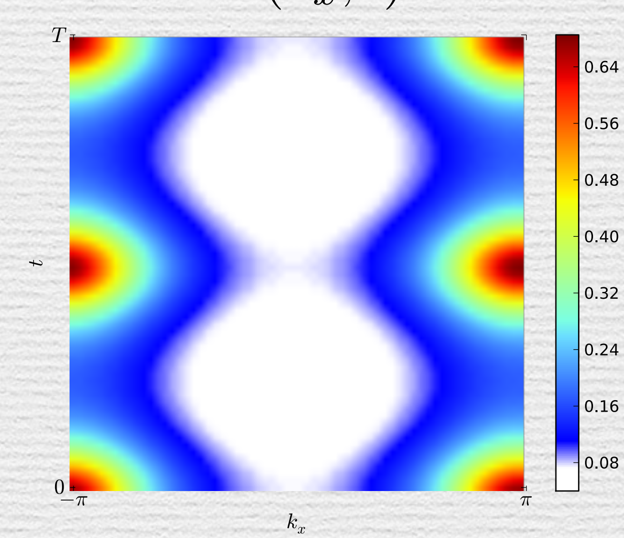

$$\Delta n = 1$$

Gap & Chern number

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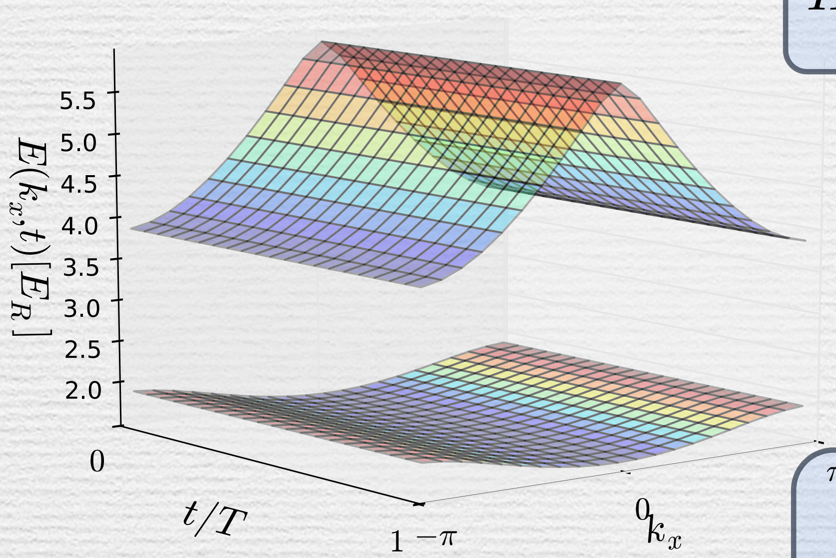

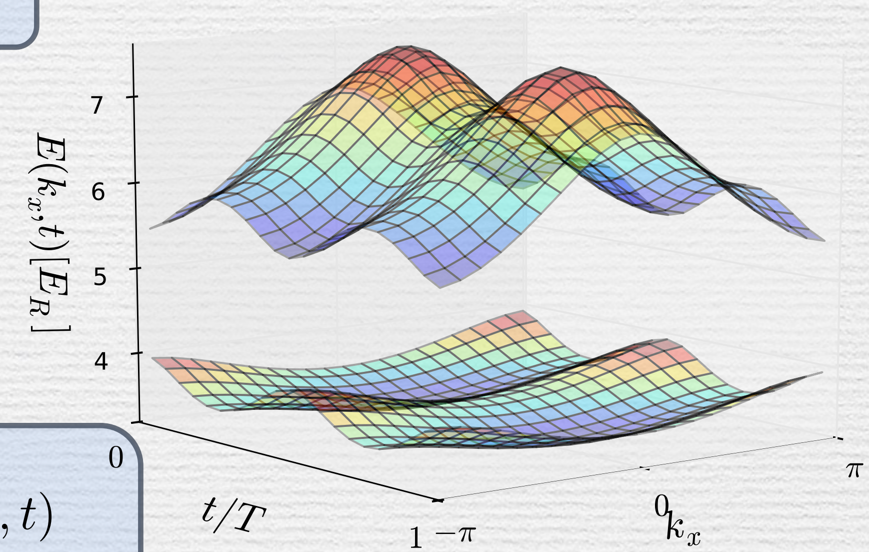

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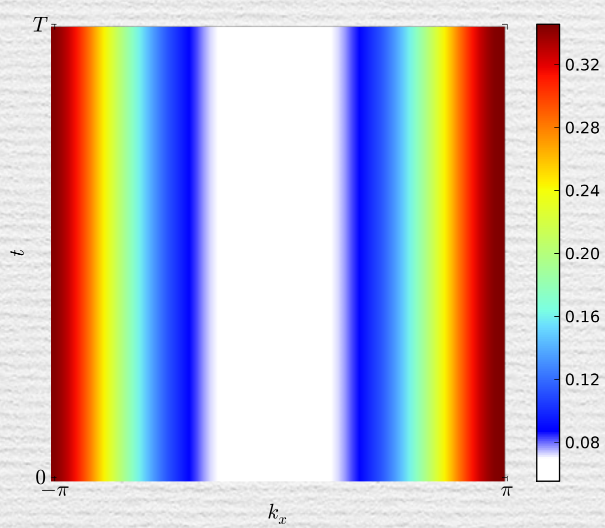

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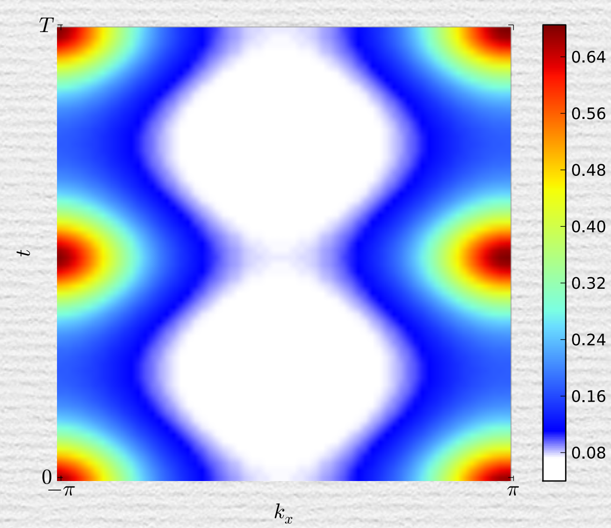


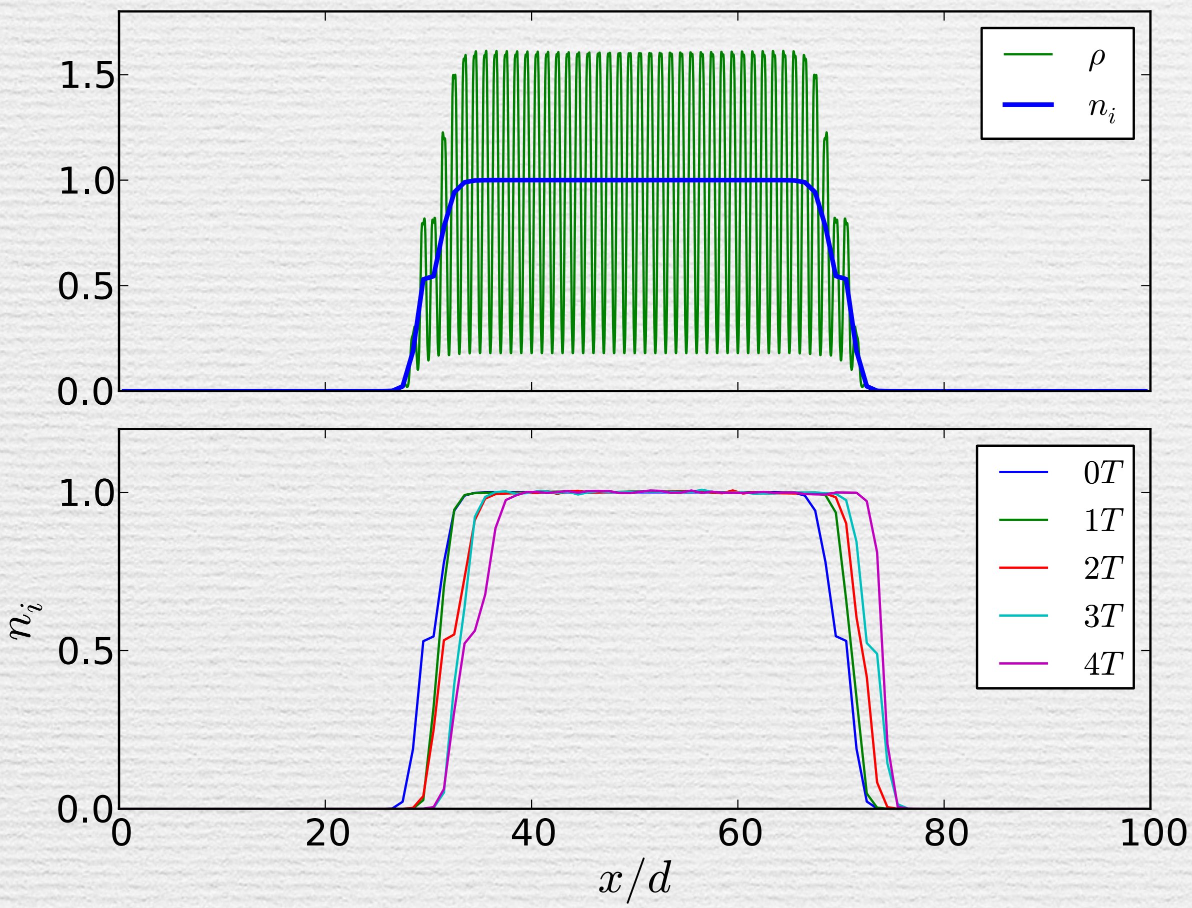
Topological pump

$$\Delta n = 1$$

Practical issues

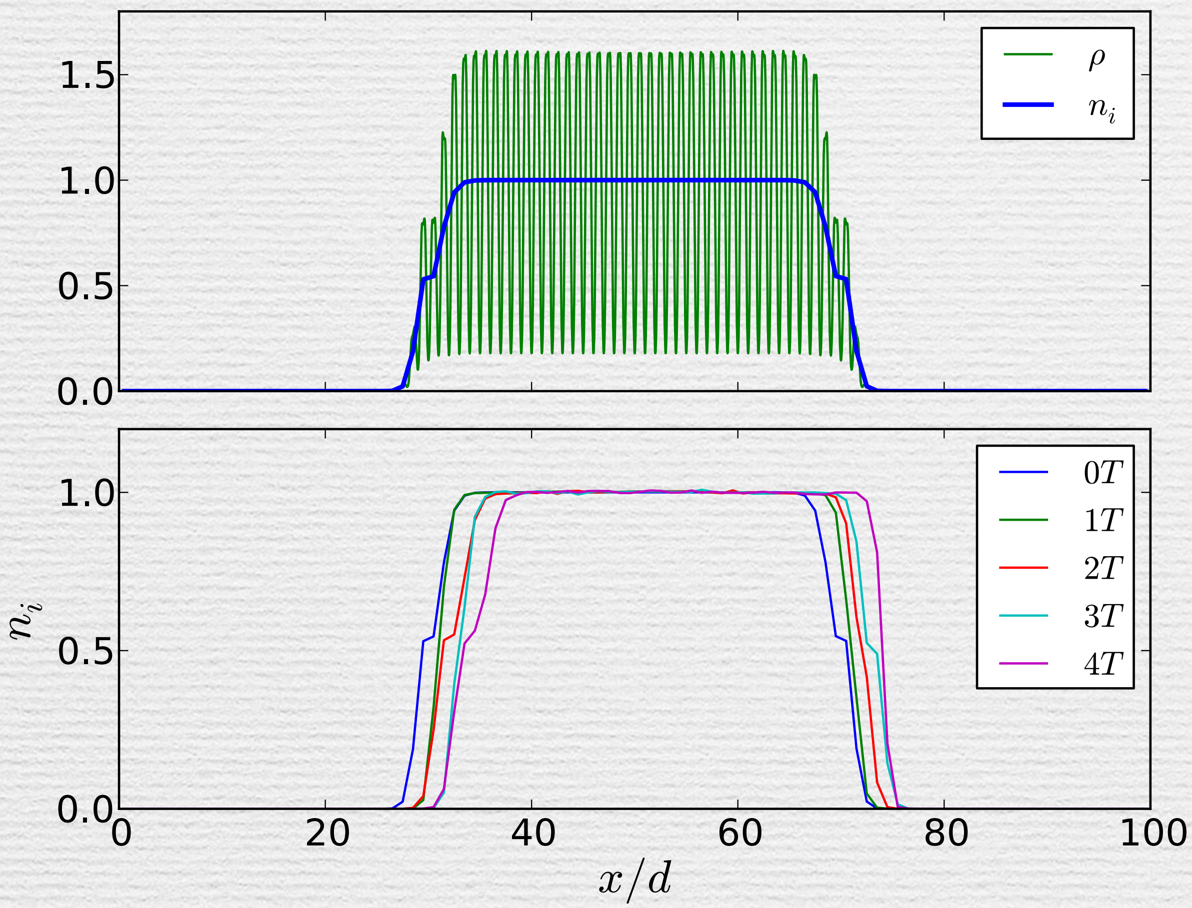
- Detection
- External trap
- Temperature effect
- Non-adiabatic effect

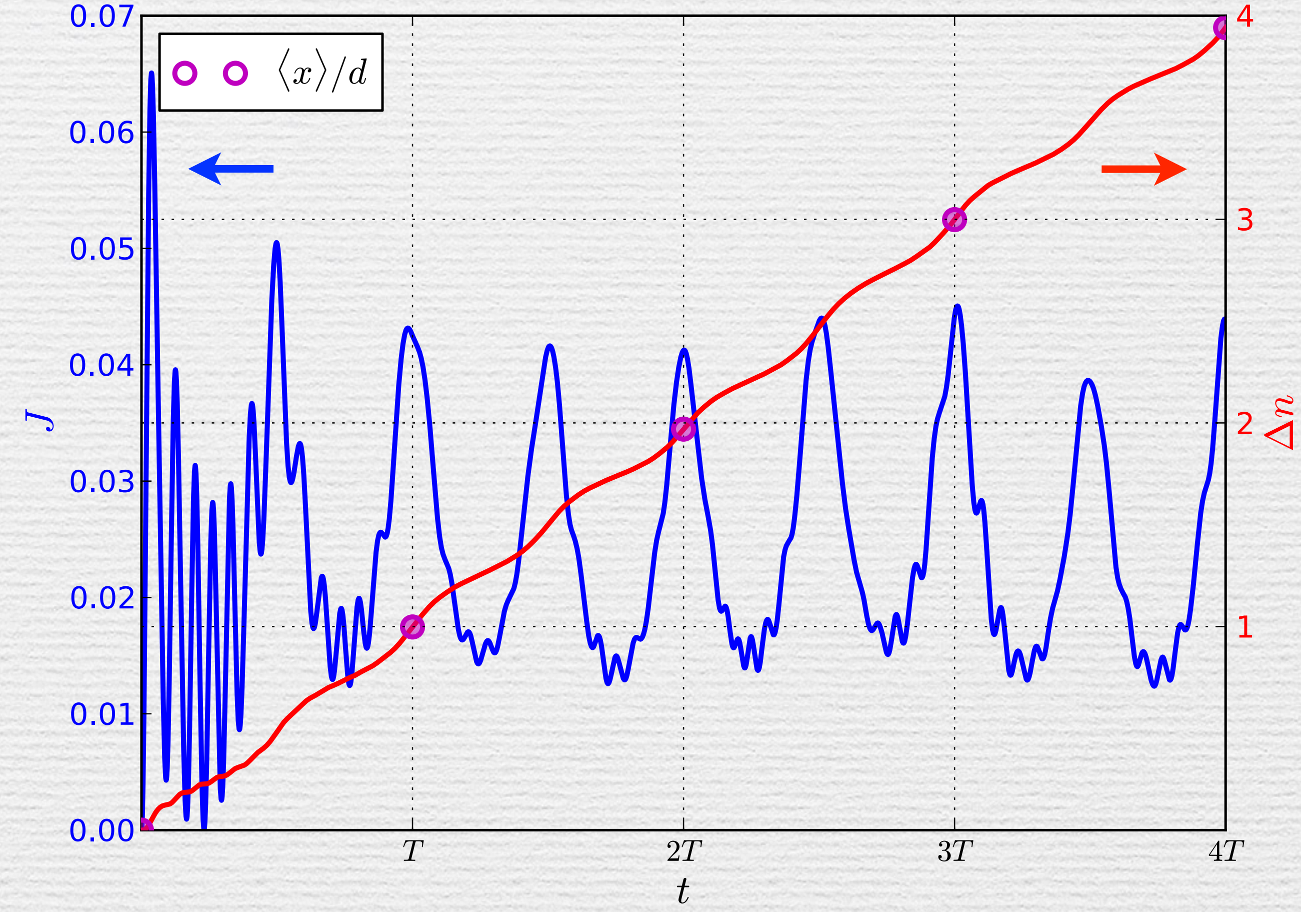
Trapping & Detection



$$\langle x \rangle / d = \Delta n$$

Trapping & Detection





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Temperature & Non-adiabatic effect

$$\text{Temperature} \ll \frac{\Delta}{k_B} \qquad\qquad T \gg \frac{\hbar}{\Delta}$$

