arXiv:1301.7435, in press
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Topological charge pumping of cold atoms

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Matthias Troyer

Xi Dai

Pumps

Pump is a device that moves fluids, or sometimes slurries, by mechanical action.

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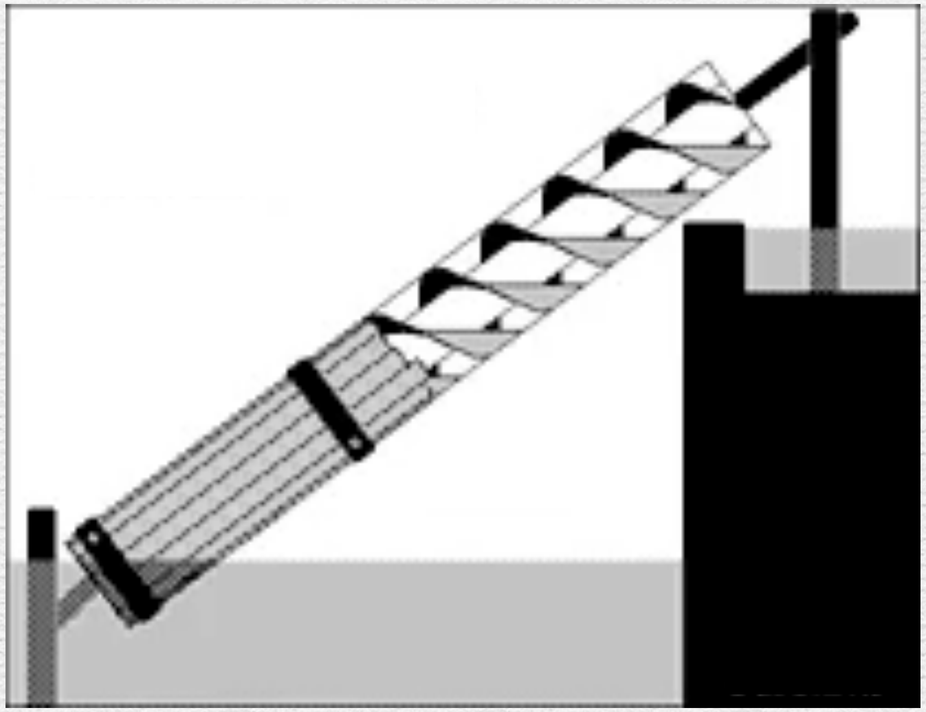

Archimedes' screw ~250 BC

Pumps

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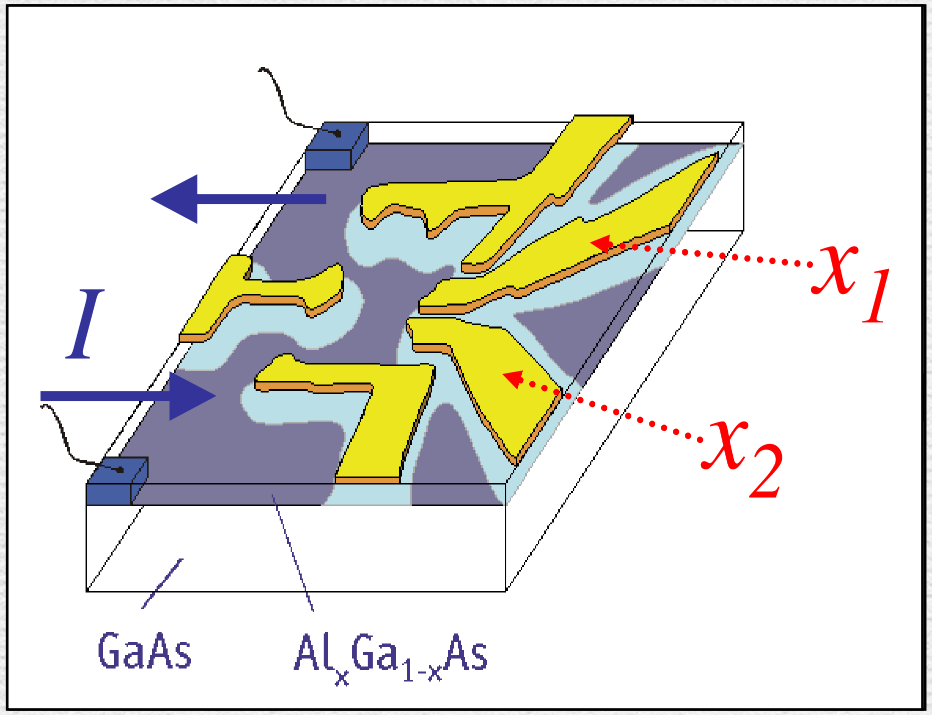
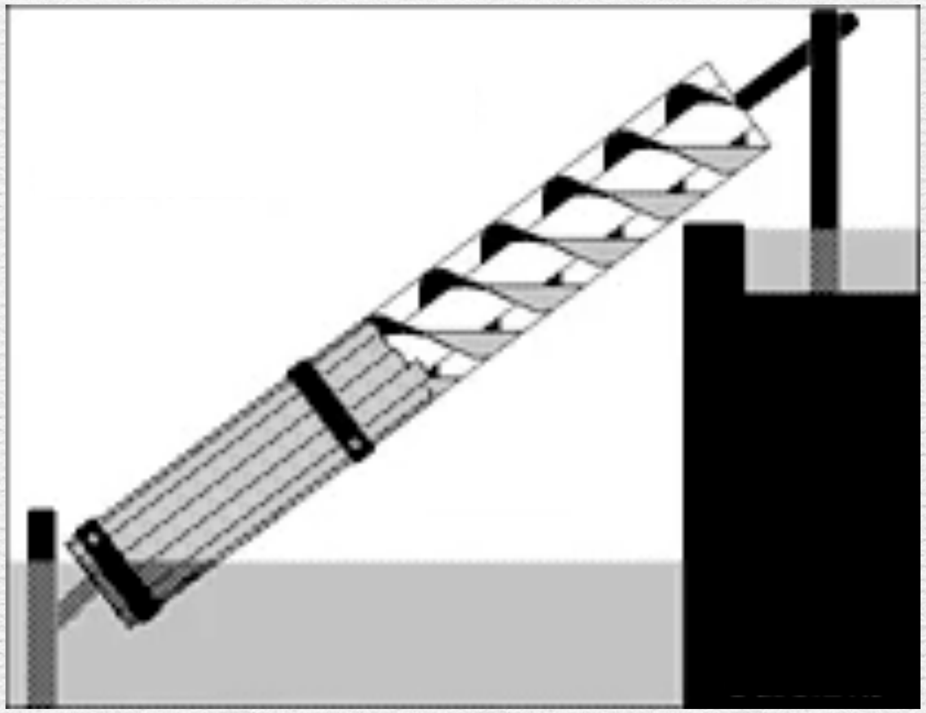


Switkes *et al* 1999

Buttiker, Brouwer, Zhou, Spivak, Altshuler …



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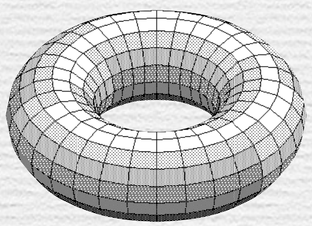
Topological pump

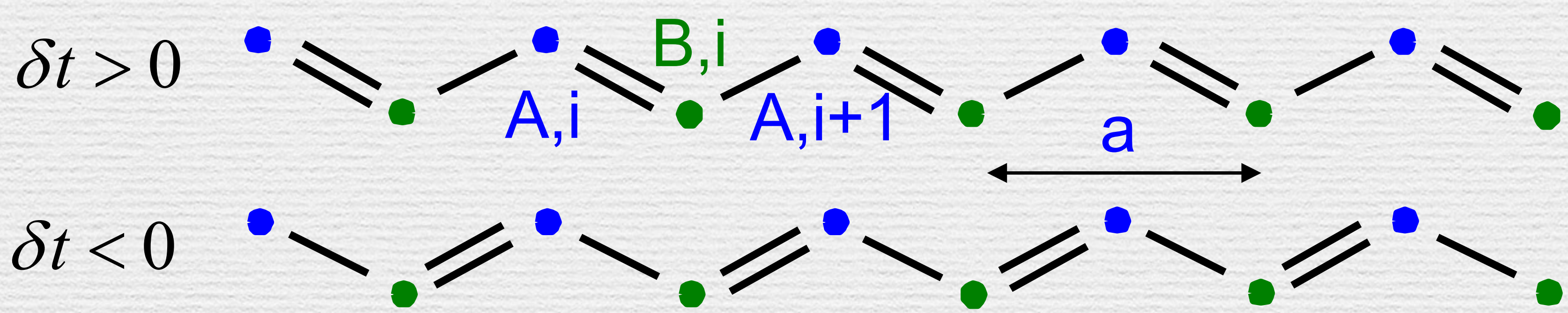


A device transfers **quantized charge** in each pumping cycle.

Thouless 1983

$$H = \sum_i (t+\delta t) c^\dagger_{Ai} c_{Bi} + (t-\delta t) c^\dagger_{Ai+1} c_{Bi} + H.c.$$

Su, Schrieffer, Heeger, 1979

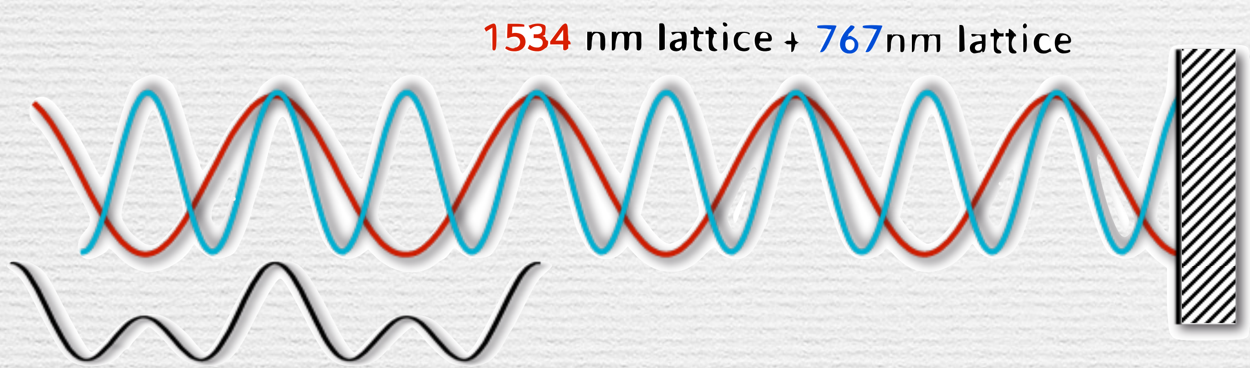


- Current flows in an insulating state
- No dissipation!
- Dynamical analog of quantum Hall effect

Experimental progresses

Optical Superlattice

Fölling *et al*, Atala *et al*

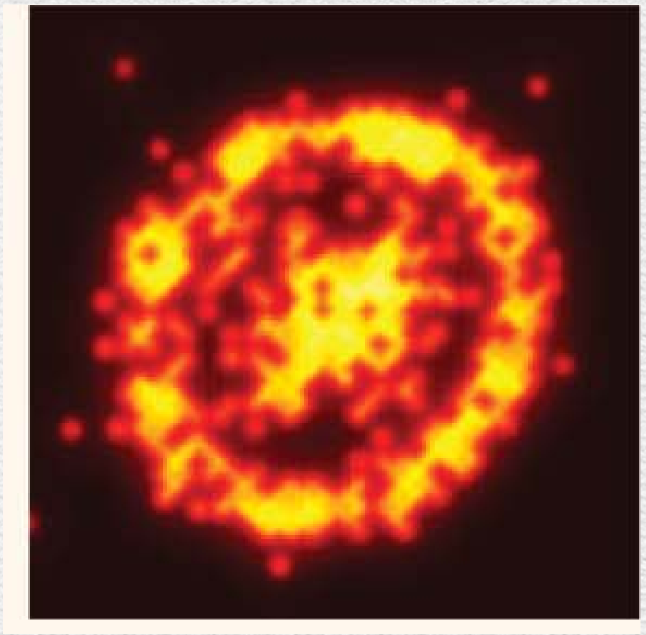


$$V_{\rm OL}(x) = V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \varphi\right)$$