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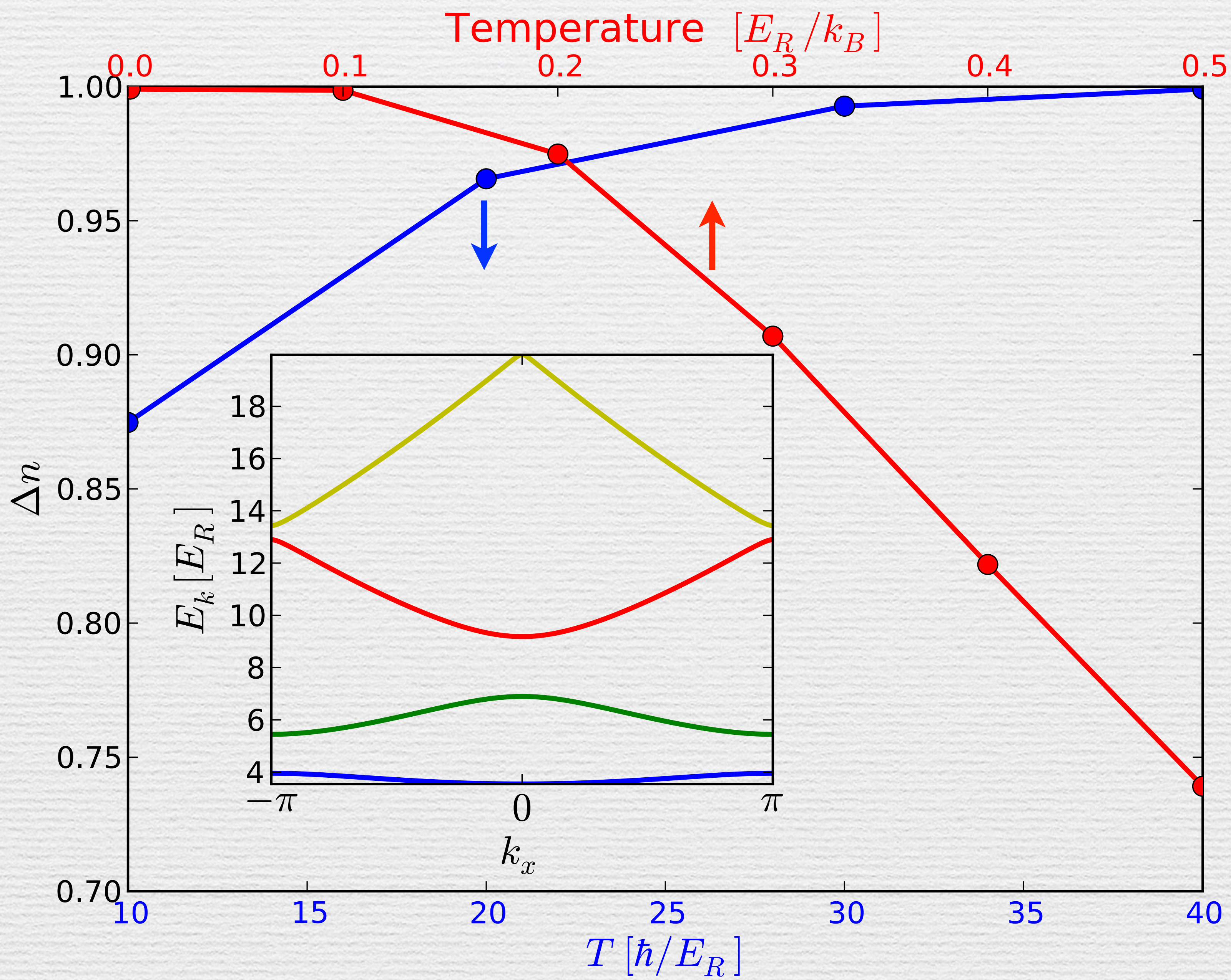
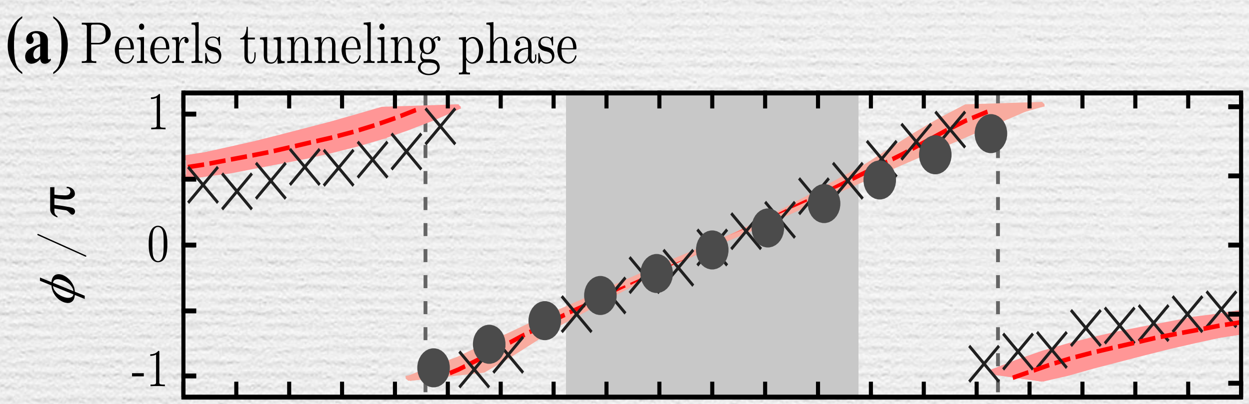


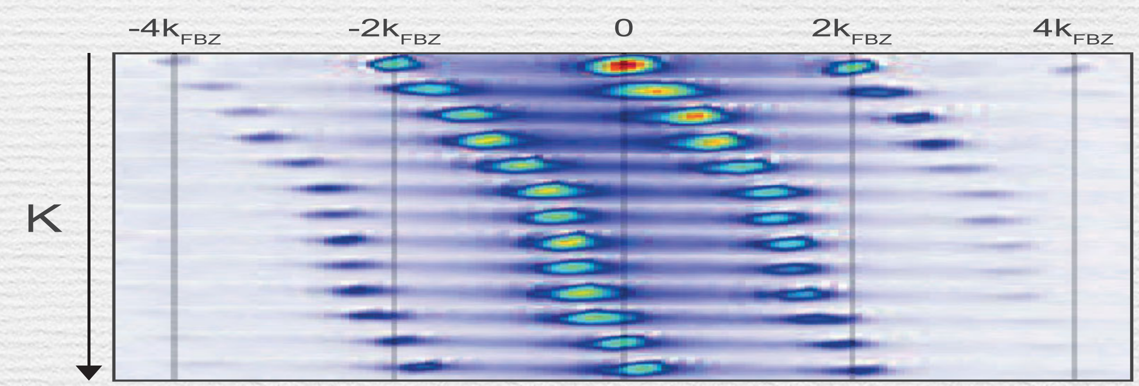
Measuring Chern number from topological charge pumping

Synthetic gauge-field in optical lattices

- Imprint complex phases to the hopping amplitude

- 1D Peierls lattice NIST, Hamburg $$H = -J\sum_m e^{i2\pi\Phi} c^\dagger_{m+1} c_m + H.c.$$

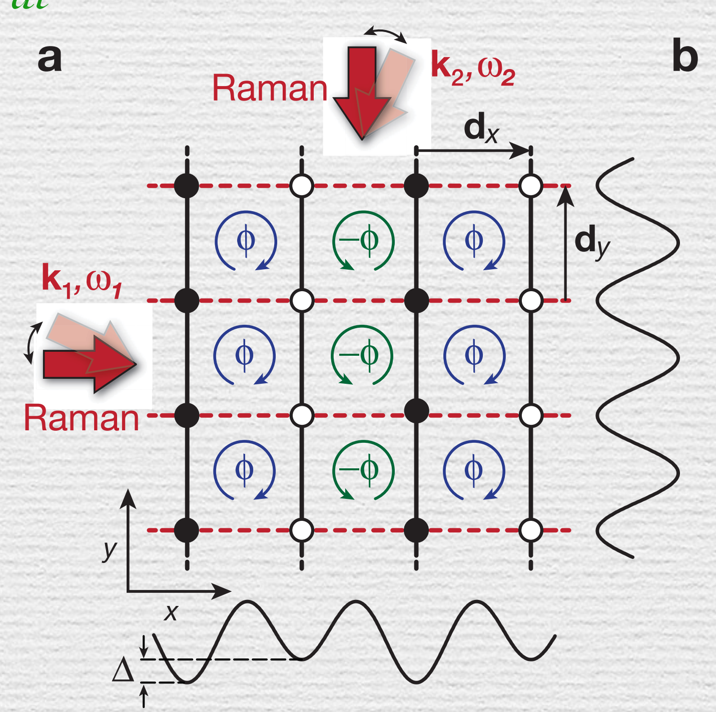




Jimenez-Garcıa *et al*

Struck *et al*

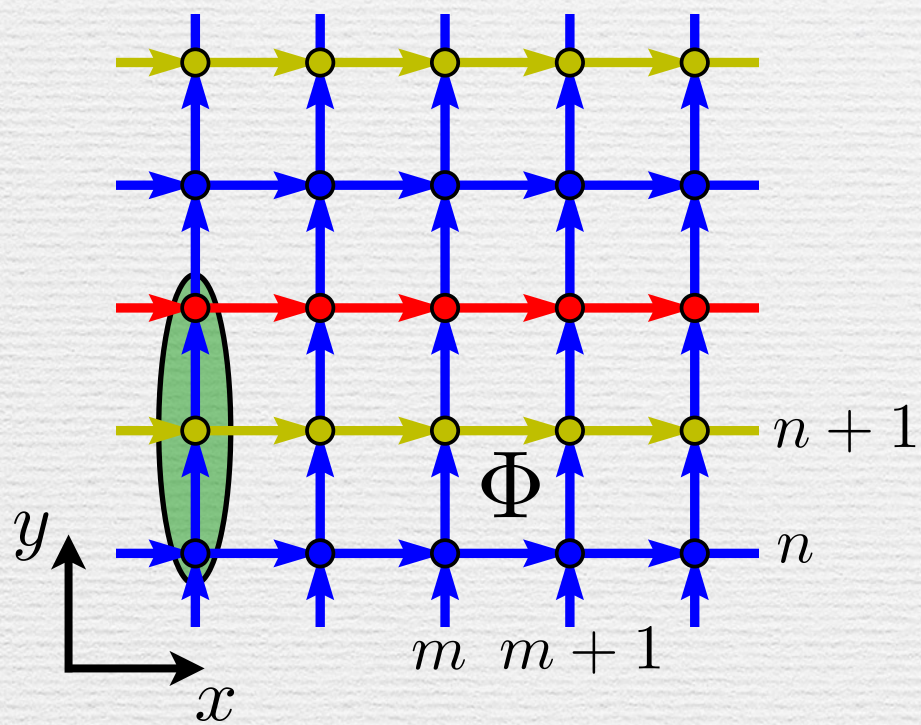
- Staggered flux lattice Munich



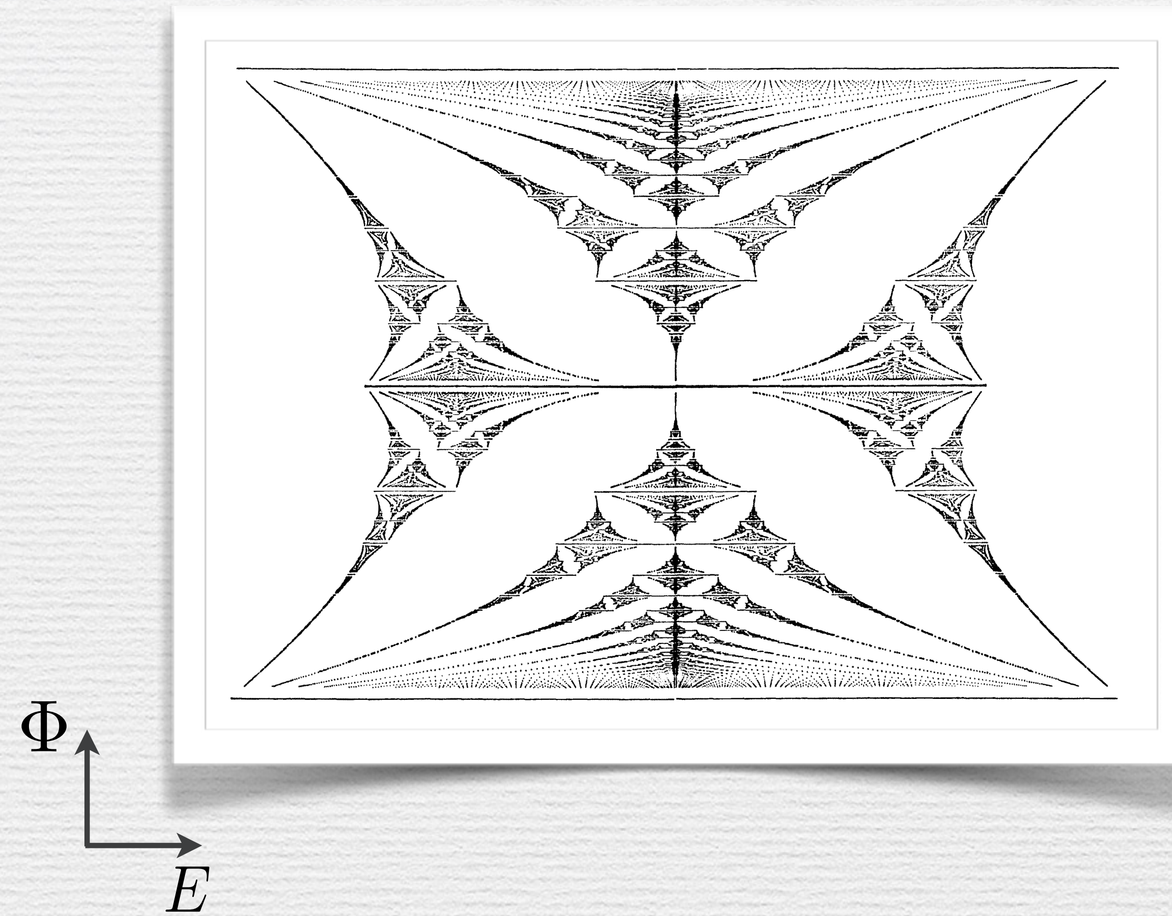
Aidelsburger *et al*

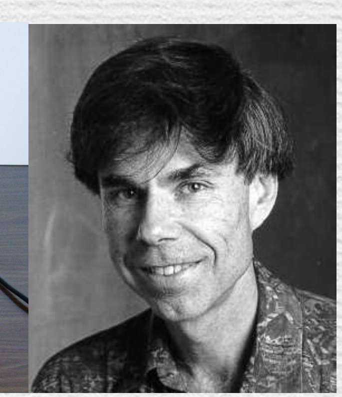
Hofstadter optical lattice

$$H = -J \sum_{m,n} e^{i2\pi n\Phi} c^\dagger_{m+1,n} c_{m,n} + c^\dagger_{m,n+1} c_{m,n} + H.c. \qquad \Phi = p/q$$