Full (independent) dynamical control over V_1, V_2 and φ

in-situ imaging

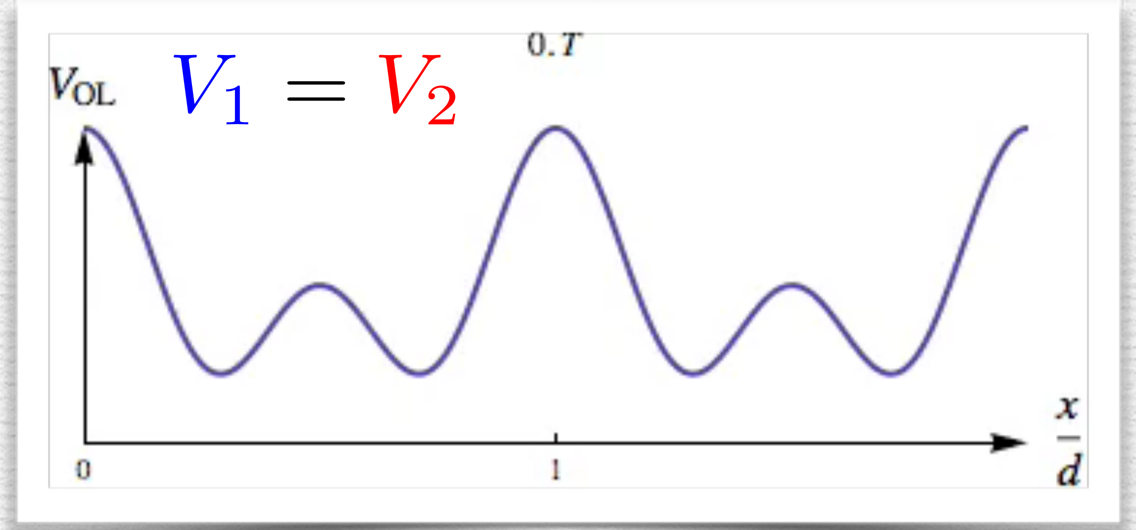
Gemelke *et al*, Sherson *et al*, Bakr *et al*



Allows to measure exact quantization of pumped charge

1D pumping lattices

$$V_{\mathrm{OL}}(x,t) = V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \frac{\pi t}{T}\right)$$



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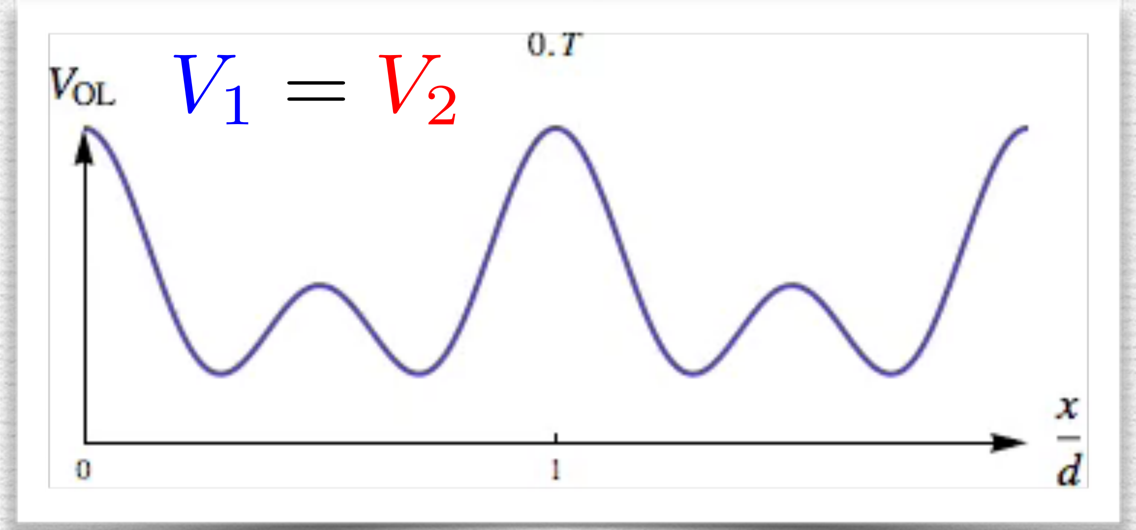

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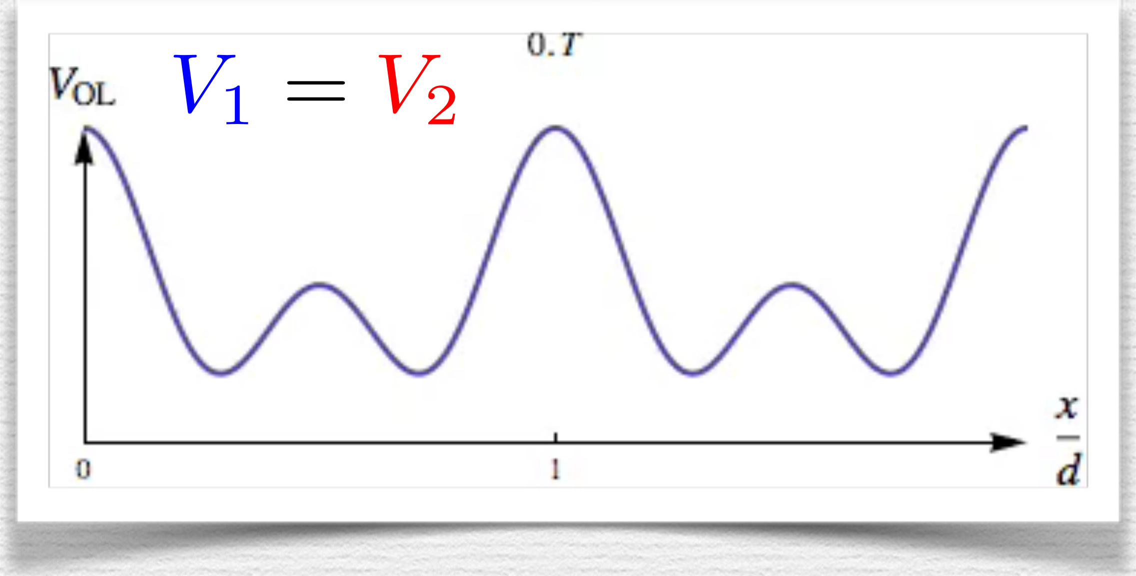

0 A ═ B — A ═ B

T/2 A — B ═ A — B

Su, Schrieffer, Heeger, 1979

1D pumping lattices

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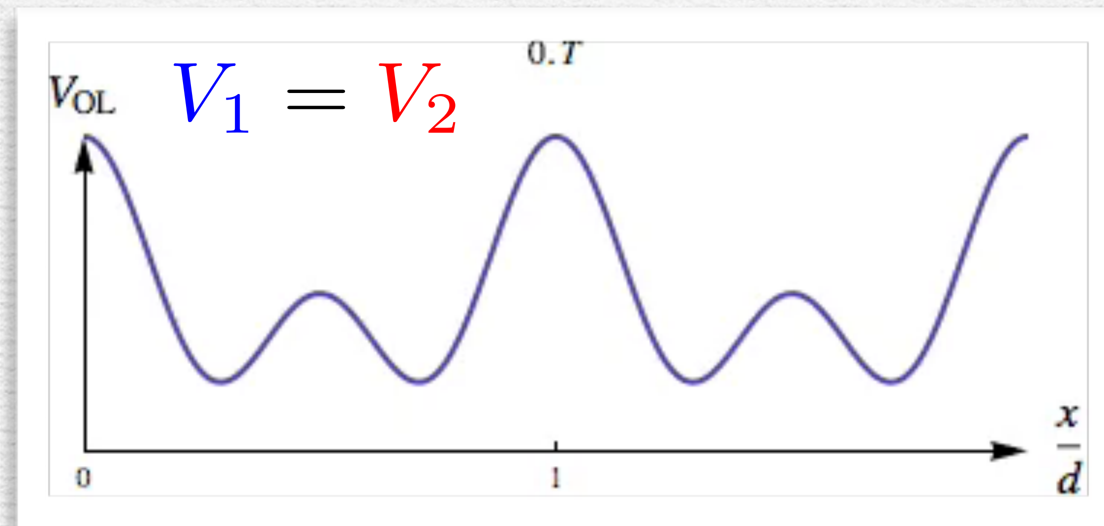


0 A ═ B — A ═ B

T/4 A ----- B ----- A ----- B

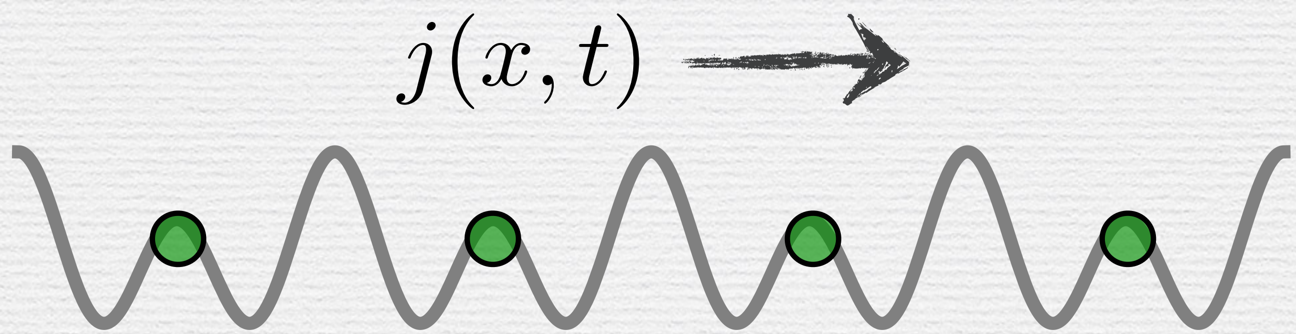
T/2 A — B ═ A — B

3T/4 A ----- B ----- A ----- B

Su, Schrieffer, Heeger, 1979

Rice, Mele, 1982

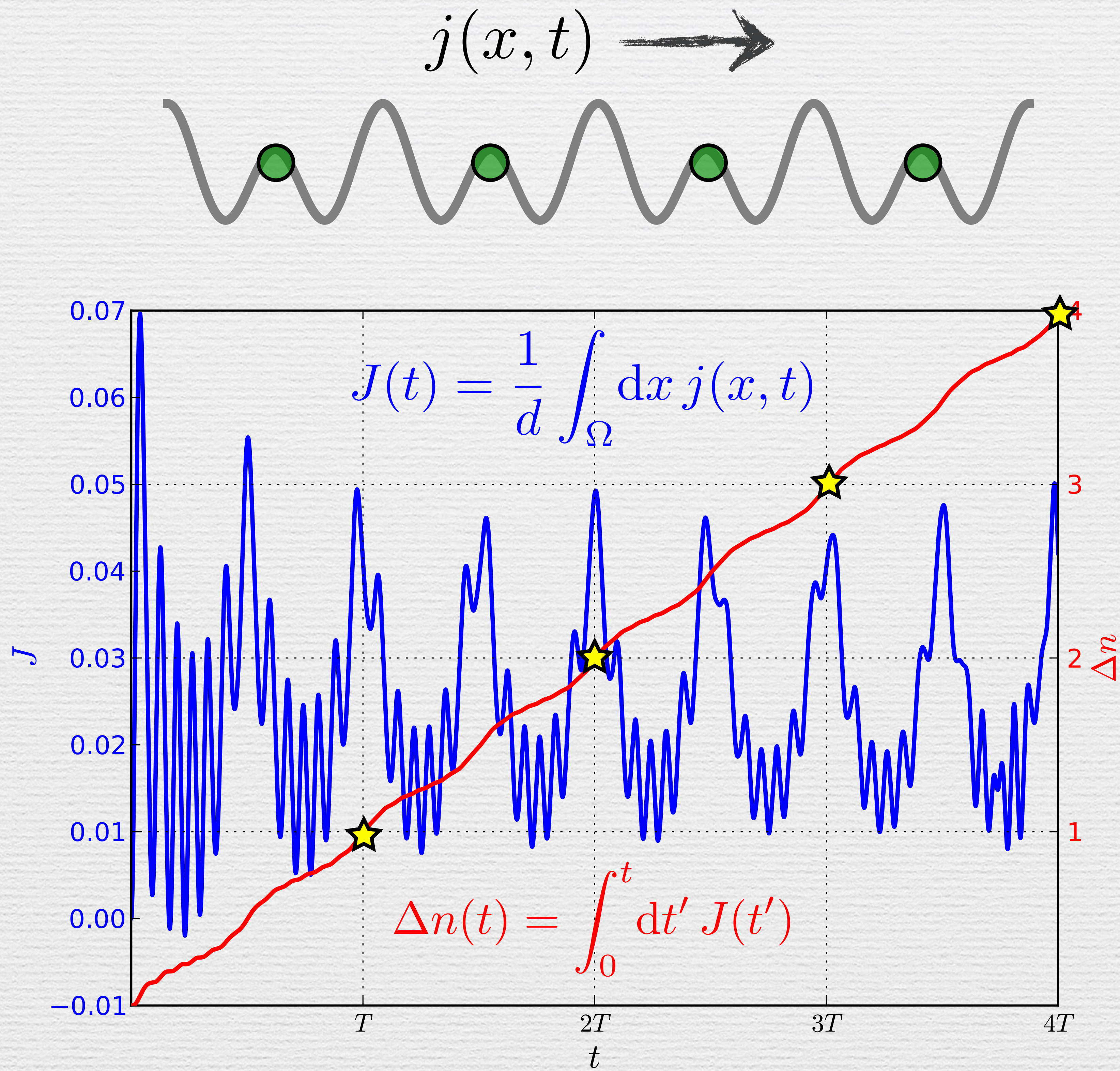
Quantization of pumped charge



$$H(x,t) = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{OL}}(x,t)$$

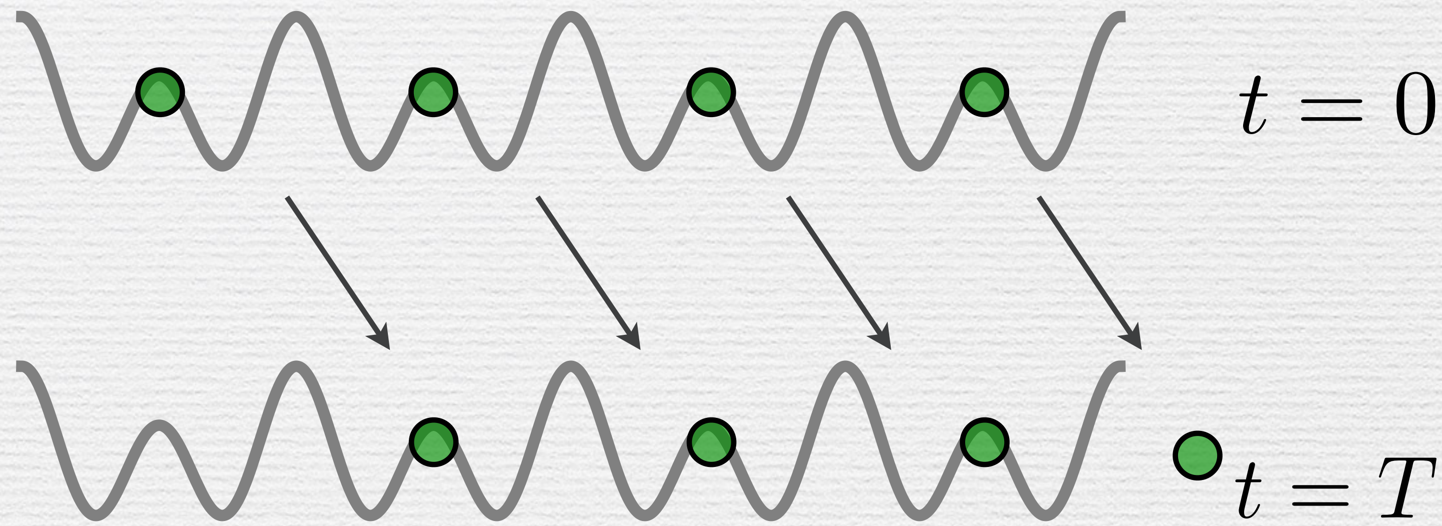
$$i\frac{\partial}{\partial t}|\Psi\rangle = H(x,t)|\Psi\rangle$$

Quantization of pumped charge

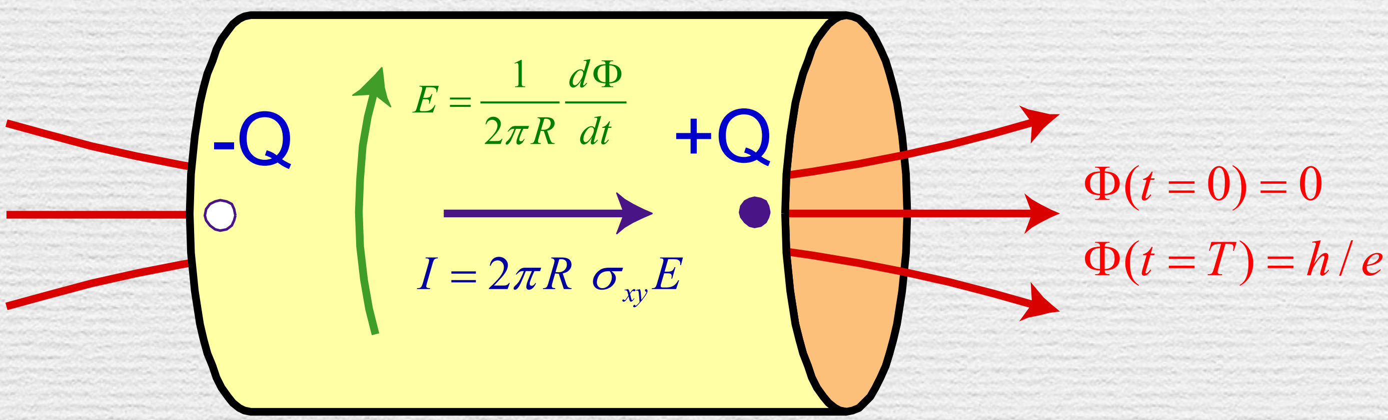


Connection to IQHE

$$\boxed{H(k_x, t) = H(k_x, t+T)}$$

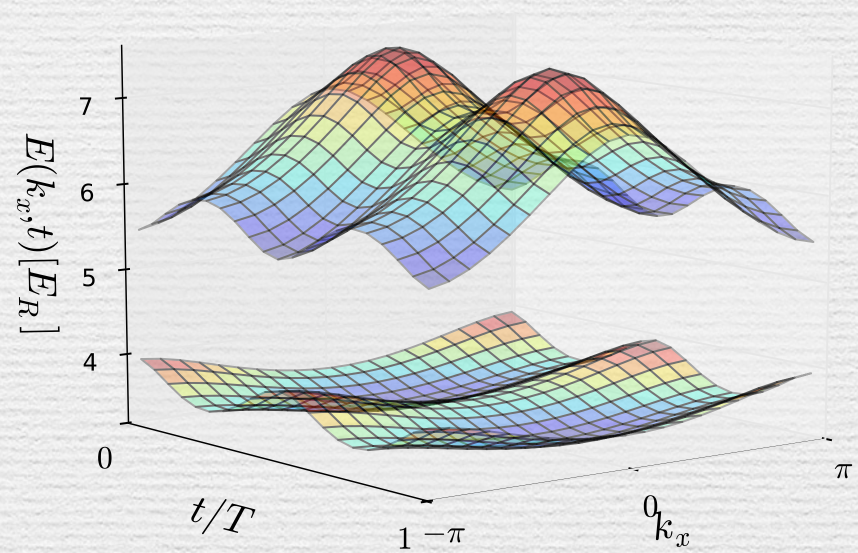


Adiabatically thread a quantum of magnetic flux through cylinder.



Laughlin, 1981

$V_1 = 4E_R \quad V_2 = 4E_R$



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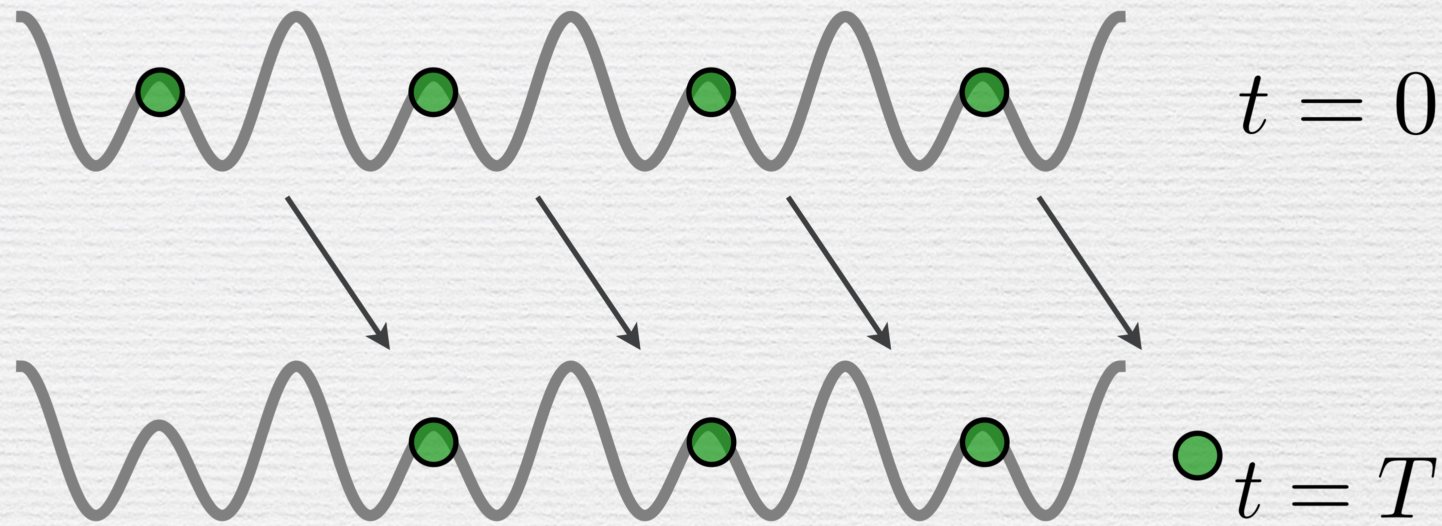

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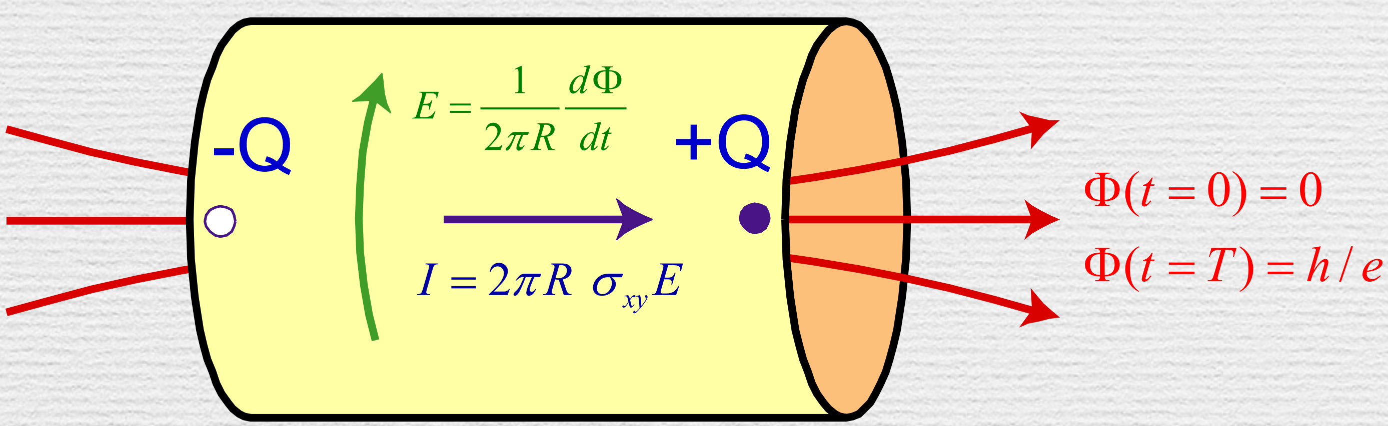