Hofstadter, 1976





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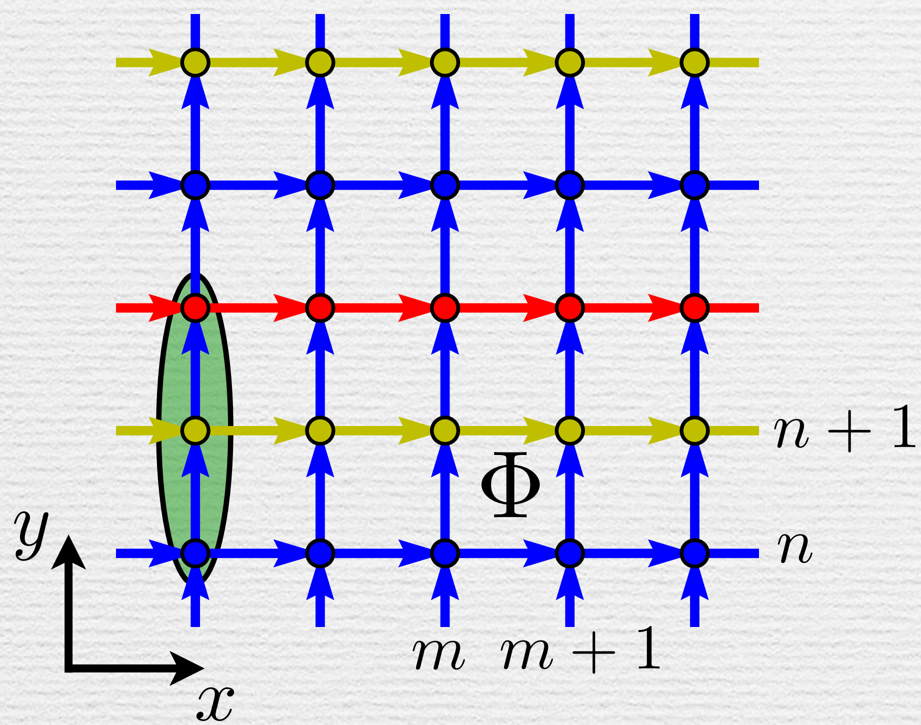

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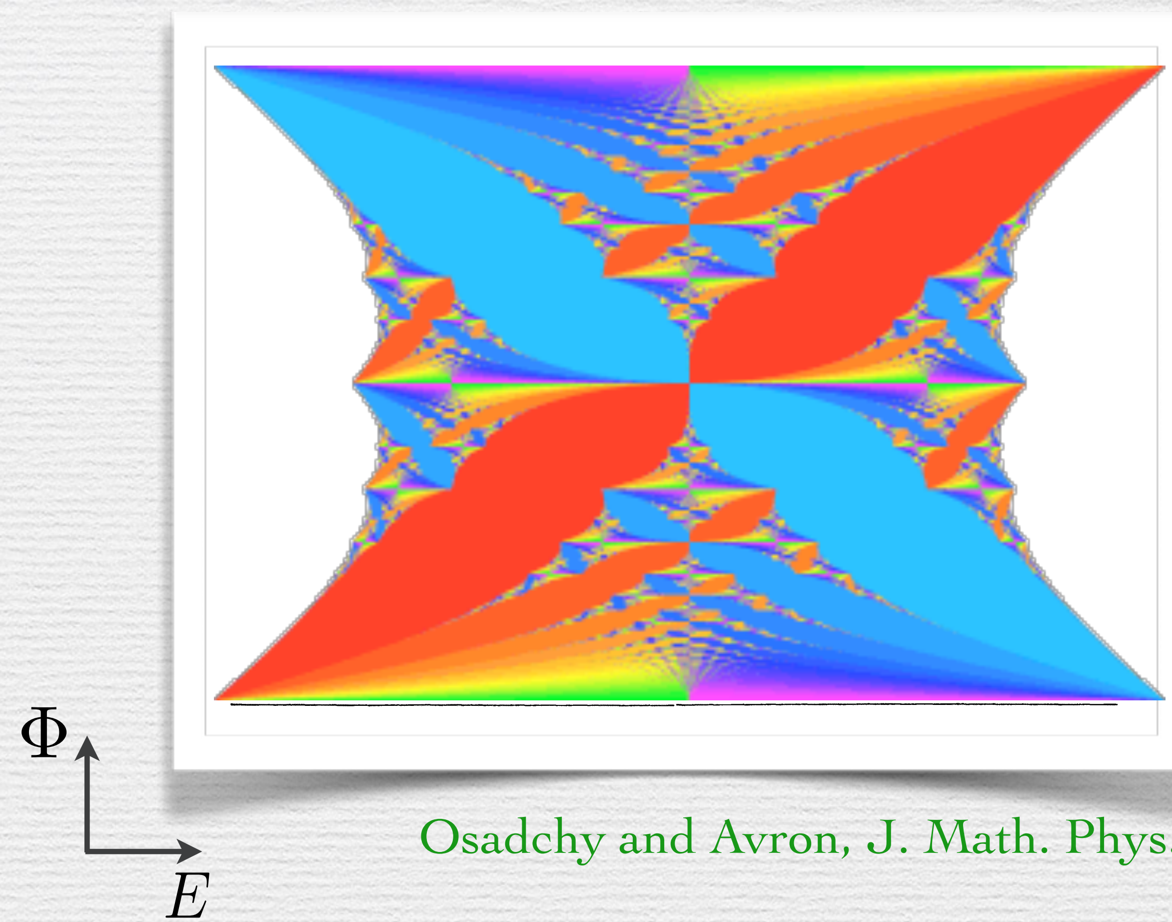
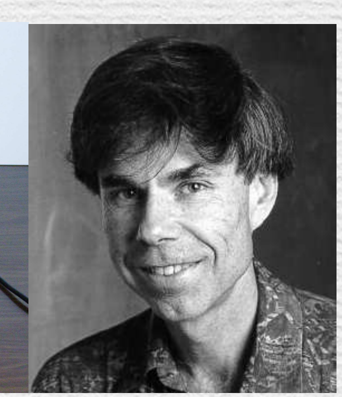


Osadchy and Avron, J. Math. Phys. 2001



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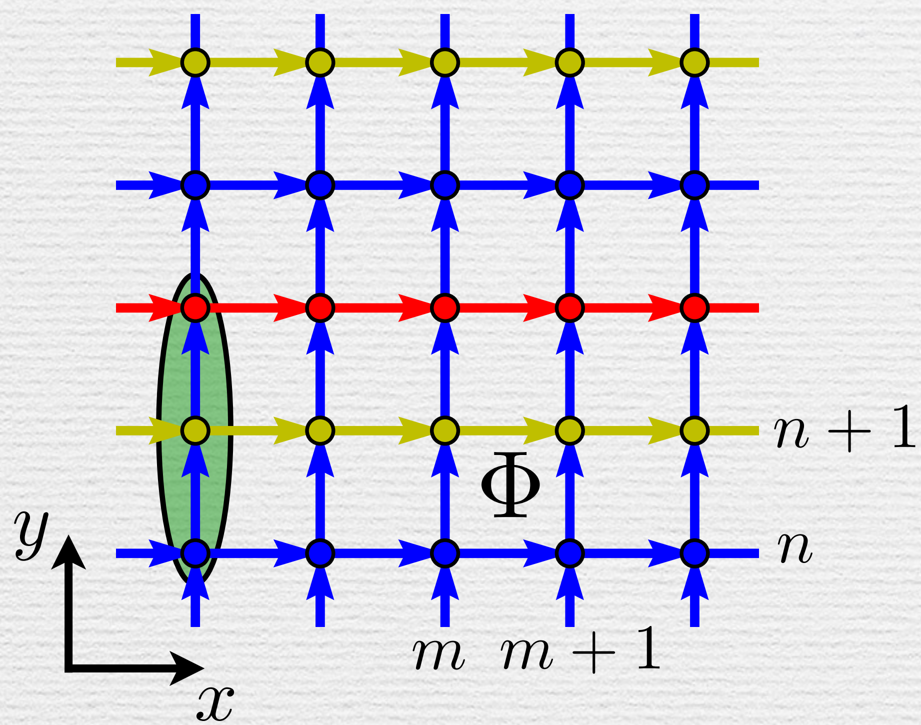

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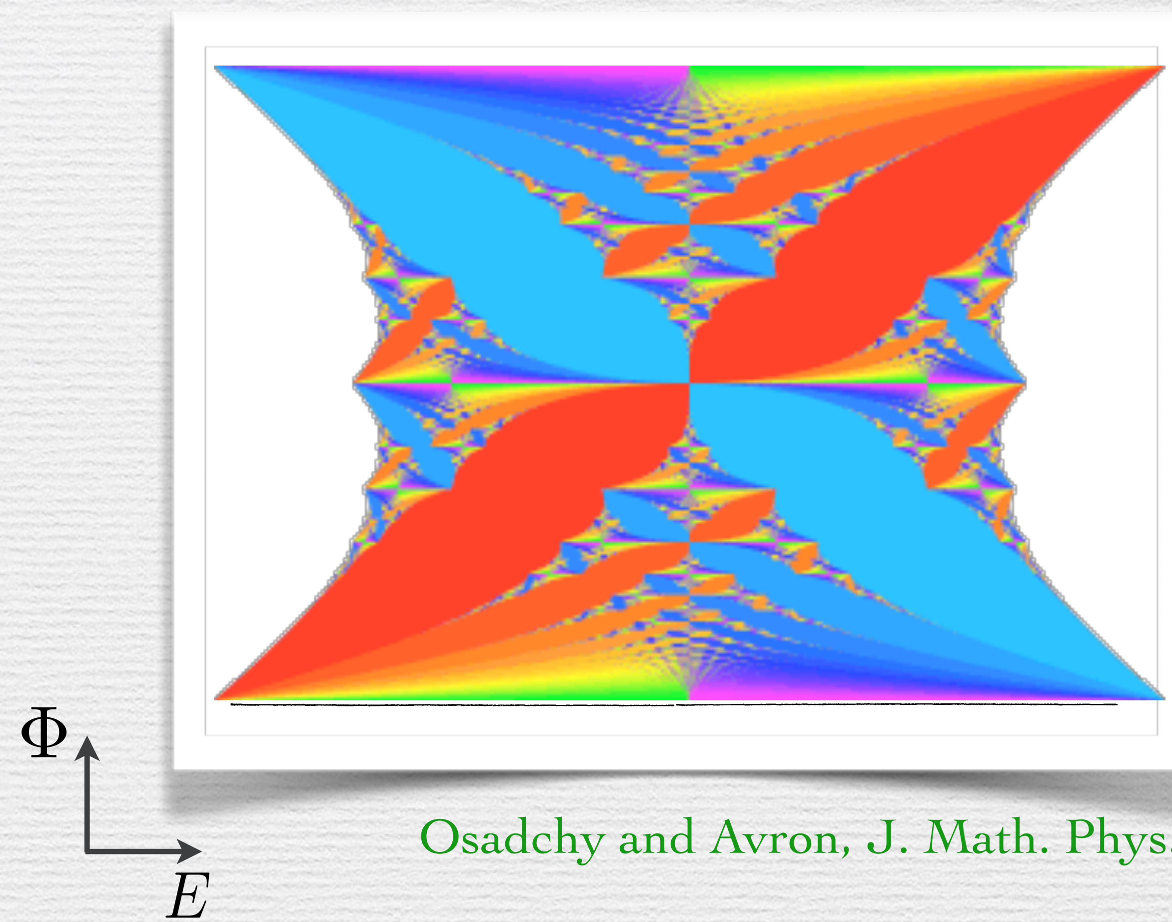

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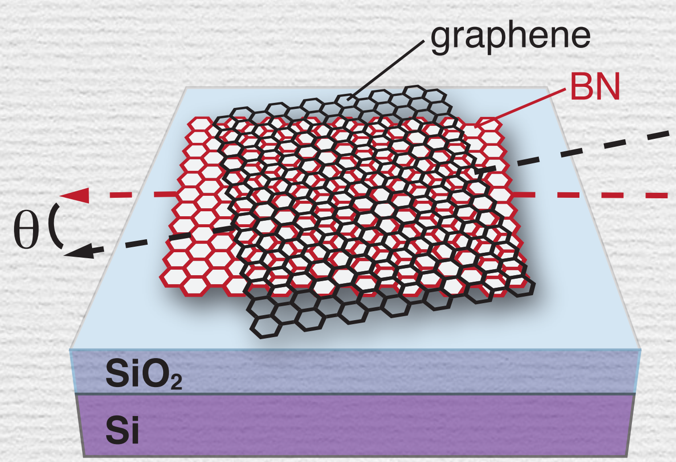
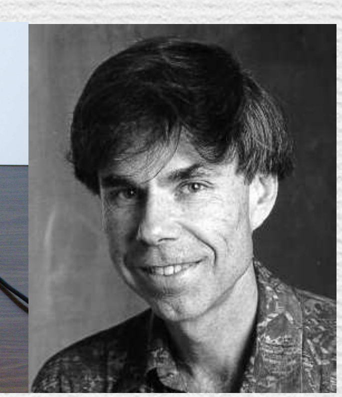


arXiv: 1212.4783 Hofstadter's butterfly in moire superlattices: A fractal quantum Hall effect



How to measure Chern # ?

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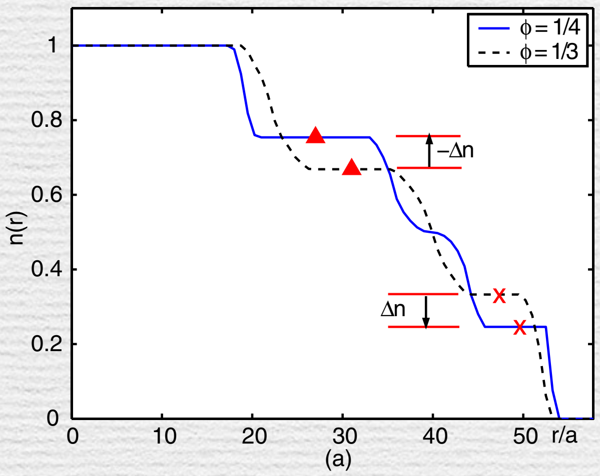
Density profile

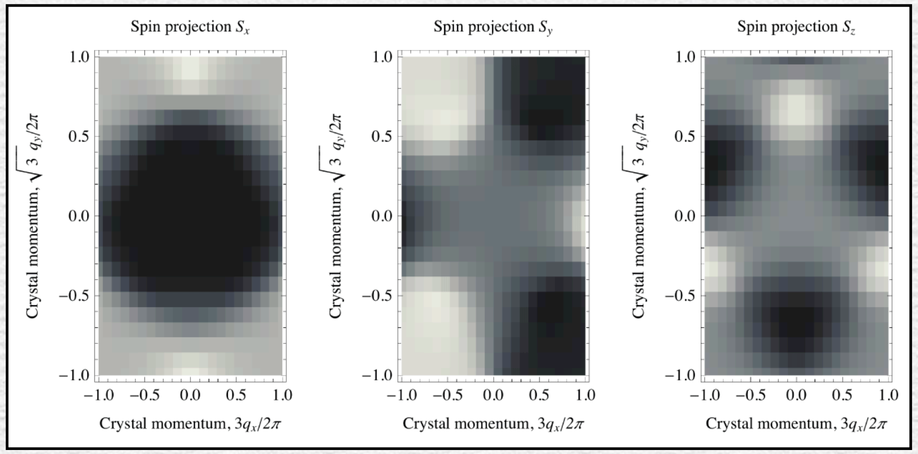
Umucalilar *et al*



How to measure Chern # ?

Time-of-flight

Alba *et al*, Zhao *et al*



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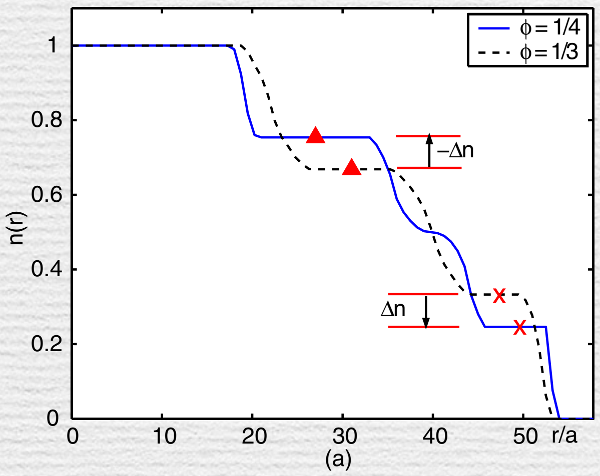

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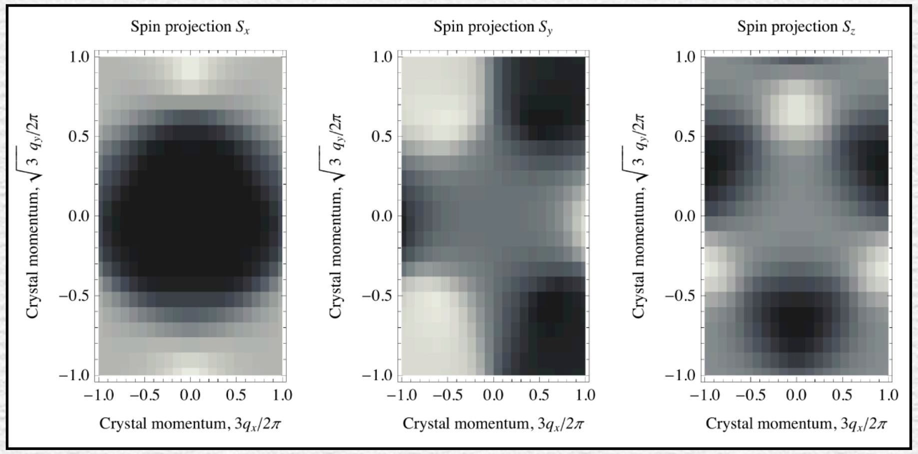

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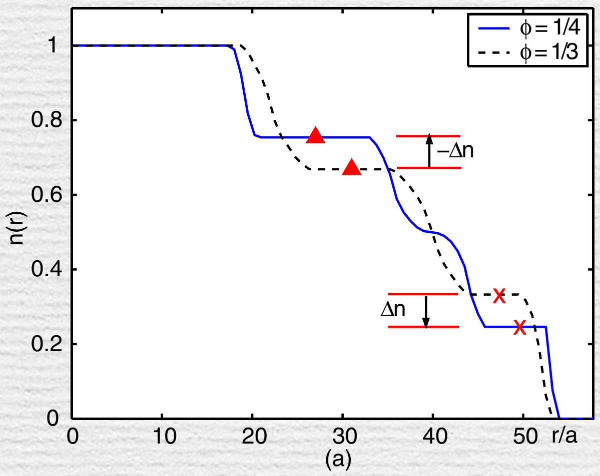
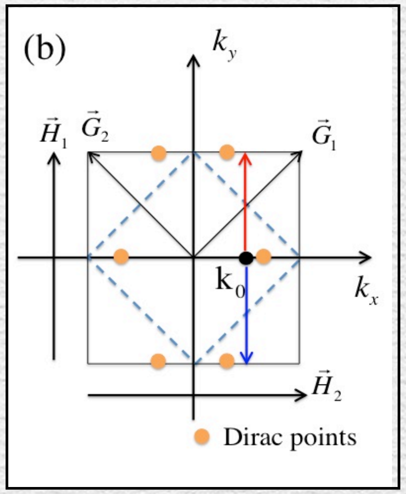


Zak phases

Abanin *et al*



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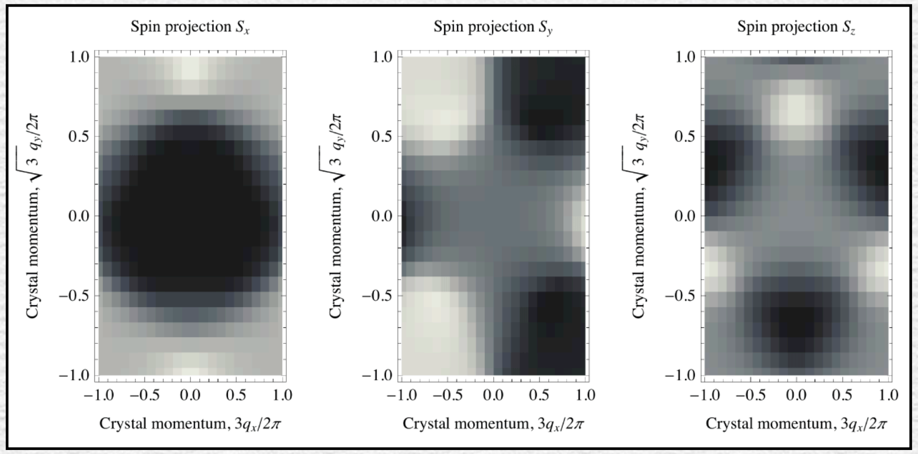

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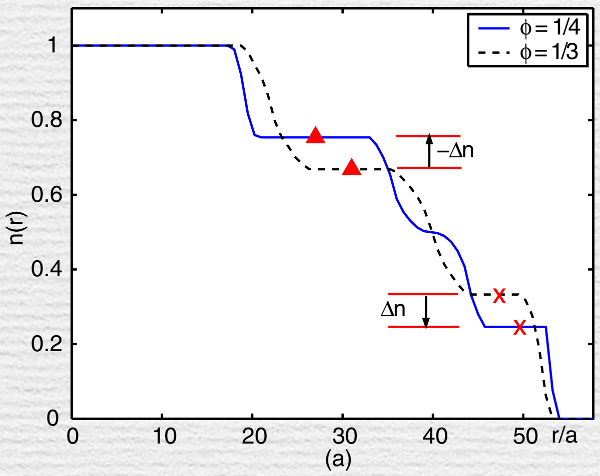

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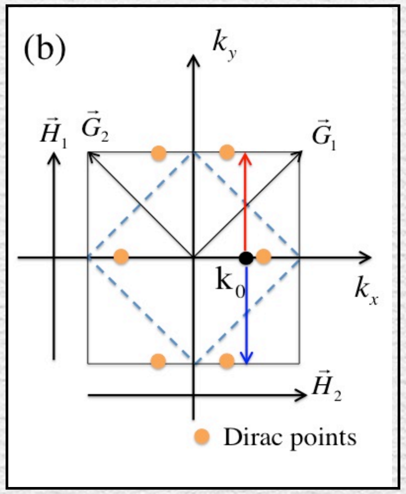


Semi-classical dynamics

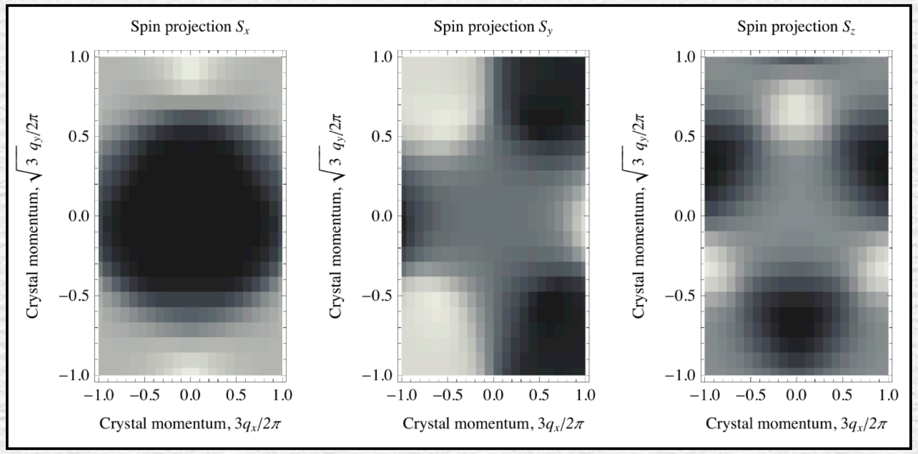
Price *et al*

$$\dot{\mathbf{r}} = \frac{1}{\hbar}\frac{\partial \varepsilon(\mathbf{k})}{\partial \mathbf{k}} - \frac{\mathbf{F}}{\hbar} \times \hat{\mathbf{z}}\Omega(\mathbf{k})$$

How to measure Chern # ?

Time-of-flight

Alba *et al*, Zhao *et al*



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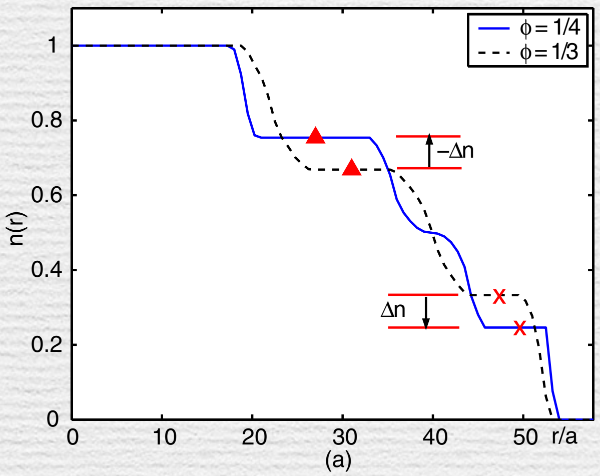

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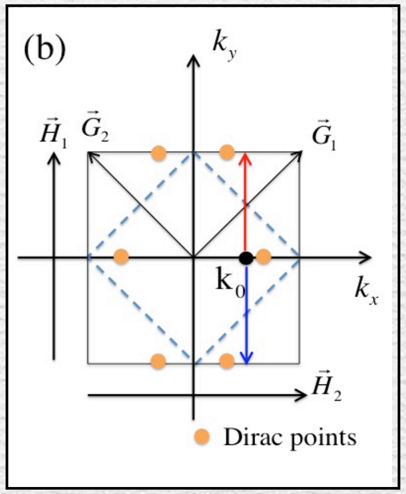

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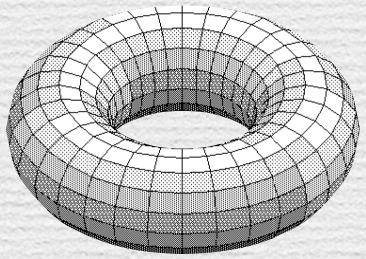
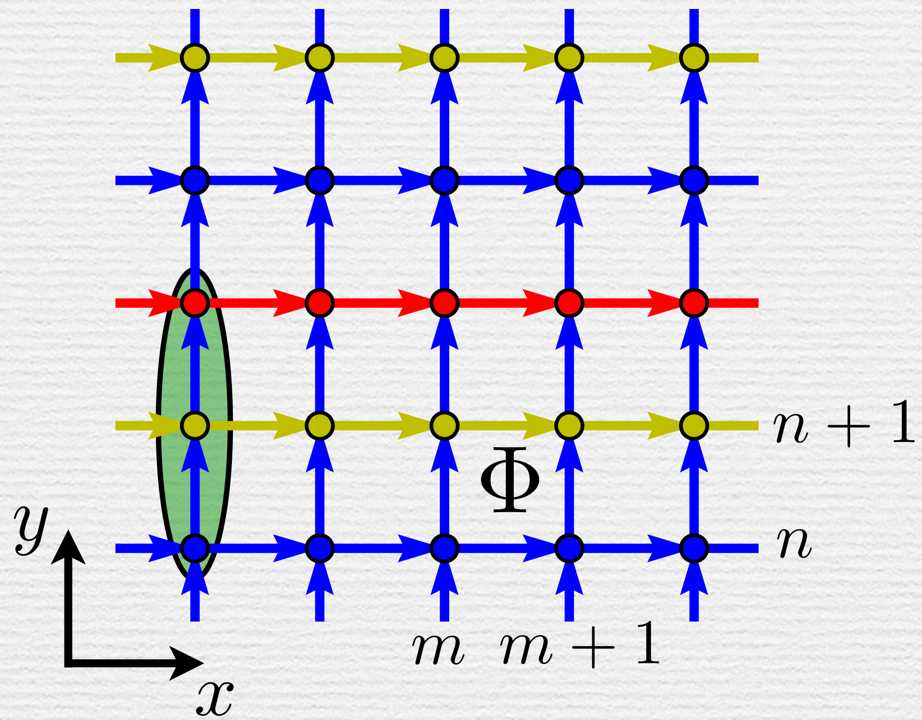


We propose a new probe based on

Topological Pumping Effect

$$\rho(k_x, y)$$

Hybrid ToF

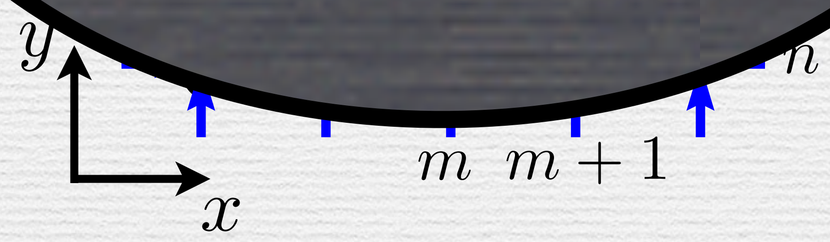


Hybrid ToF

H[redacted]ToF

[redacted]