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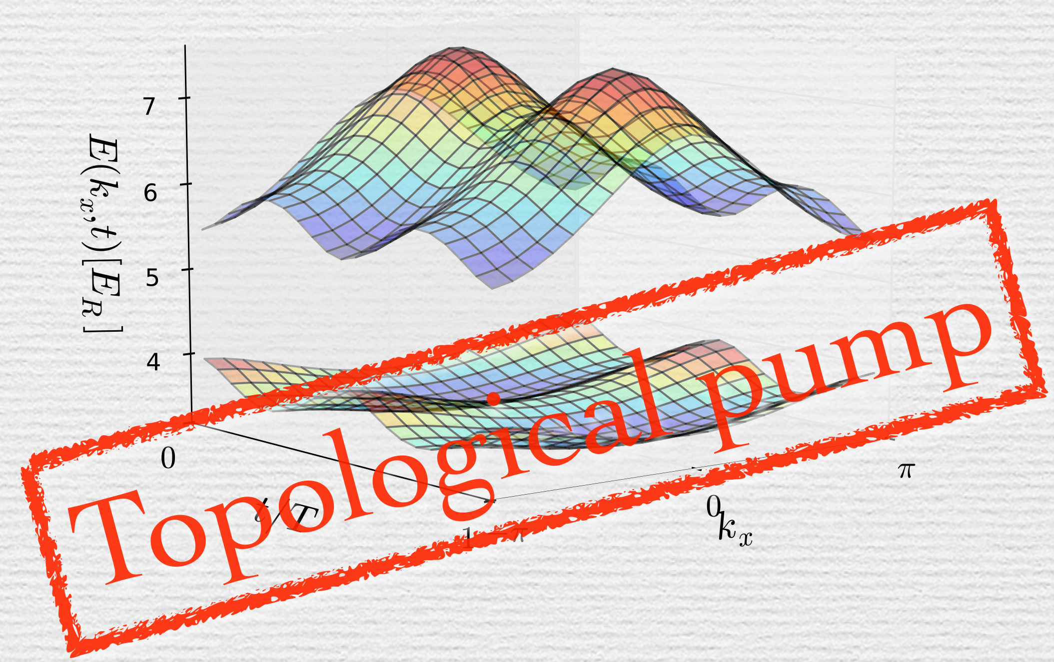

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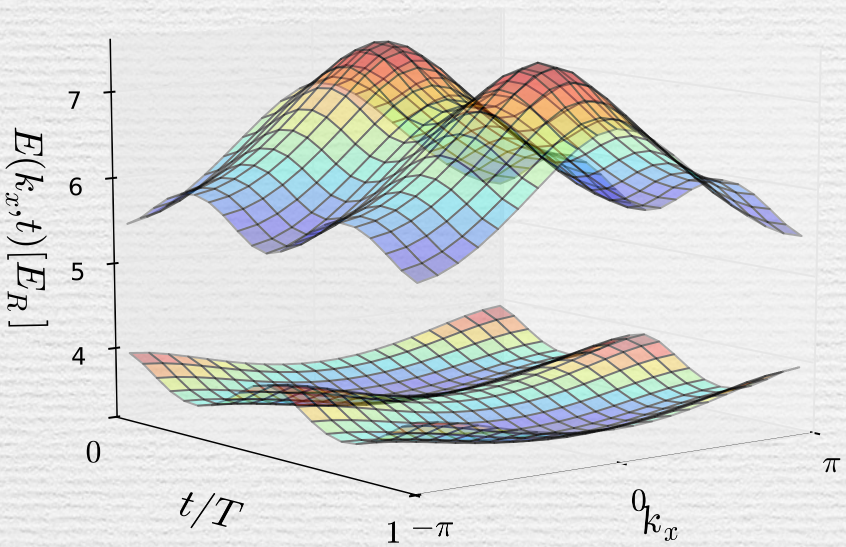

$$V_1 \cos^2\left(\frac{2\pi x}{d}\right) + V_2 \cos^2\left(\frac{\pi x}{d} - \frac{\pi t}{T}\right)$$

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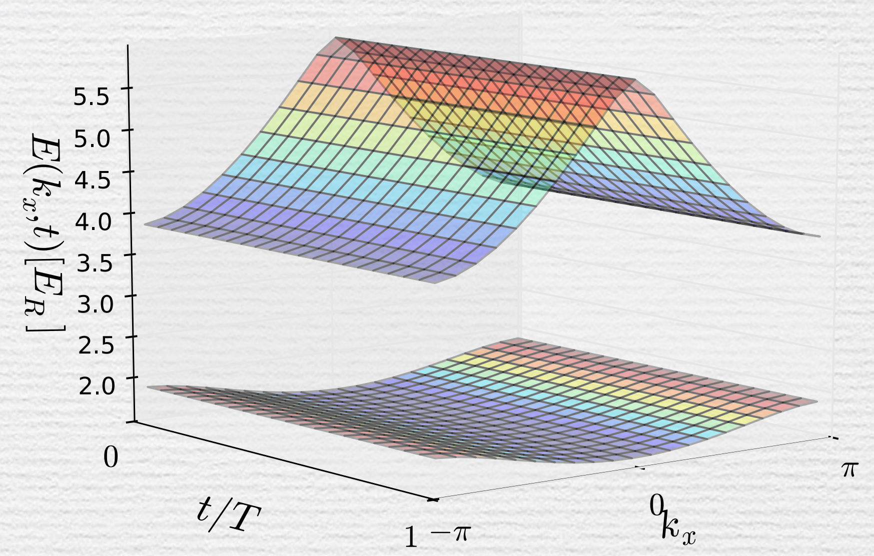

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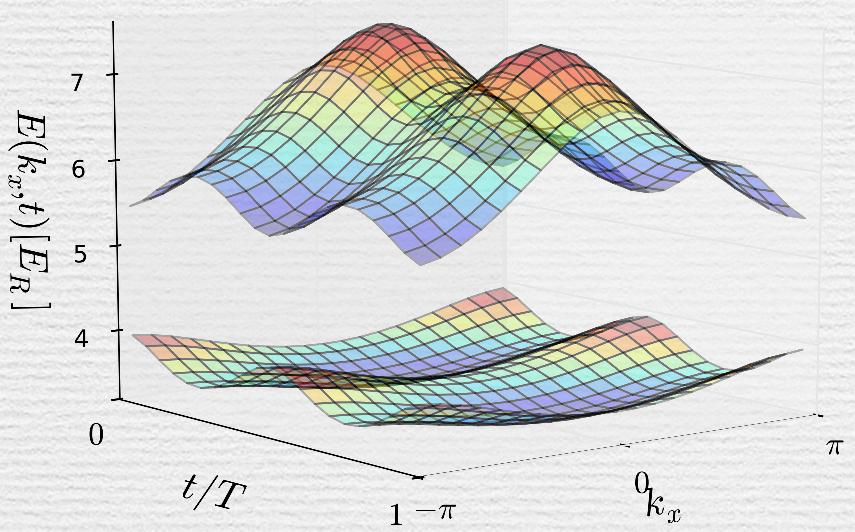

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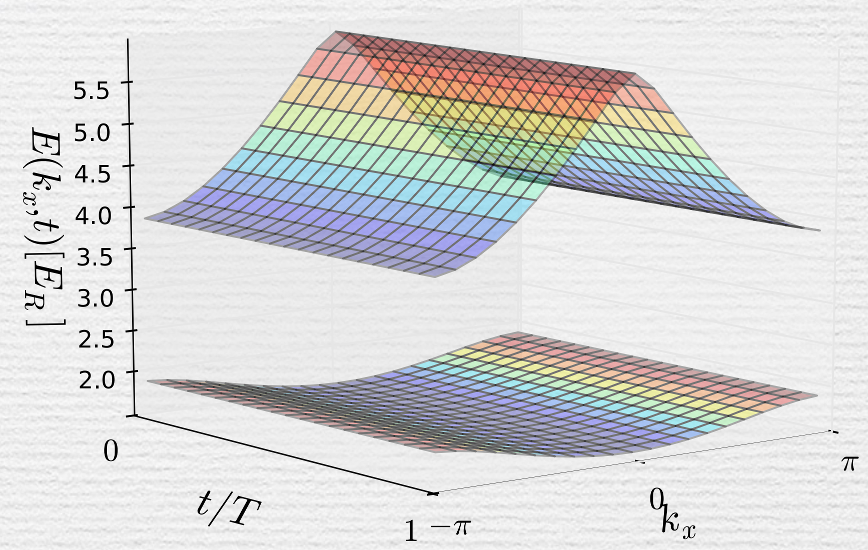

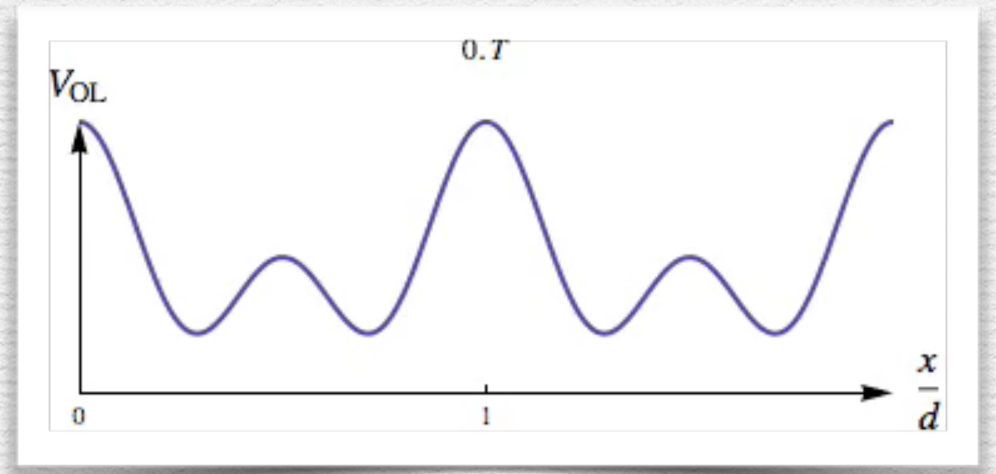

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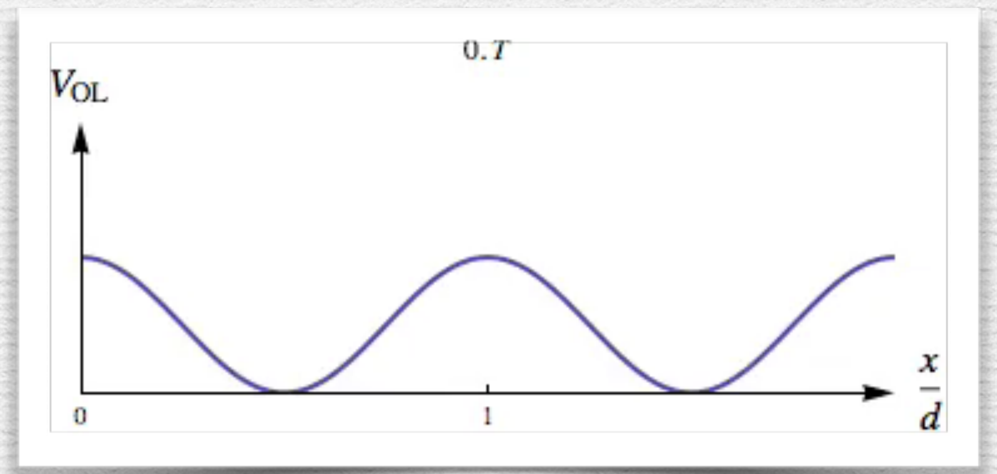
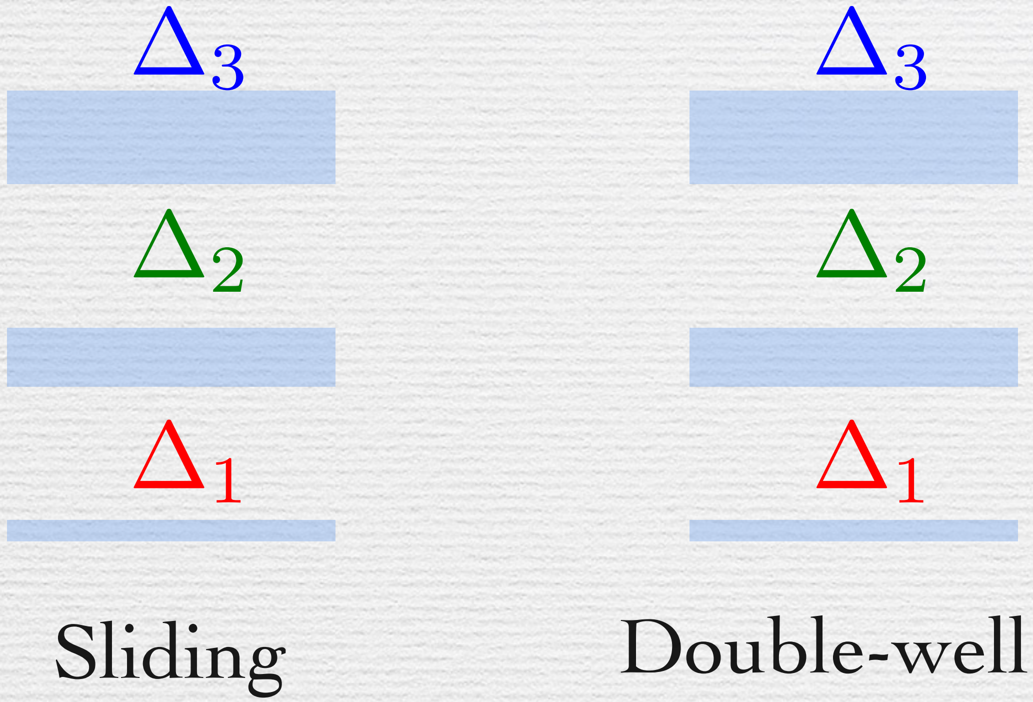
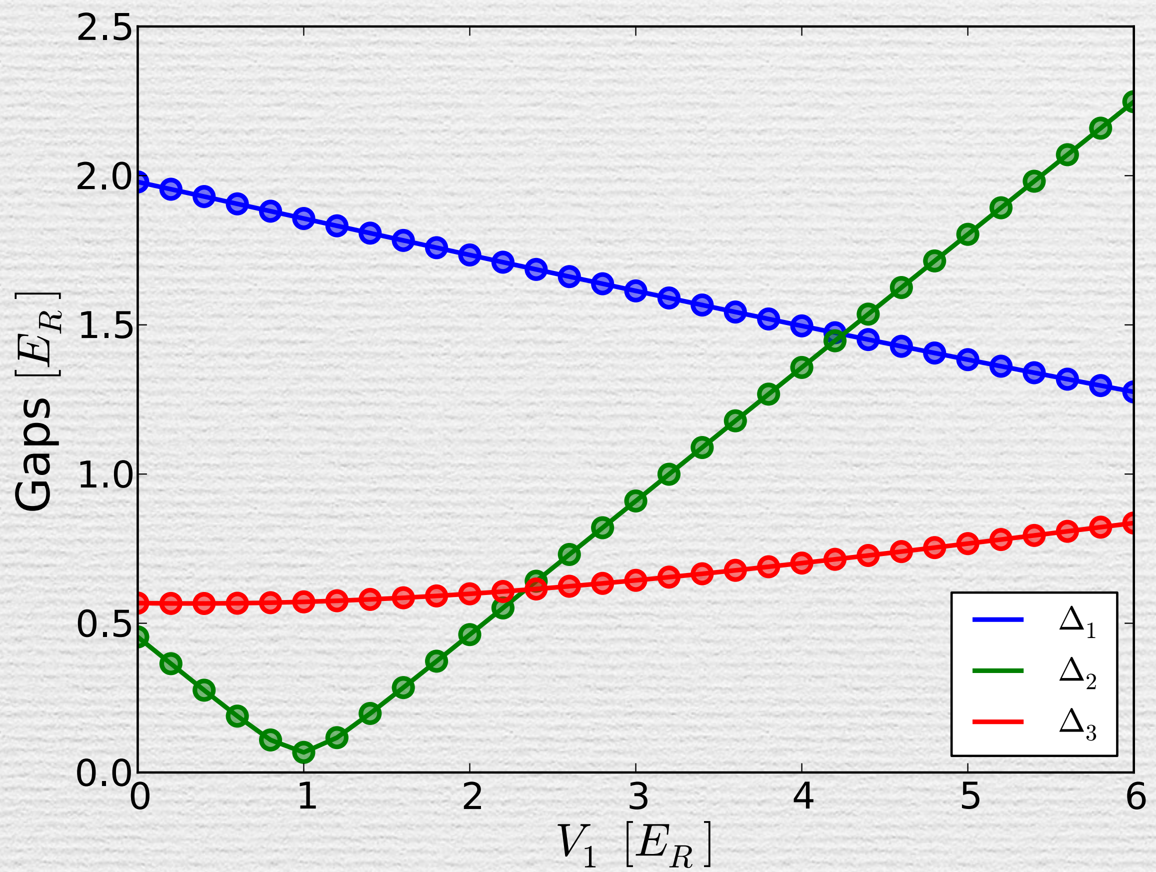