H[redacted]F



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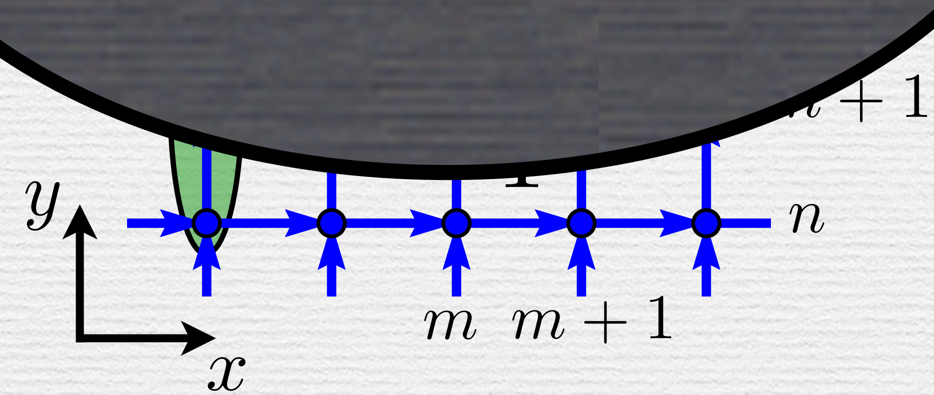

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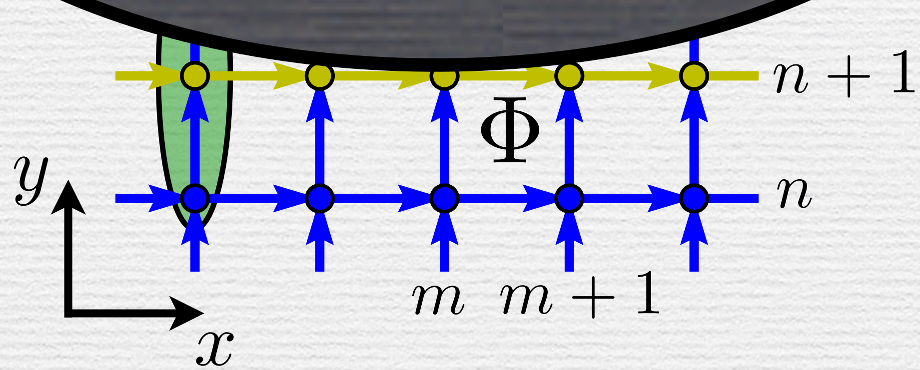

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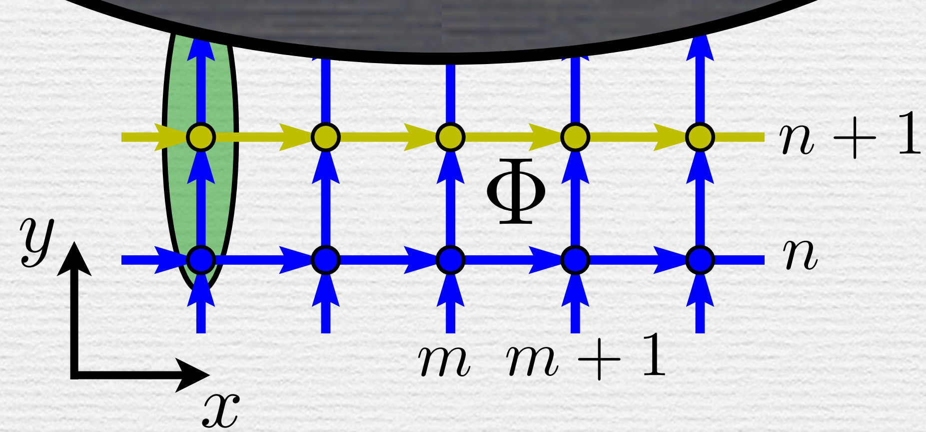

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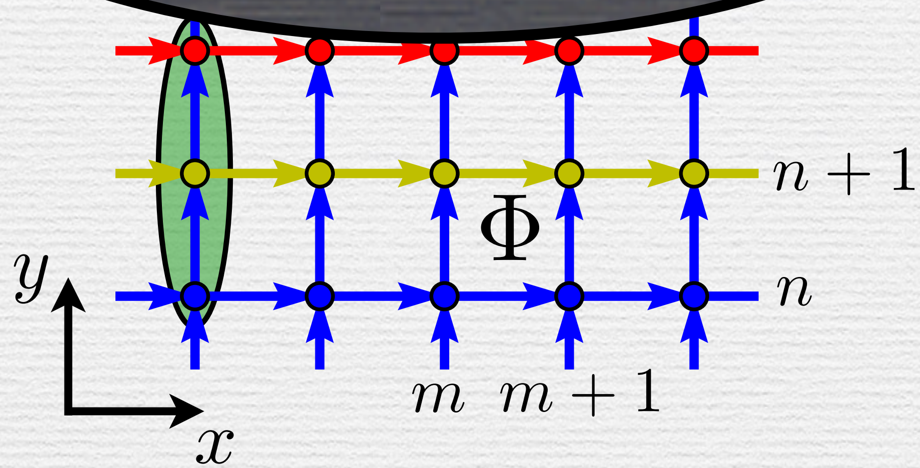

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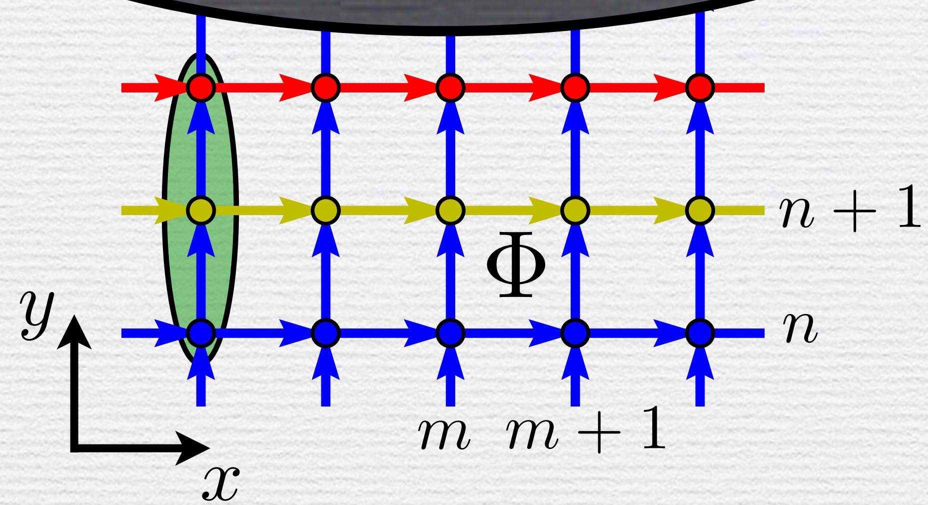

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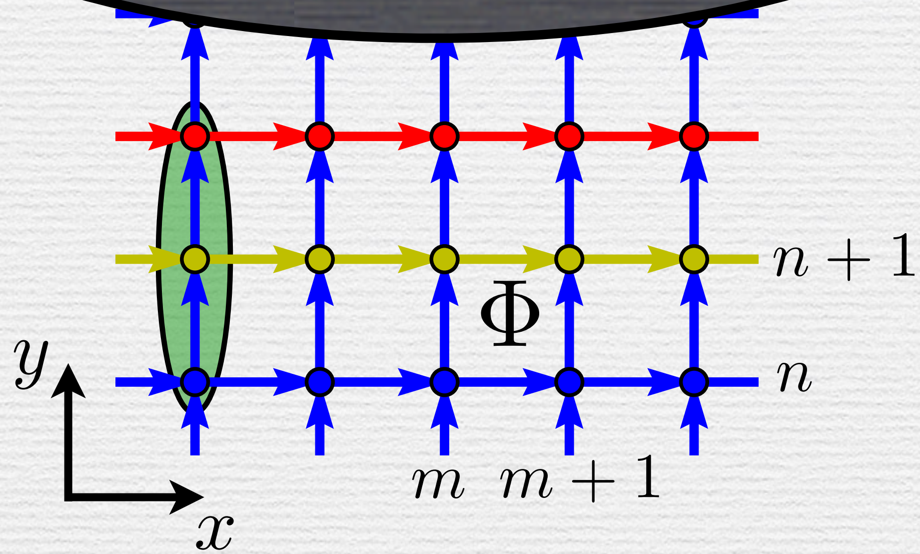

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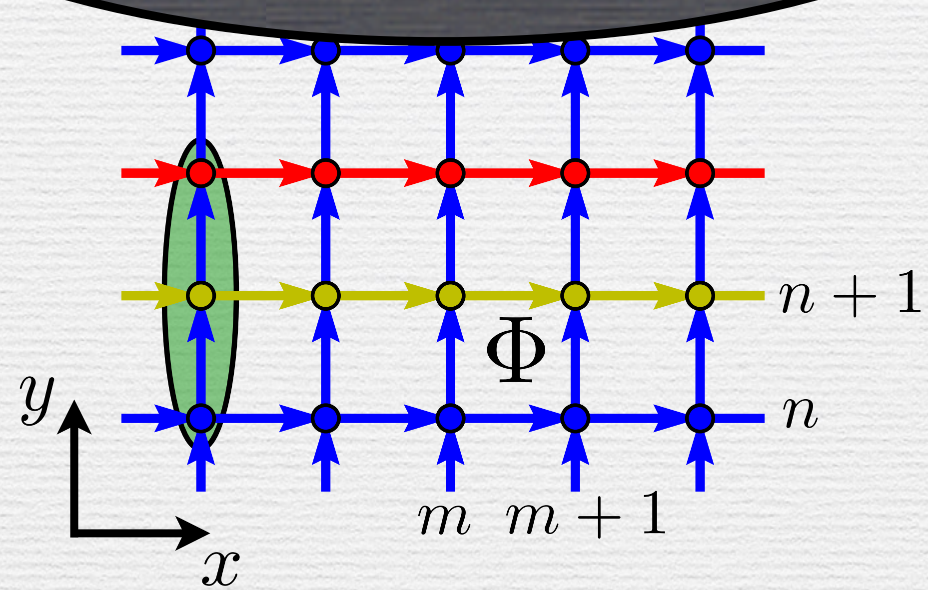

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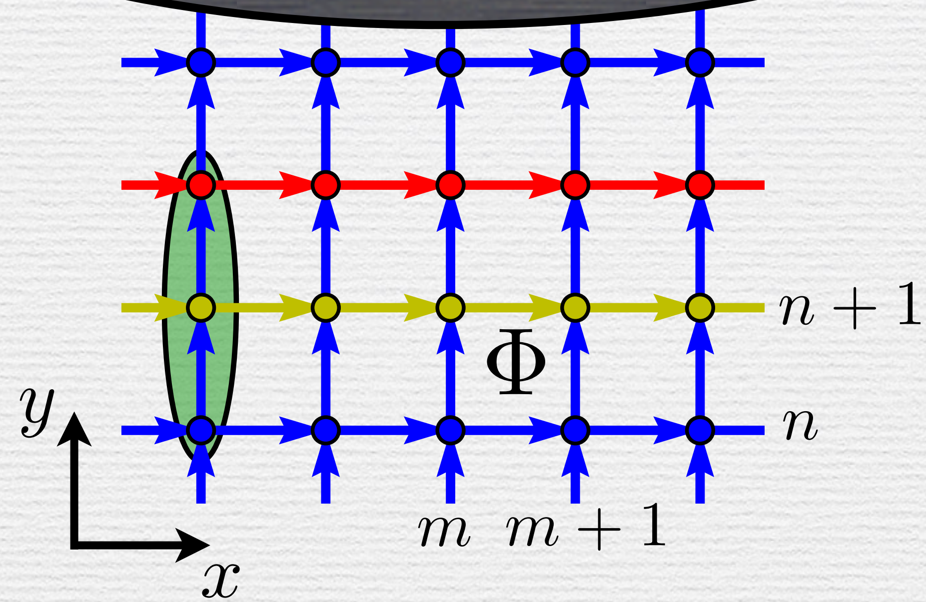

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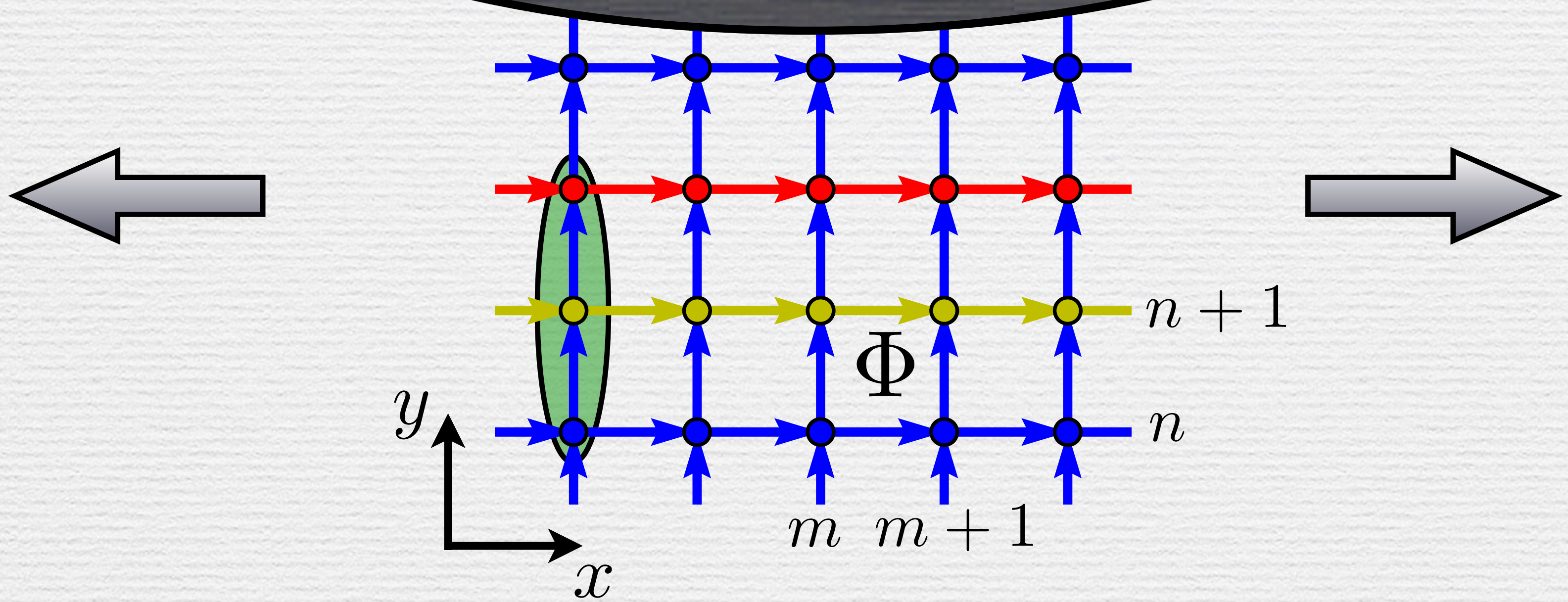

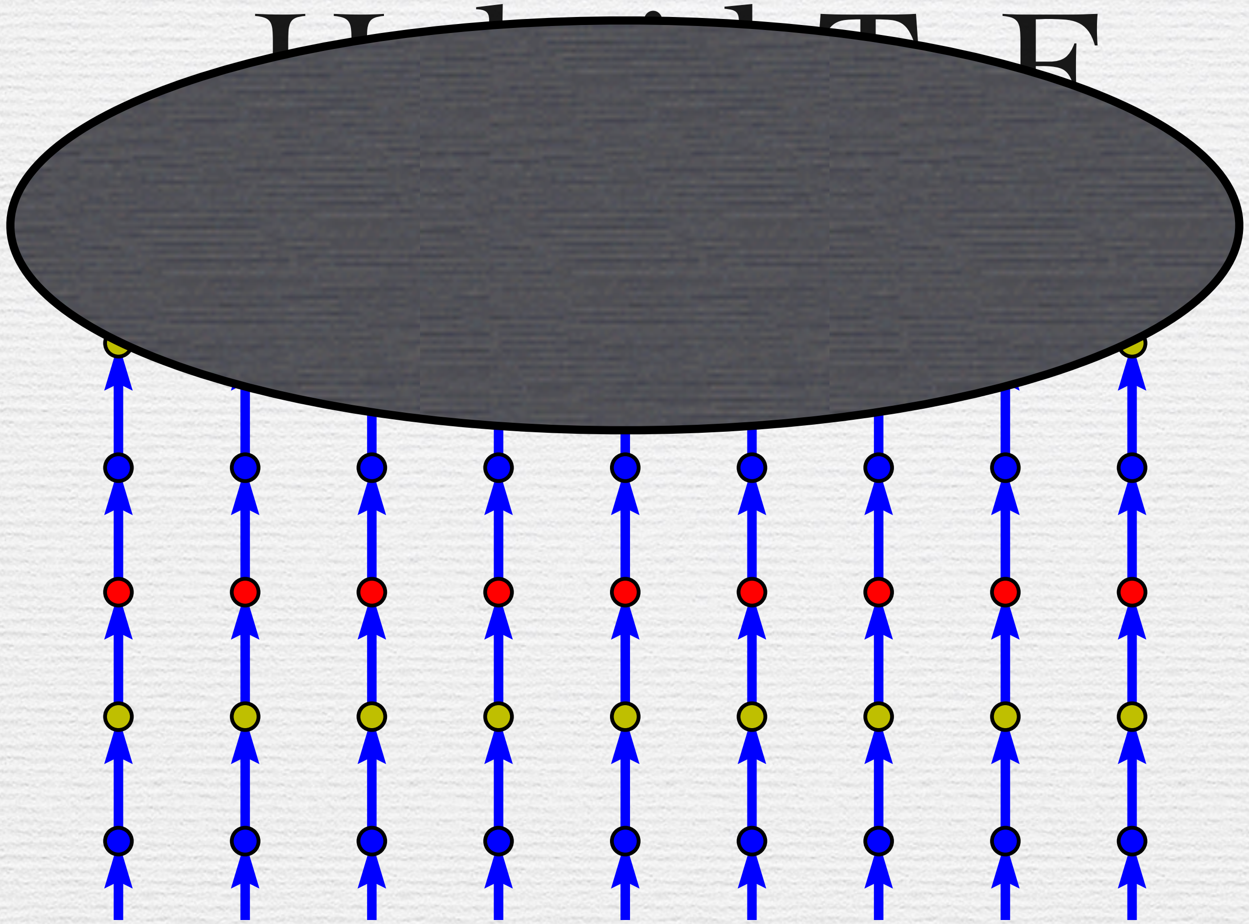

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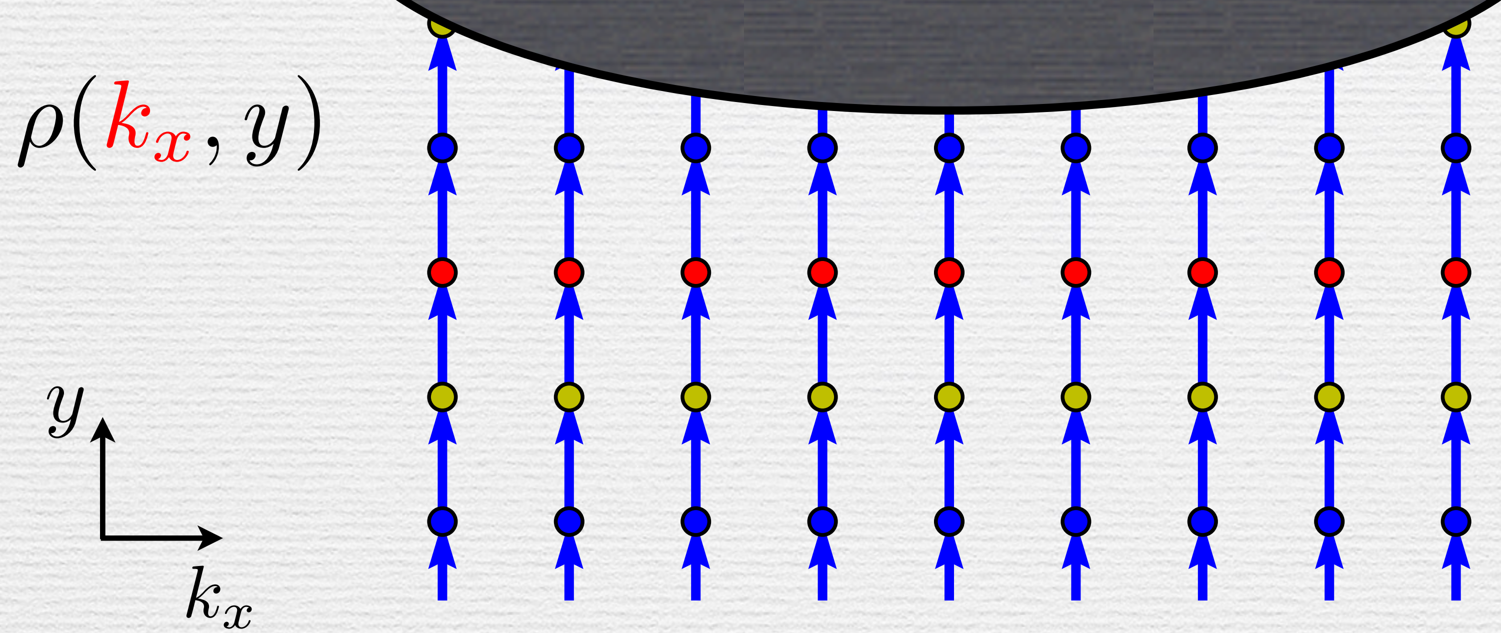

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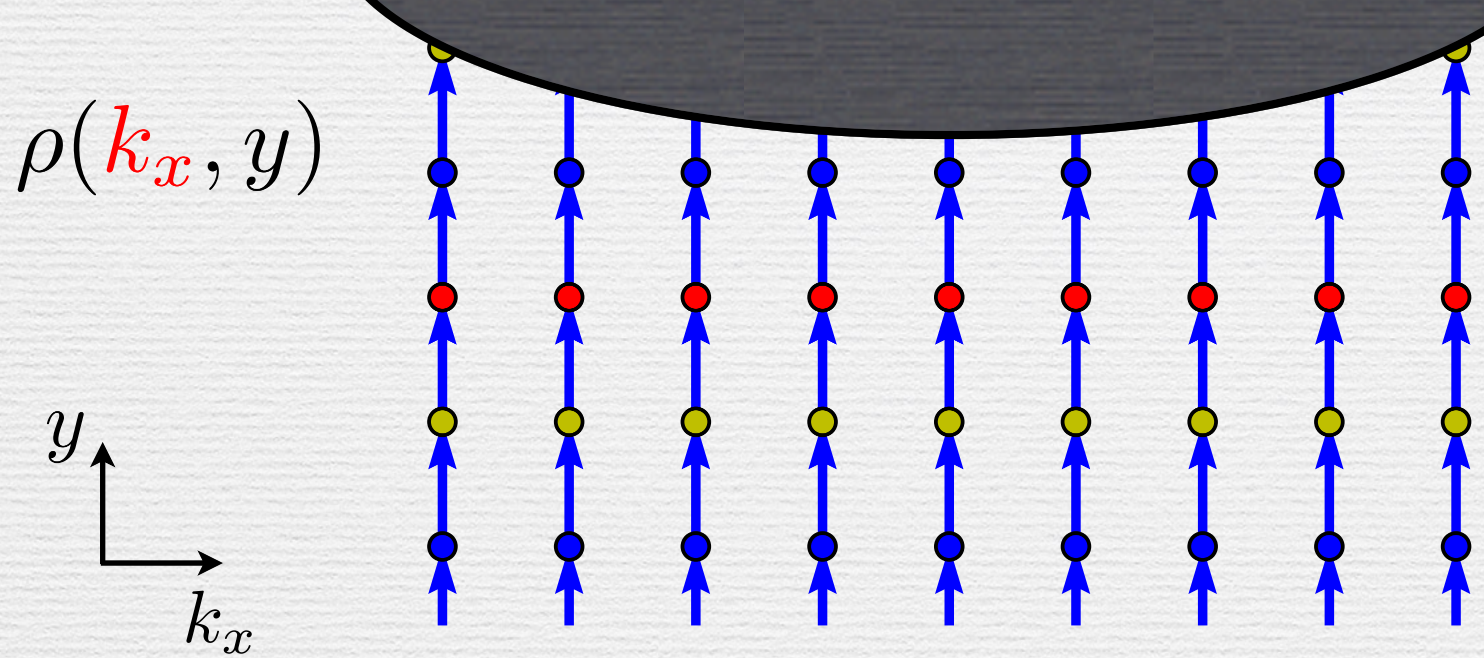
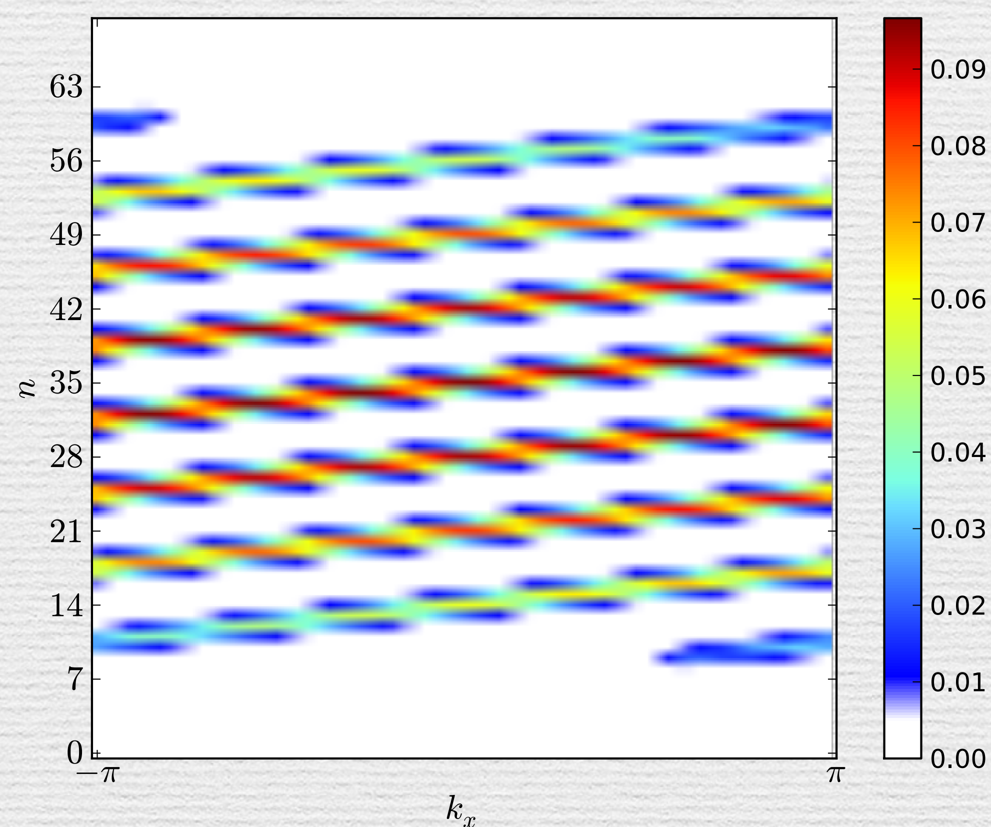
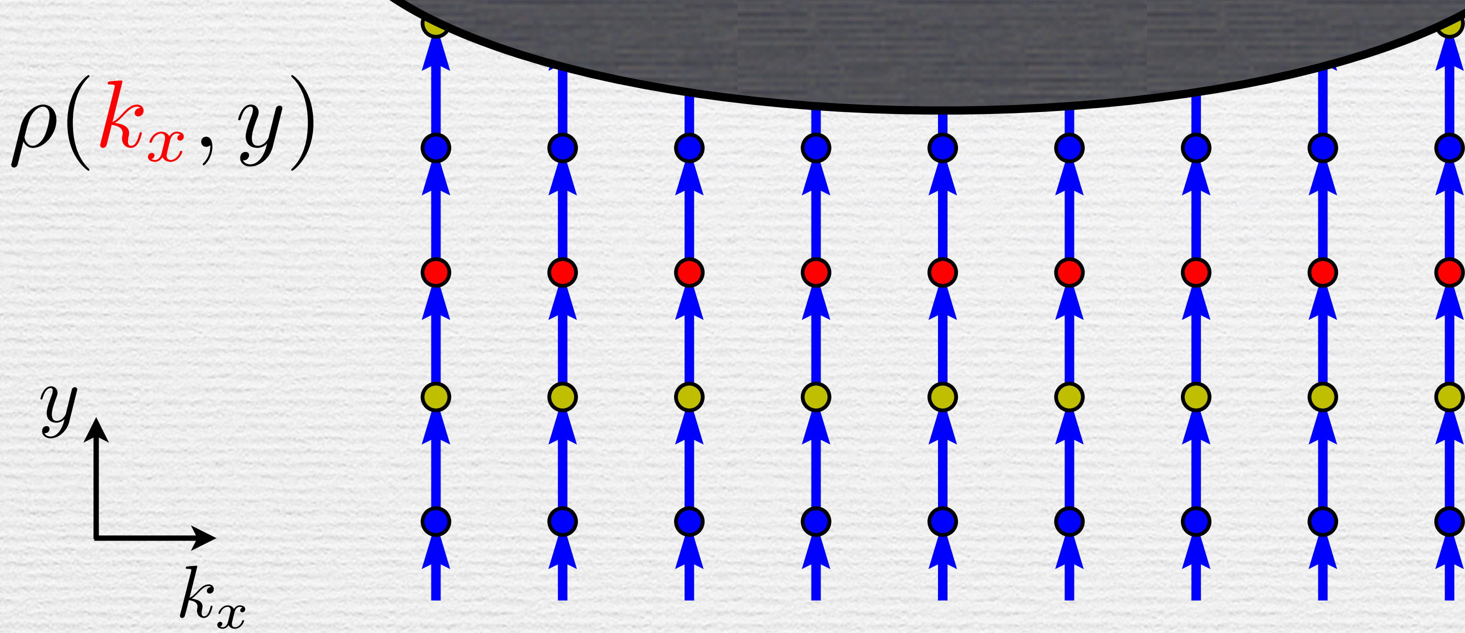