Higher bands





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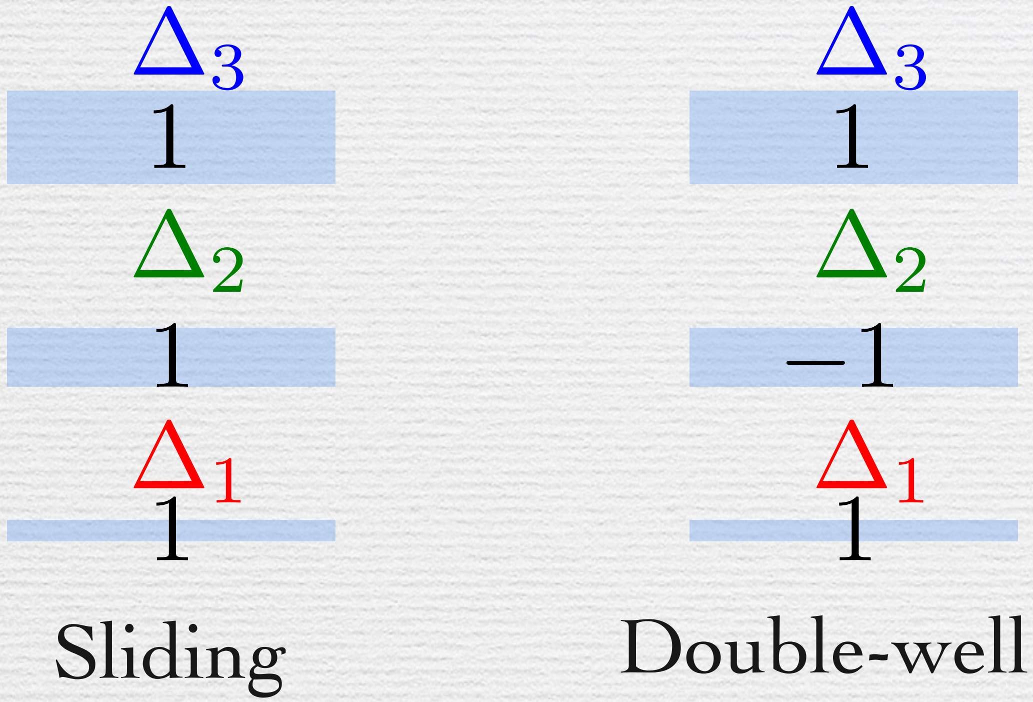

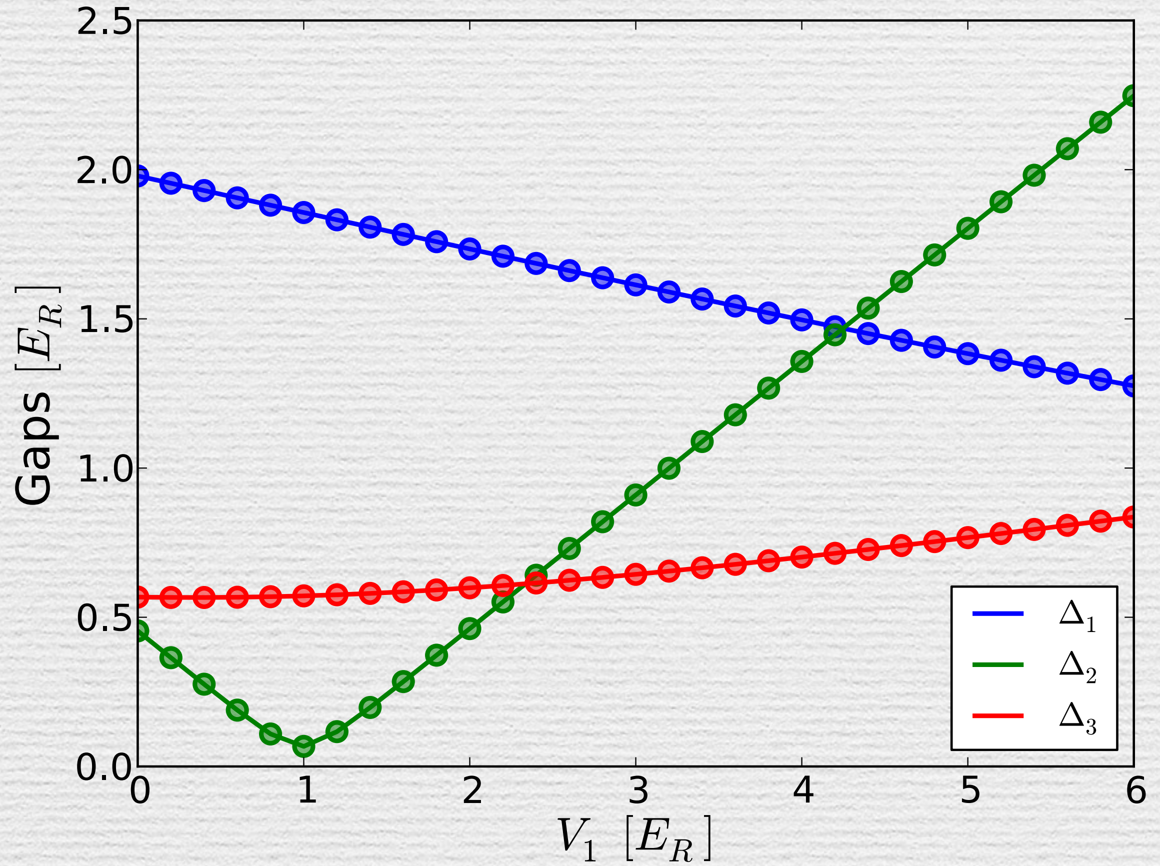

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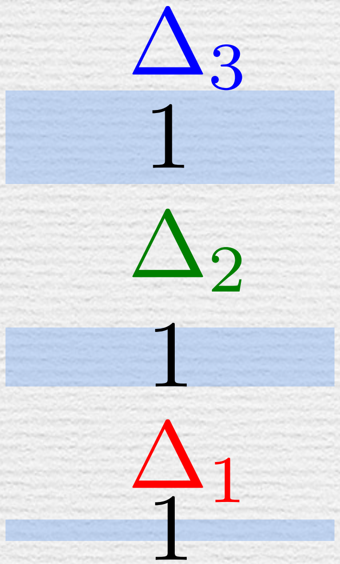
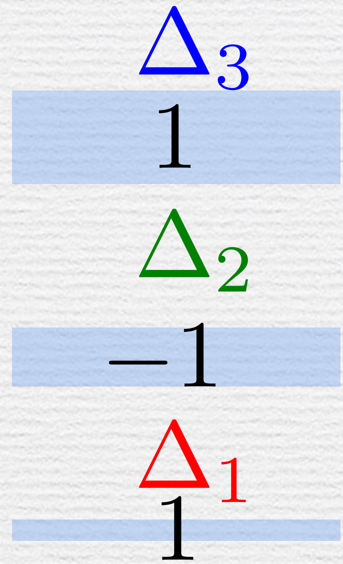
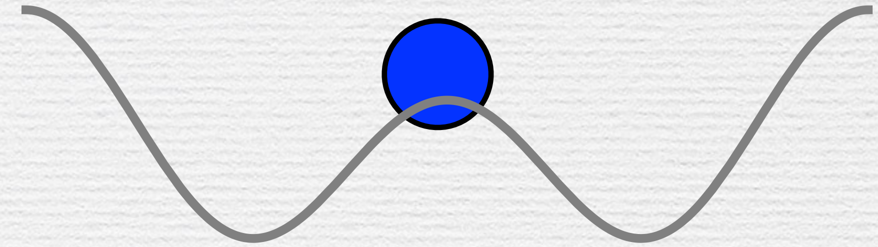
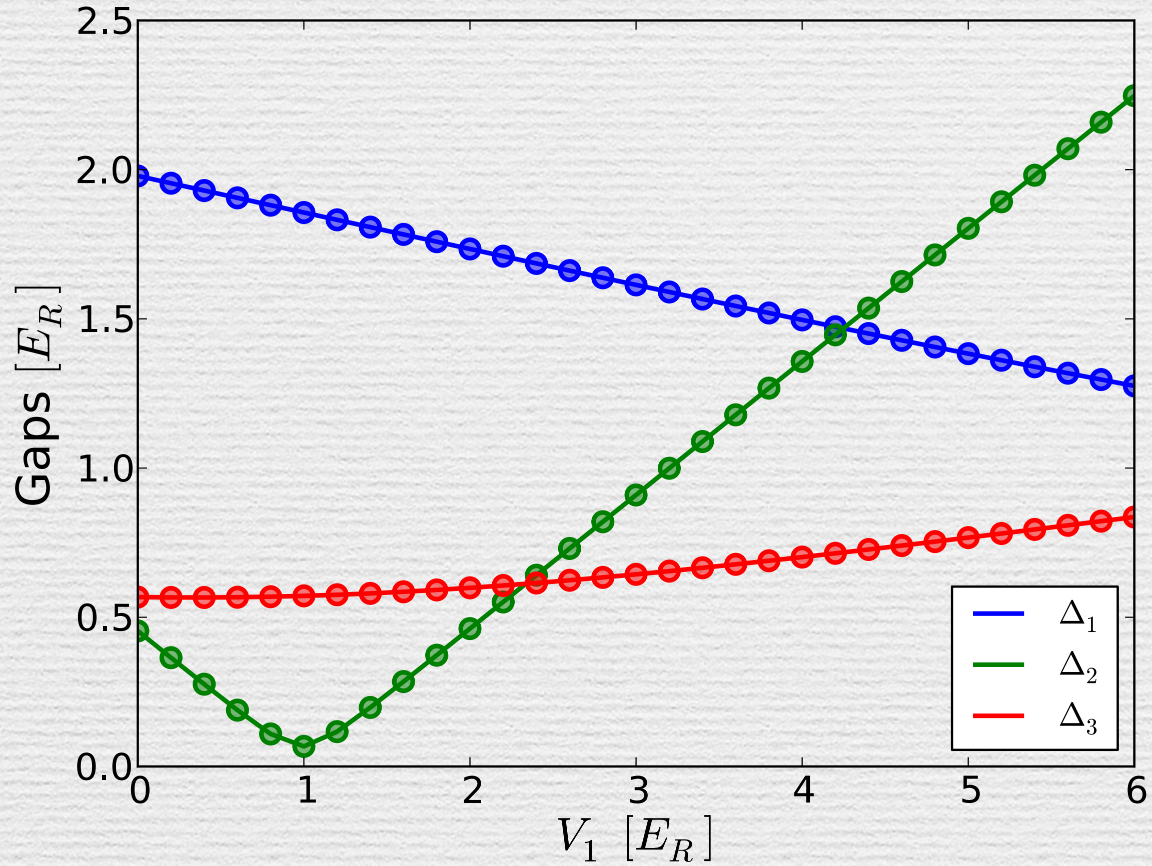


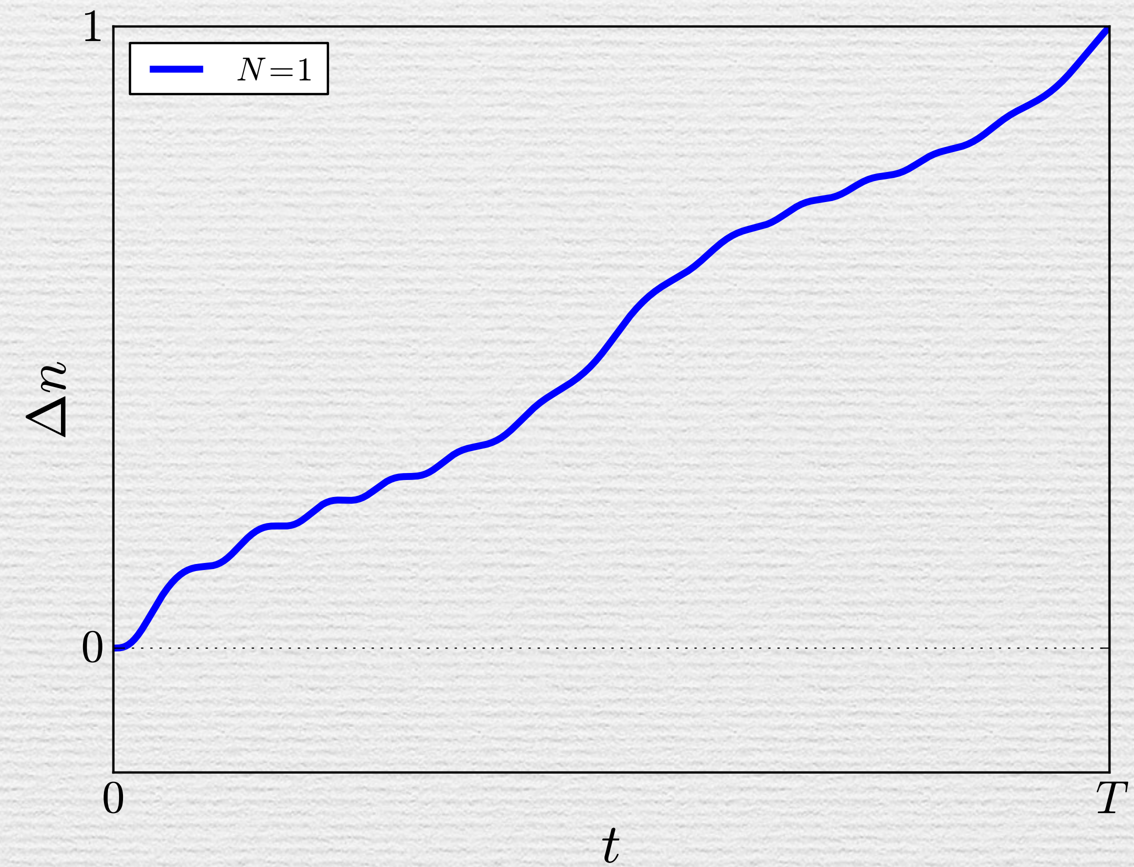
Sliding



Double-well







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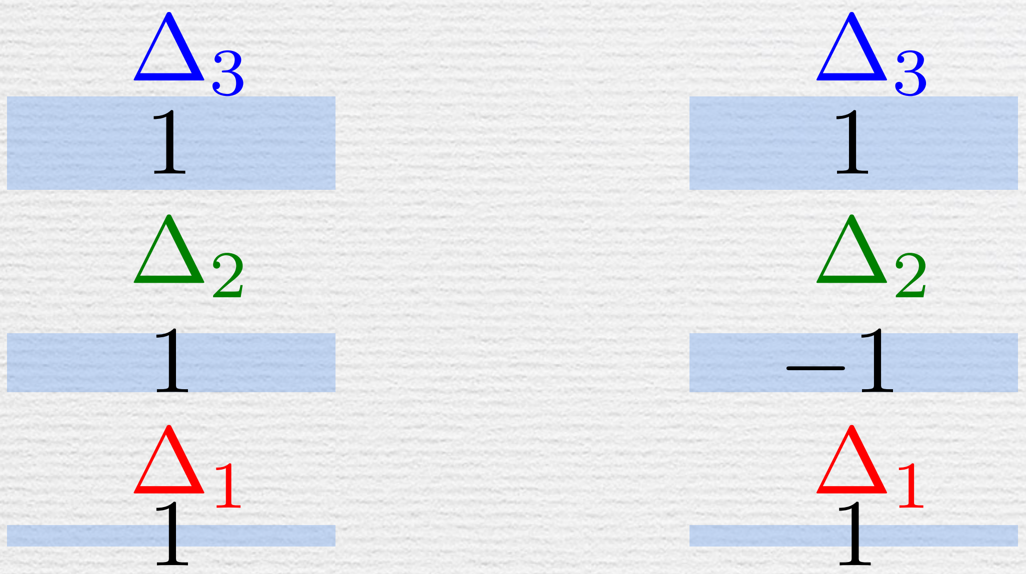

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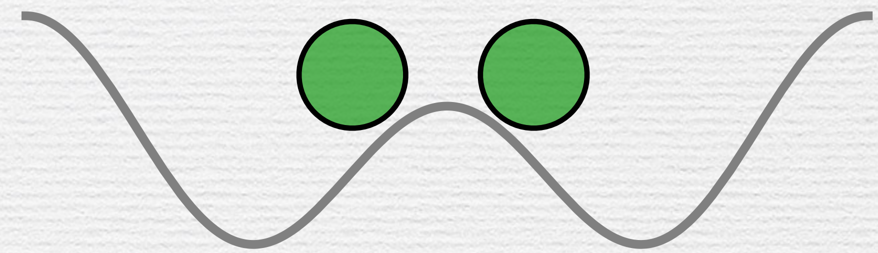