ρ(kx, y)
y
kx

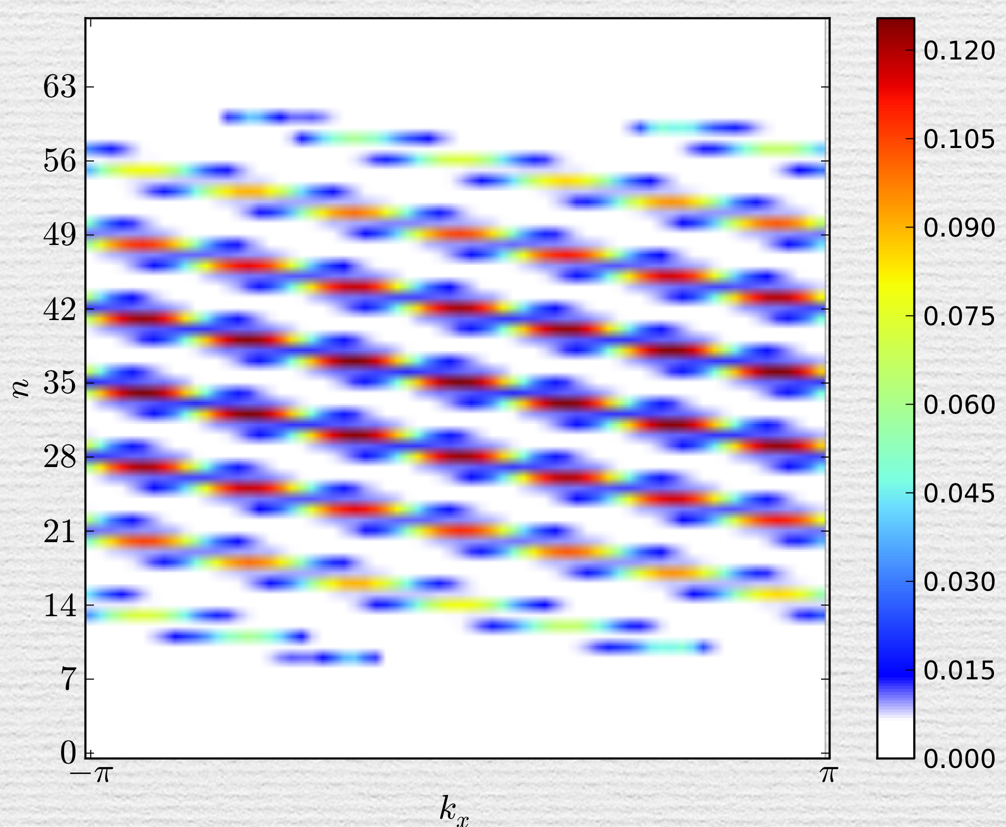


63
56
49
42
35
28
21
14
7
0
n
−π
π
kx
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00

[redacted]

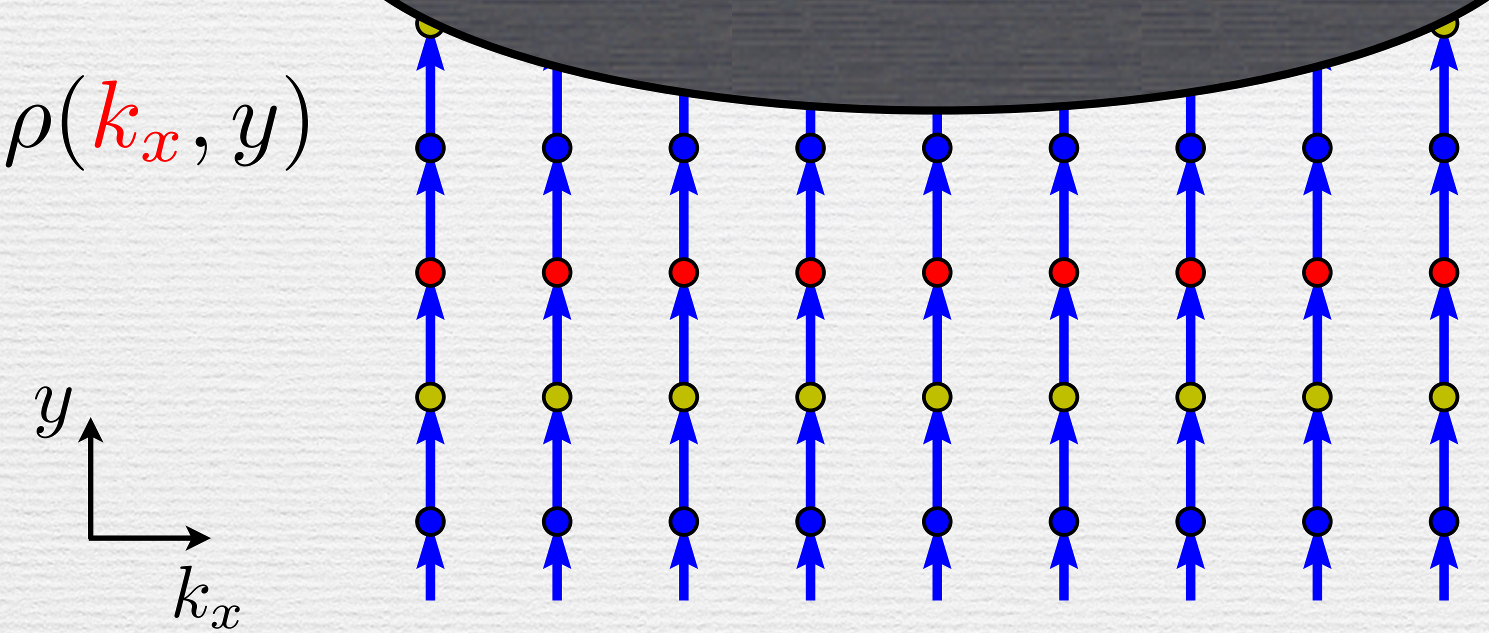


$$\Phi = 3/7 \quad C = -2$$



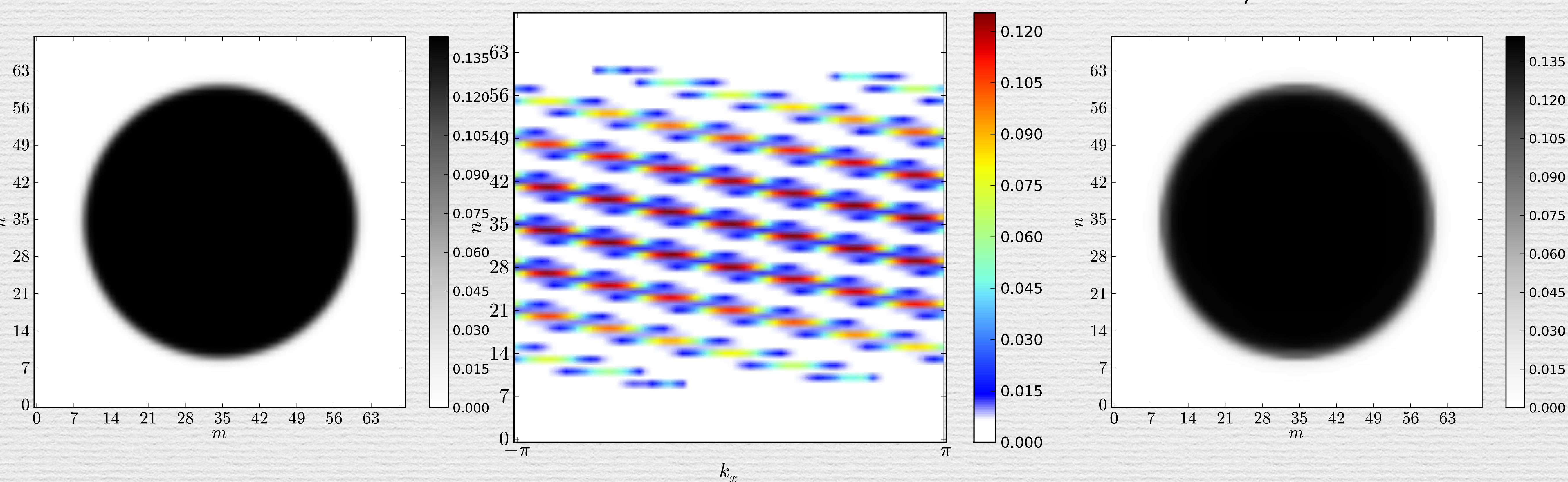
[redacted]

$\rho(k_x, y)$



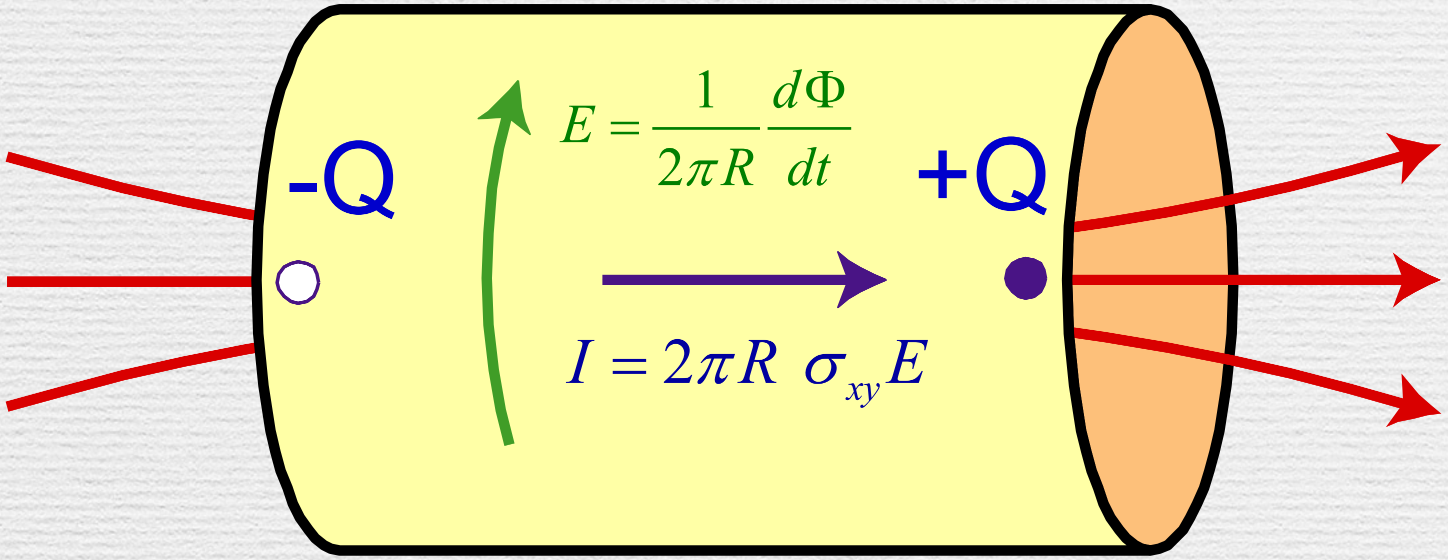
$\Phi = 1/7 \quad C = 1$

$\Phi = 3/7 \quad C = -2$



Why it works?

Topological charge pumping



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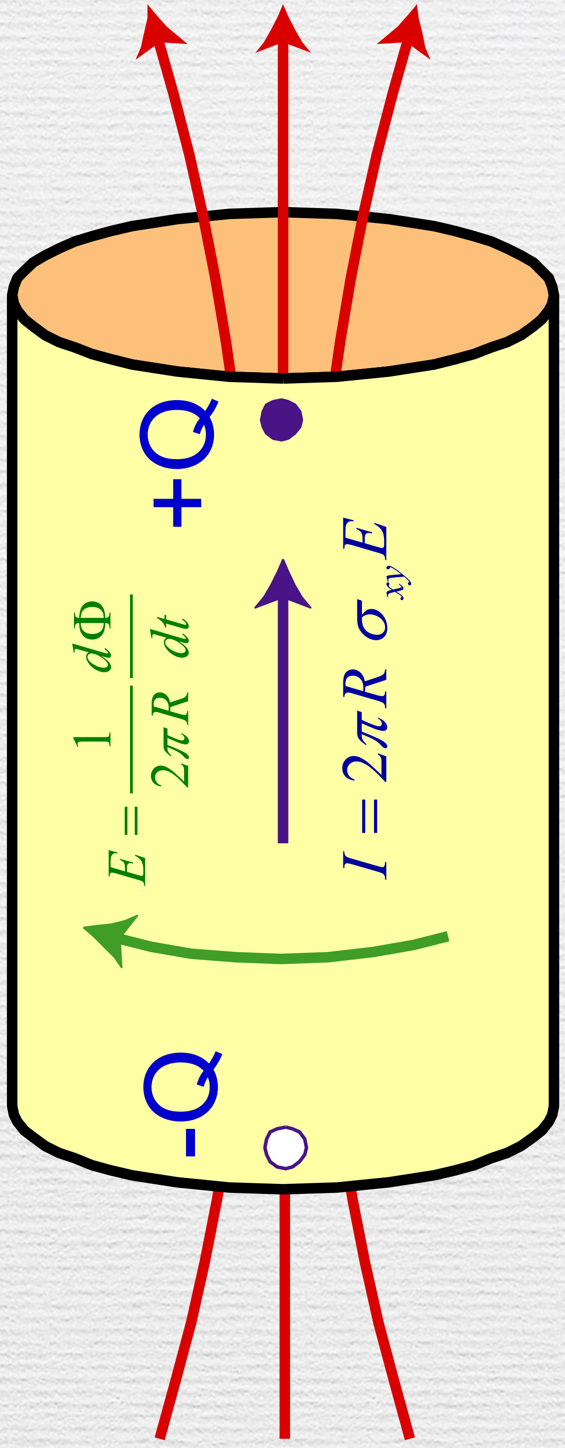
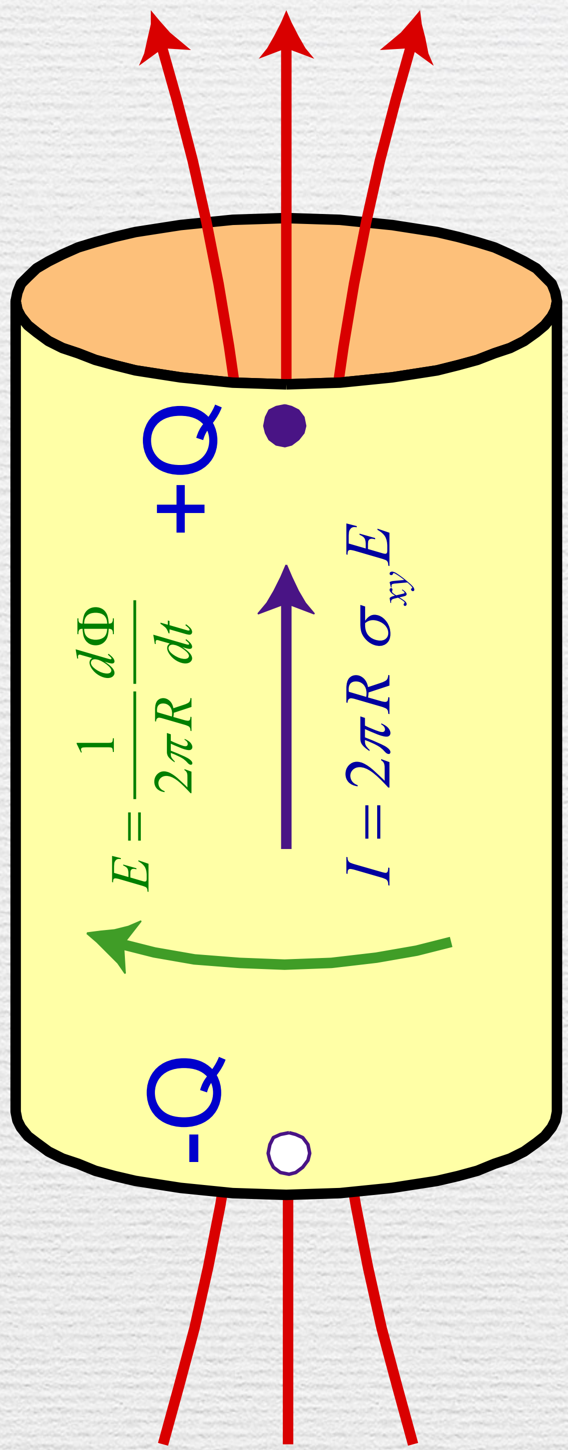
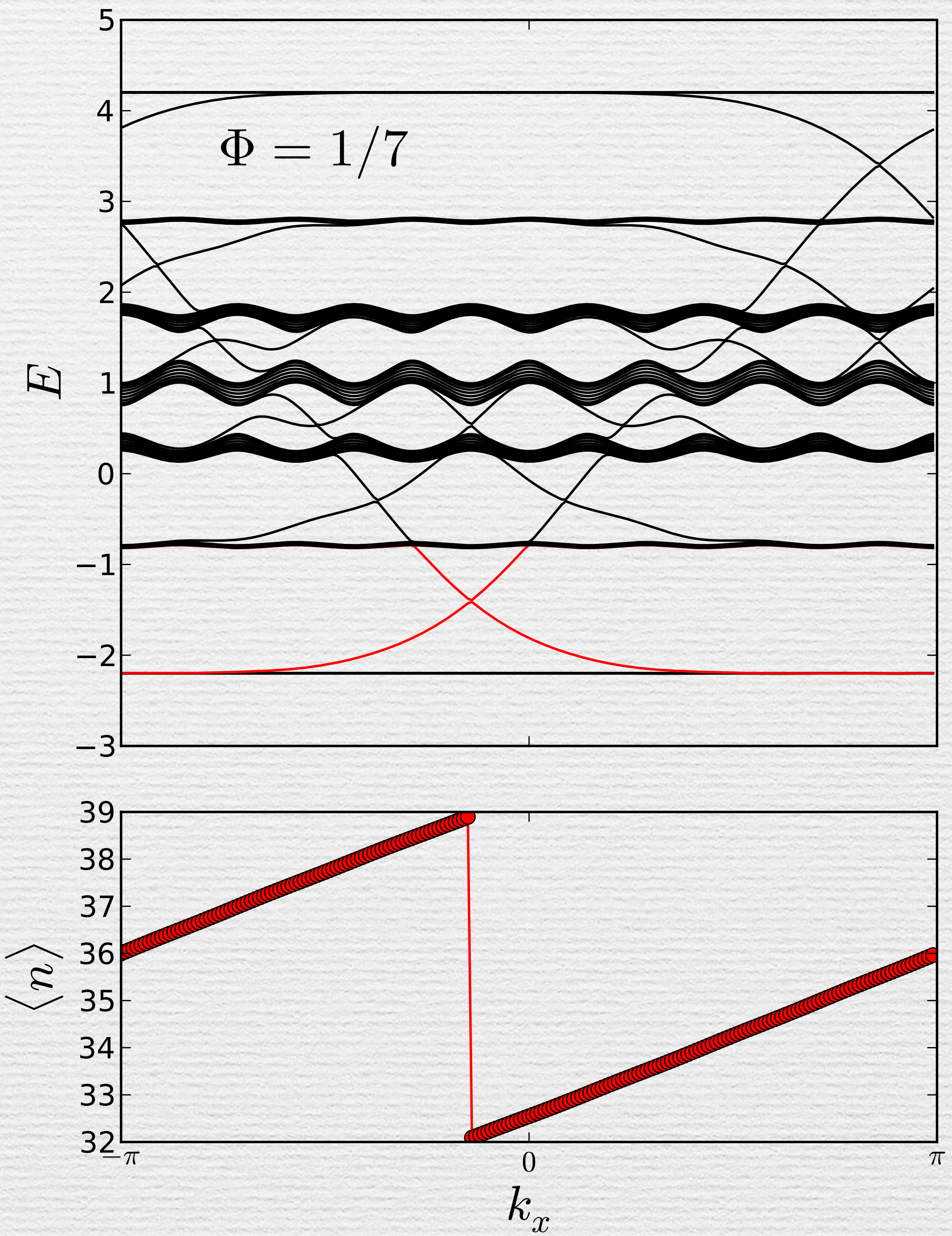
Topological charge pumping