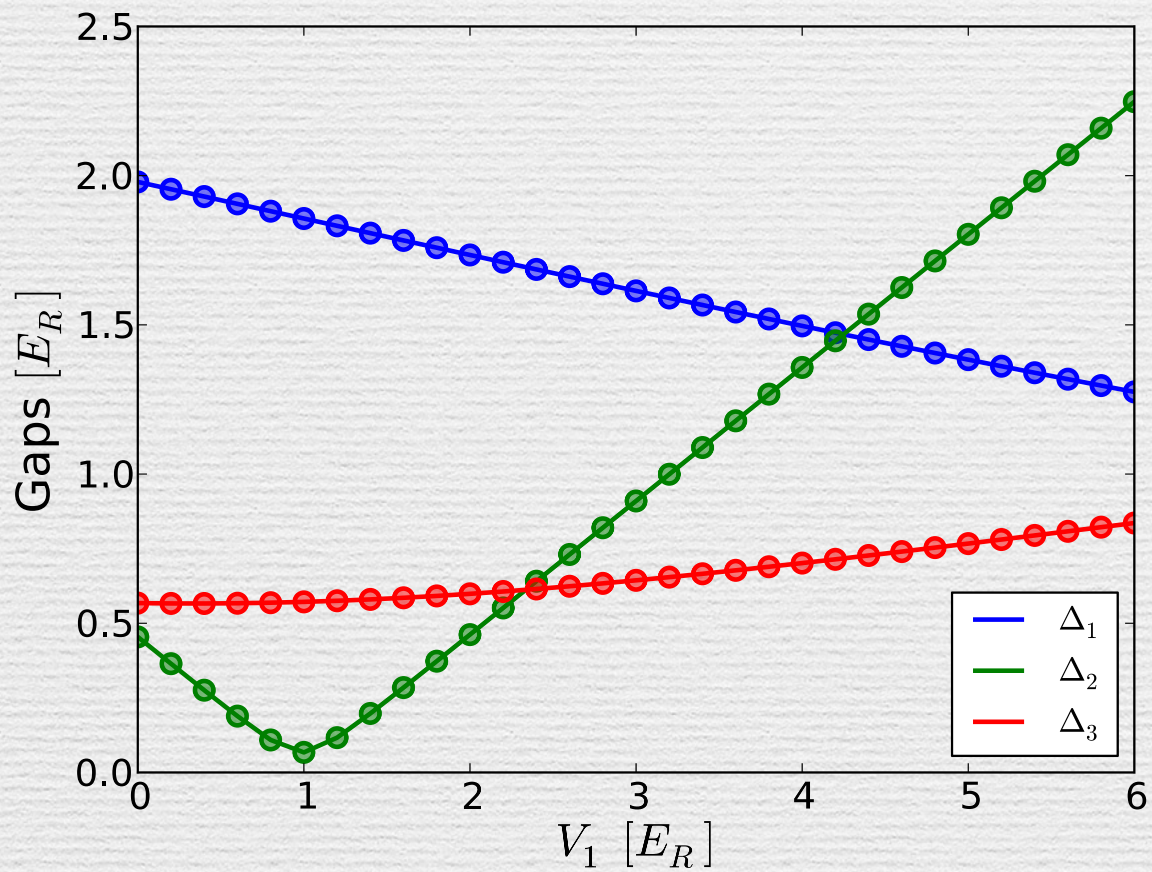

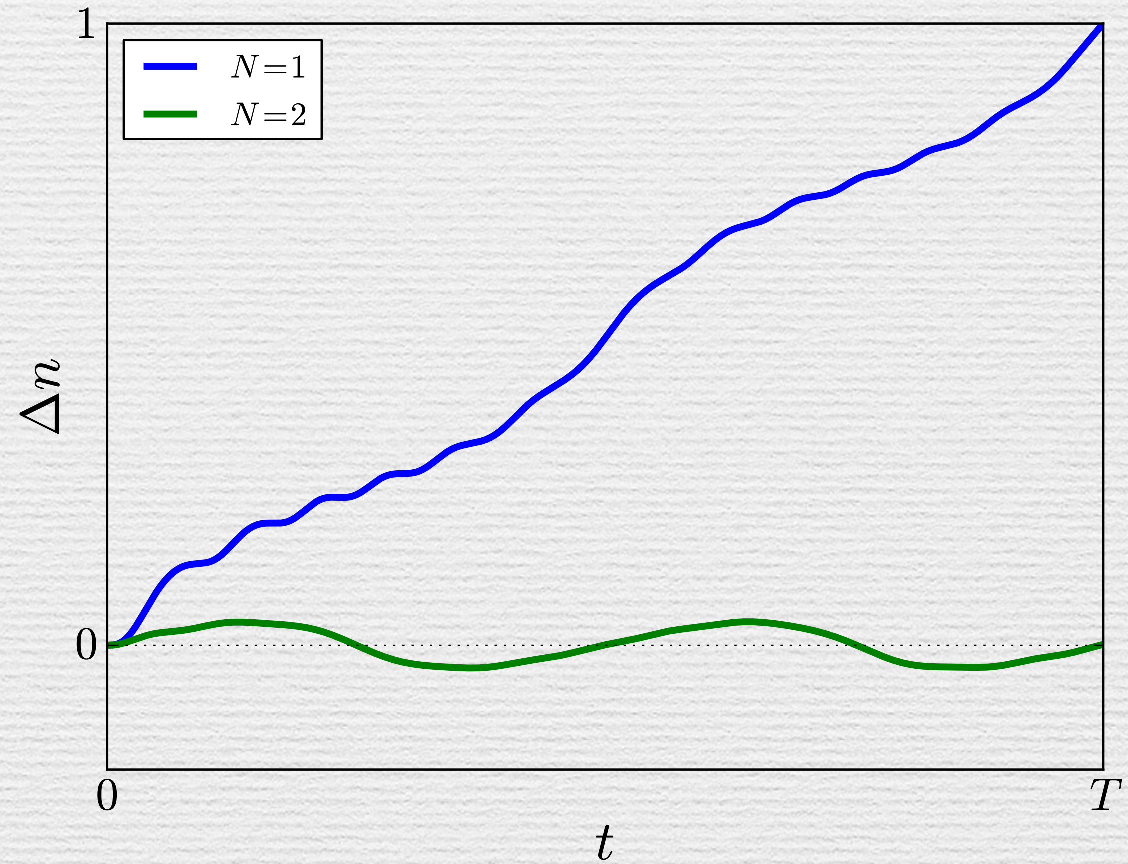

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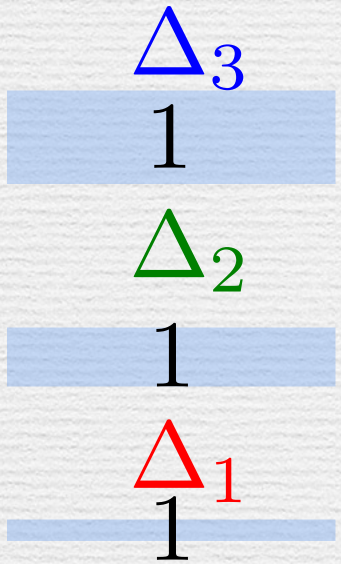

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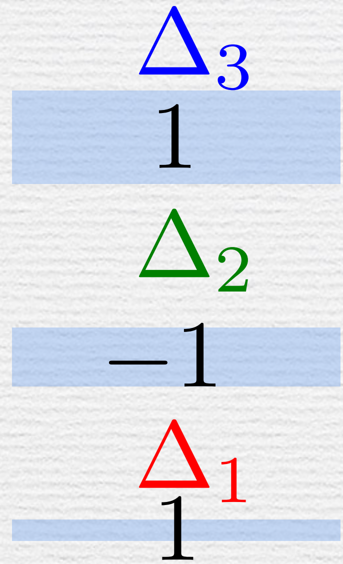

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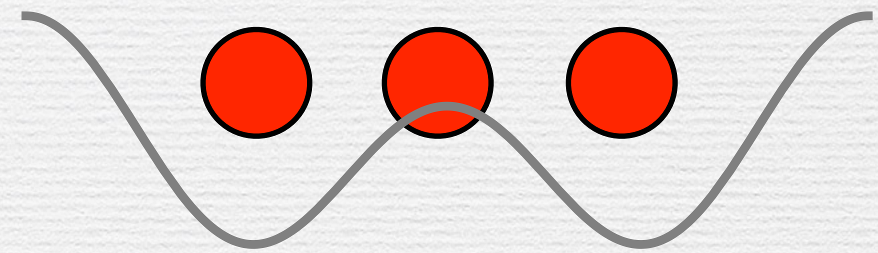
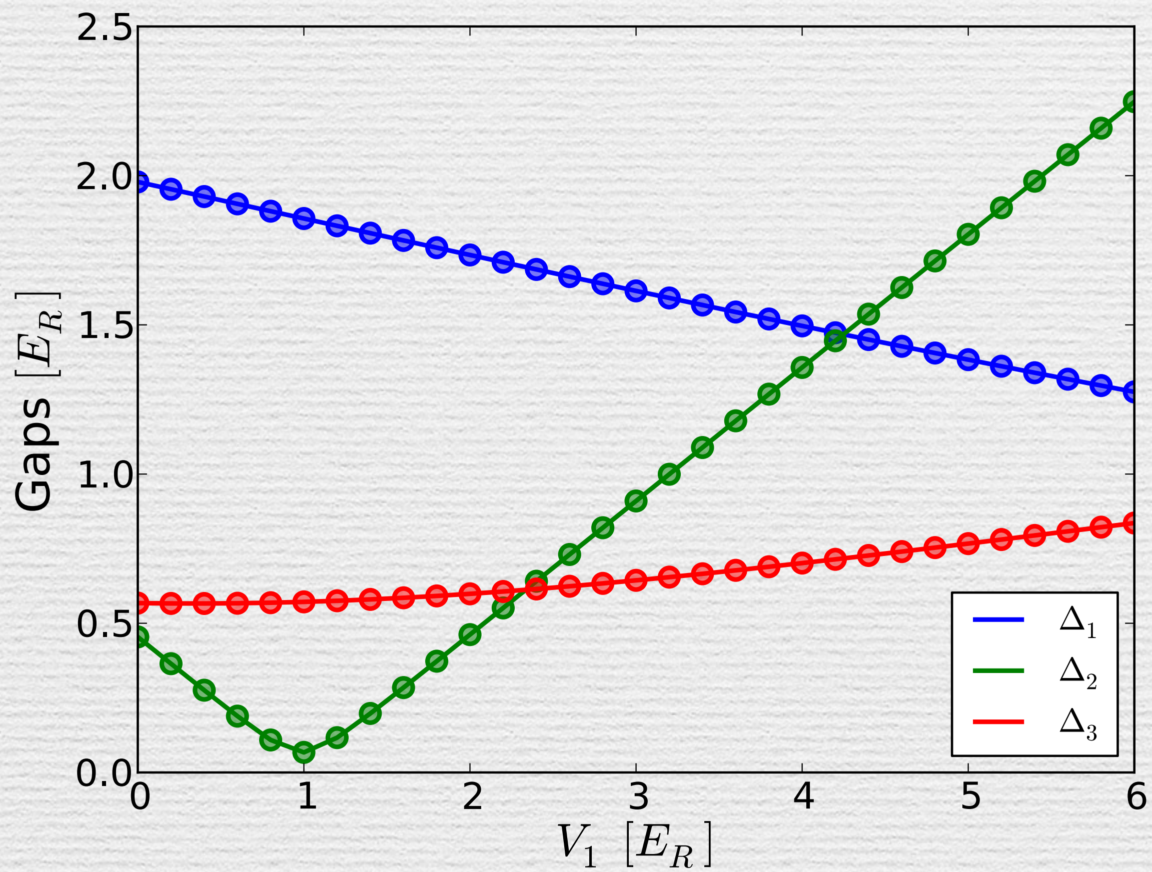
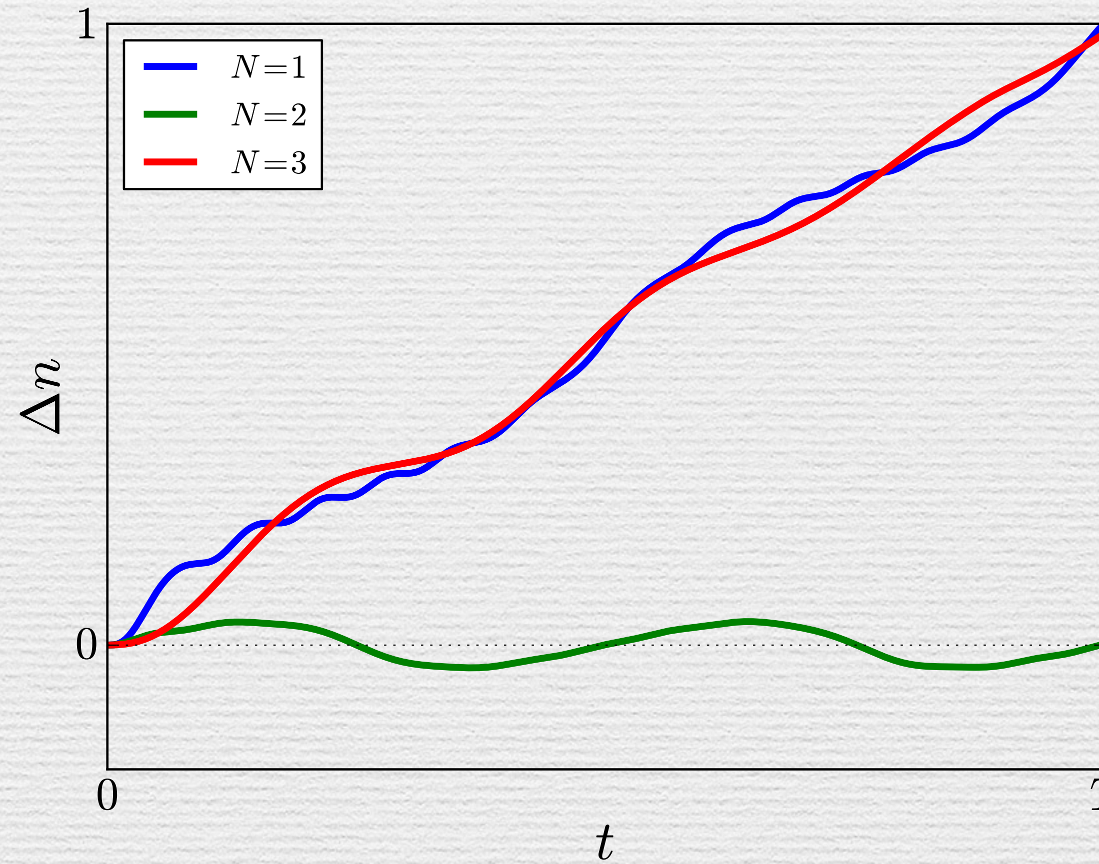