Why it works?

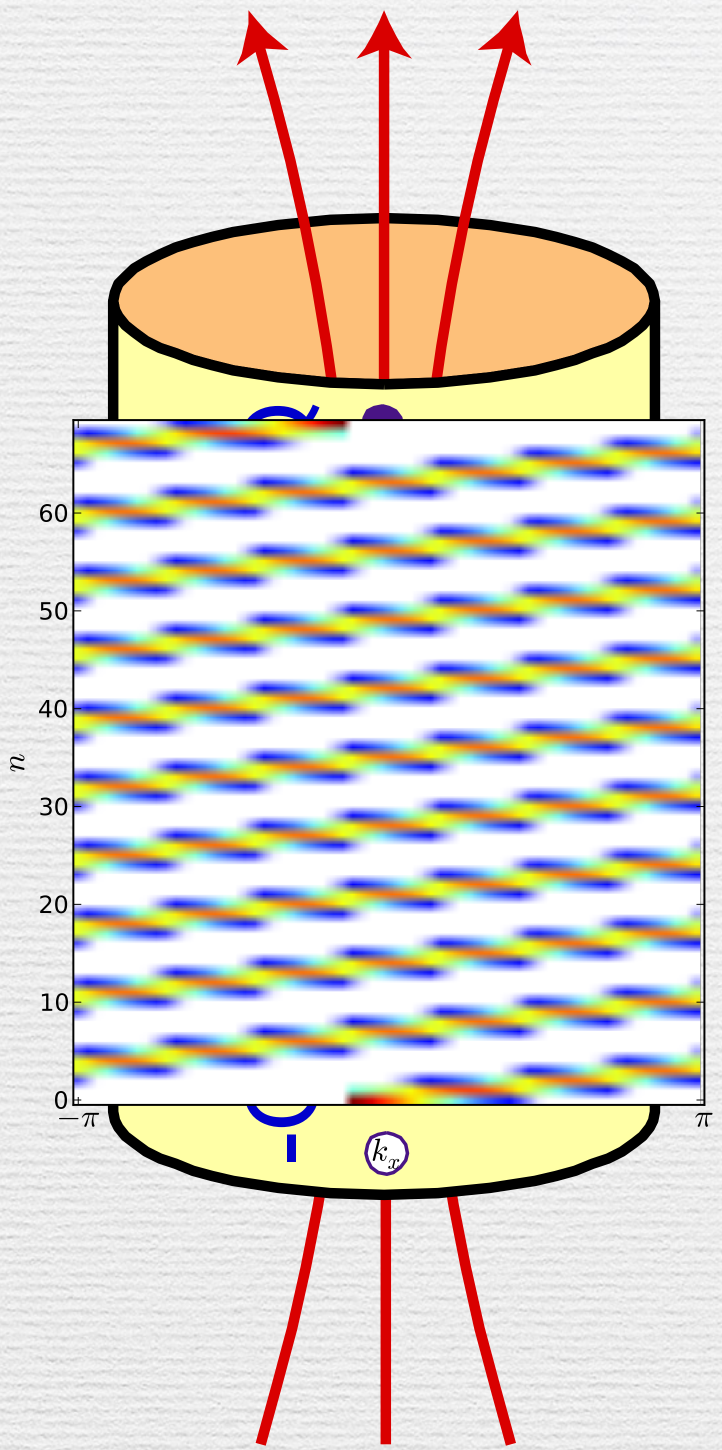
Topological charge pumping

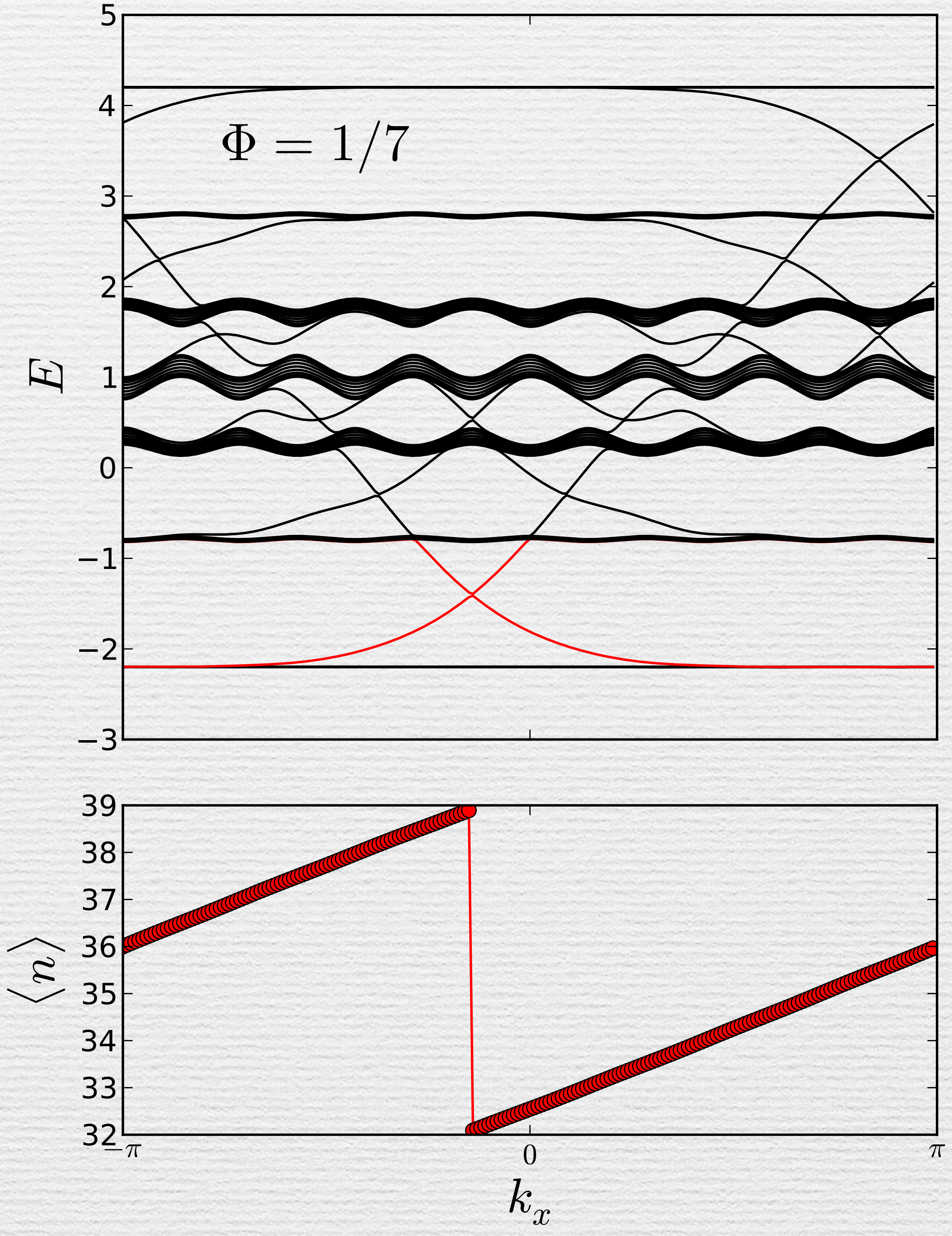




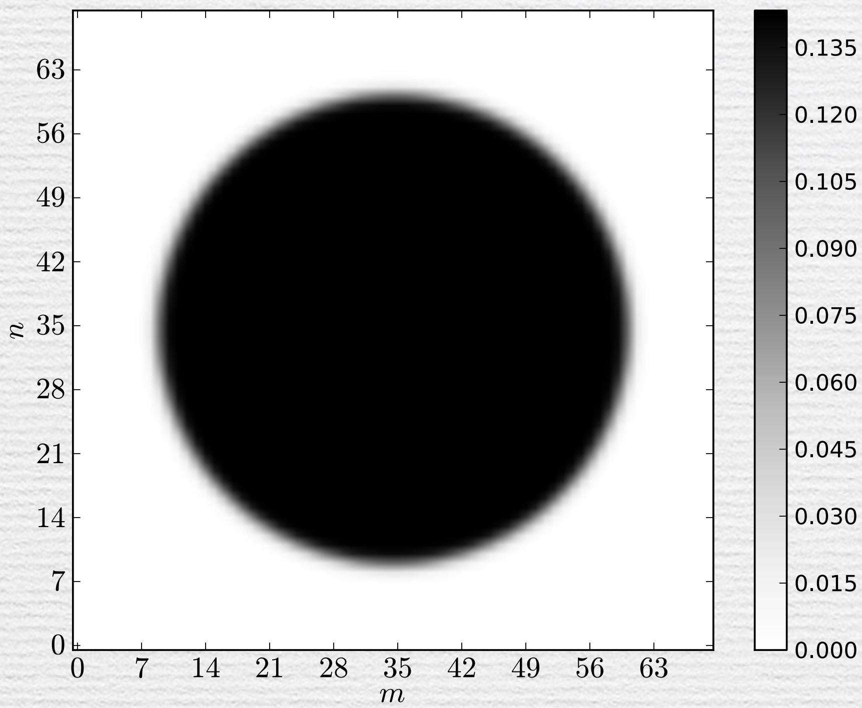
Why it works?

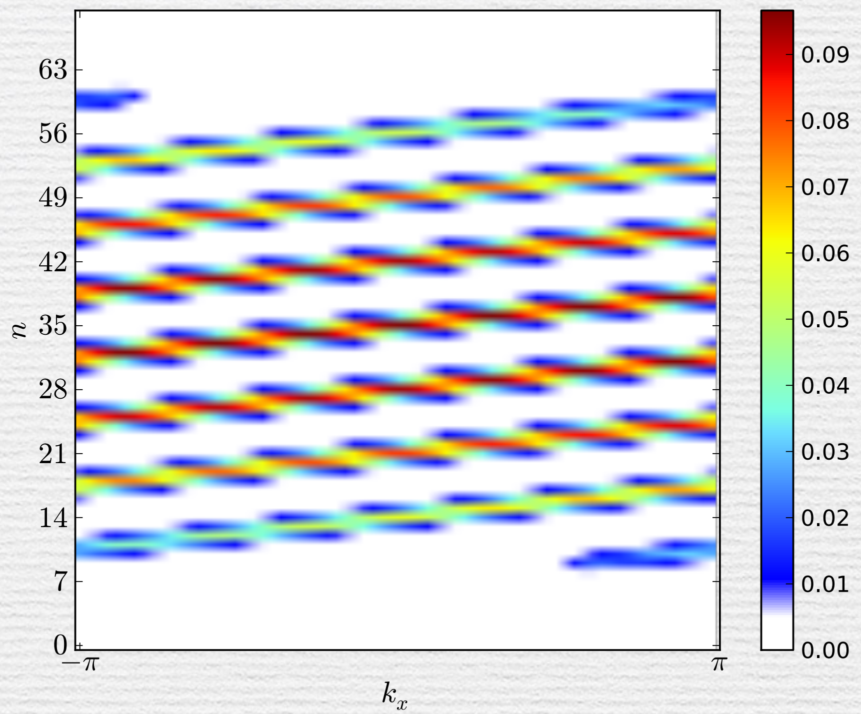
Topological charge pumping





Summary





Topological charge pumping is a common thread unifies many features of topological states

Guideline for design and detection of topological phases in cold atom systems

Thank you!

FAQ

Tight binding limit? Do not need
Edge state modes, fractionalized charge ? Do not need
Is sliding topological ? Yes