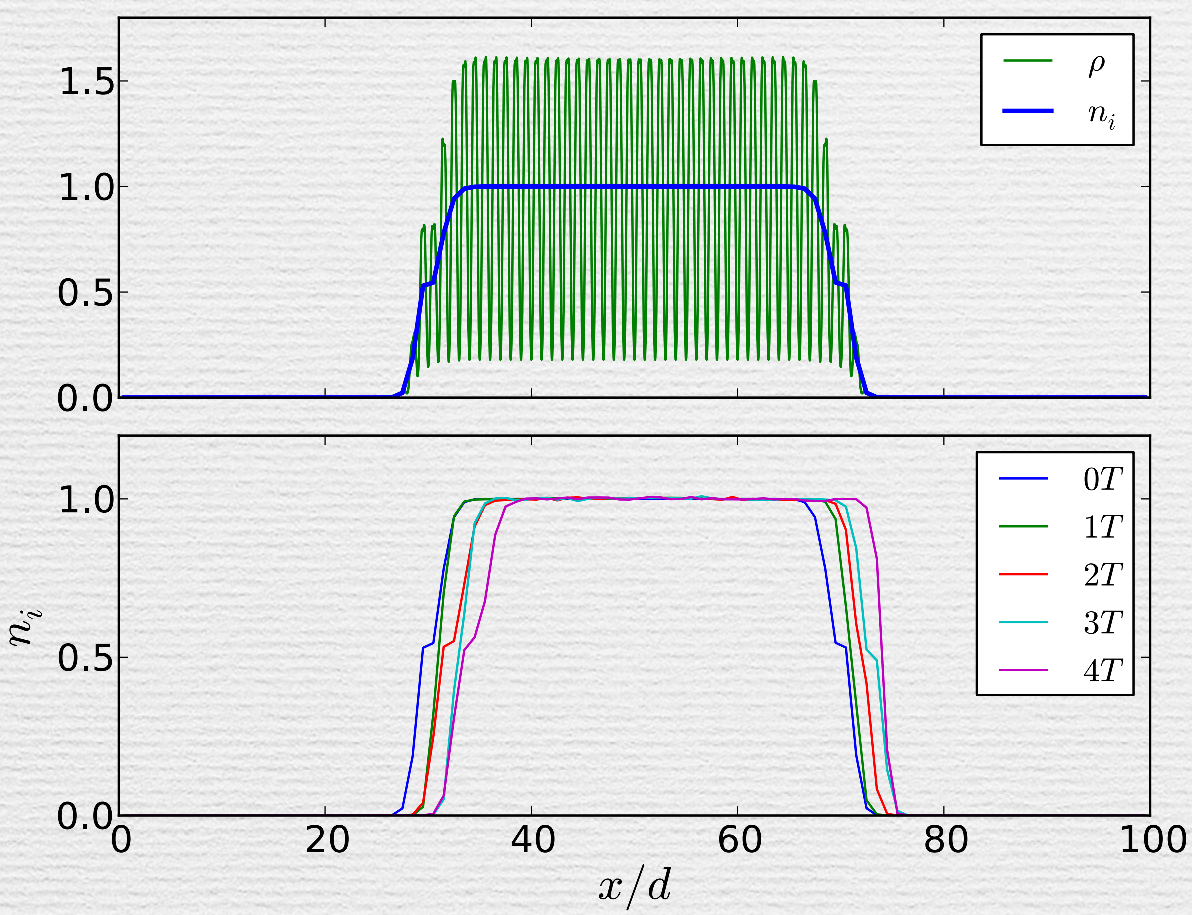


Practical issues

- Detection
- External trap
- Temperature effect
- Non-adiabatic effect

Trapping & Detection

LW, Troyer and Dai, 1301.7435
PRL in press



$$\boxed{\Delta n = \langle x \rangle / d}$$

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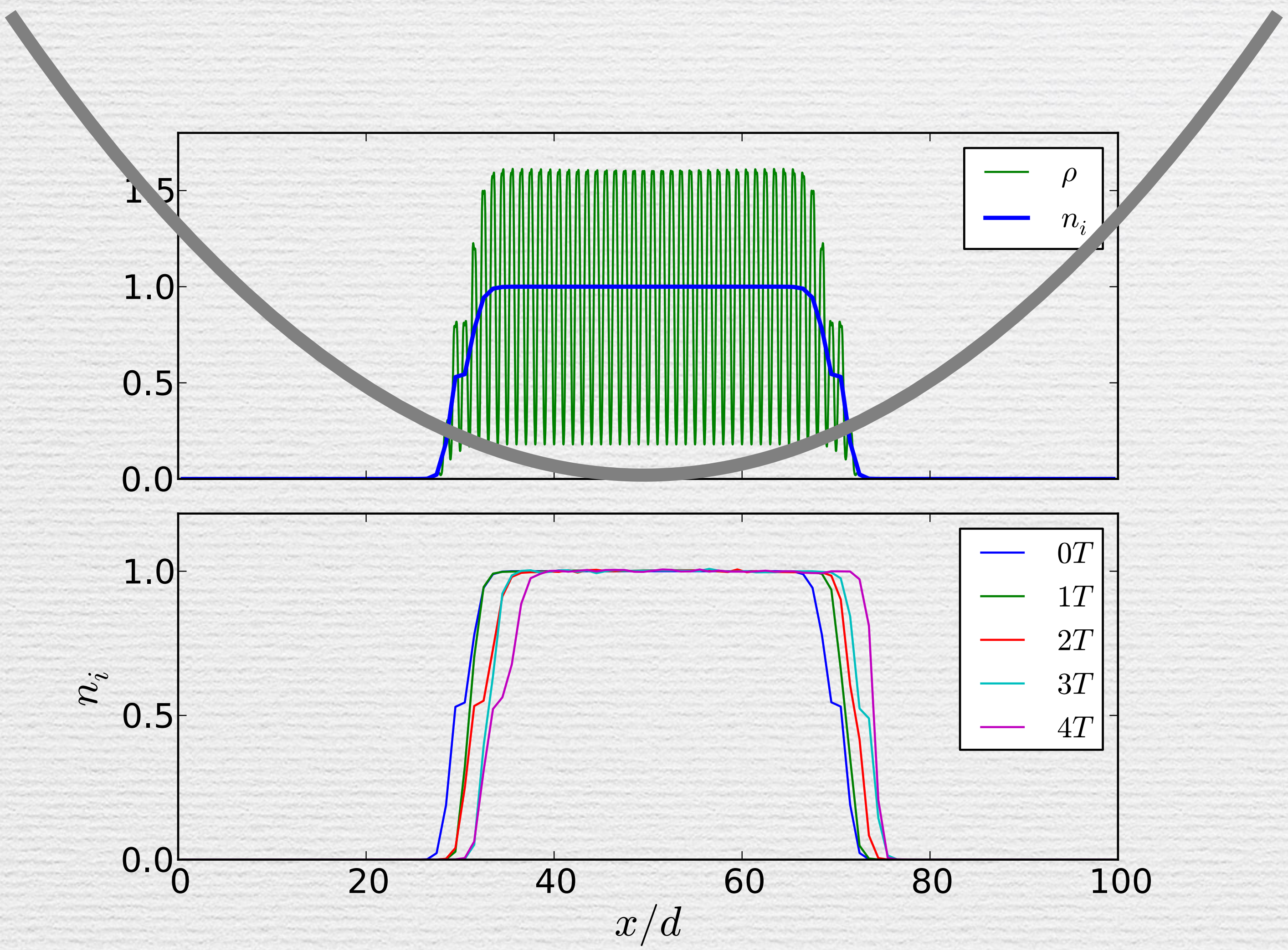

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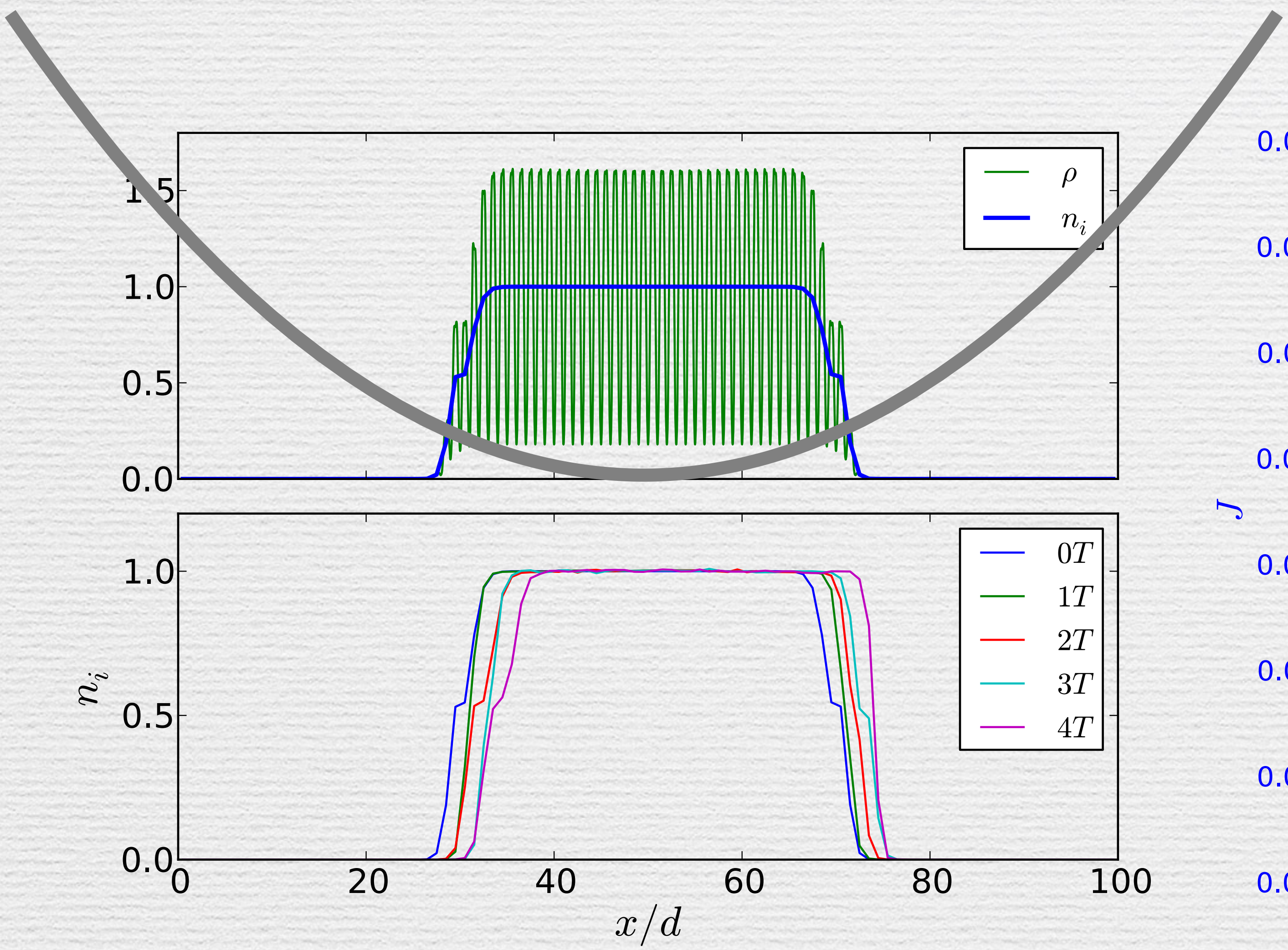

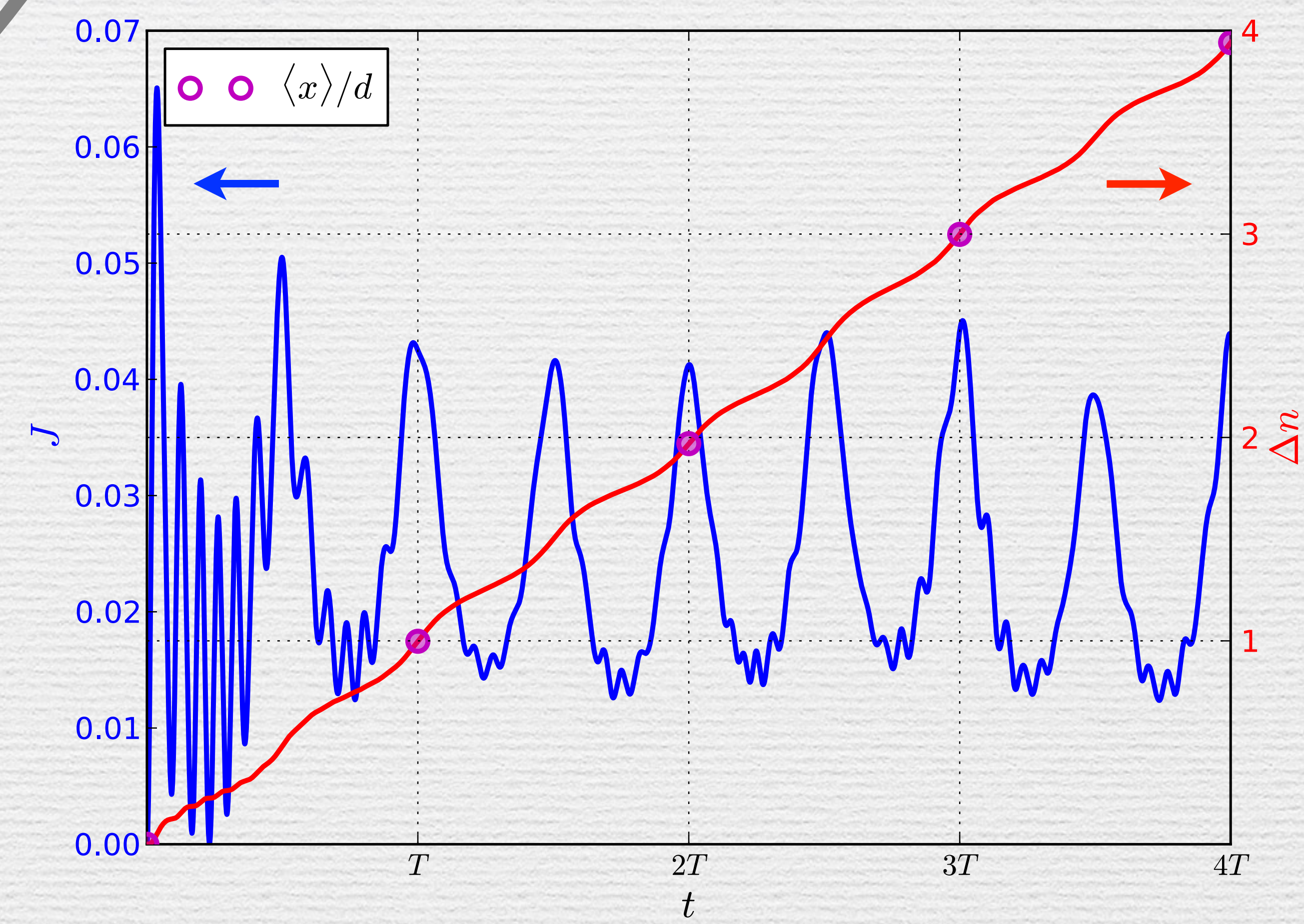

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Temperature & Non-adiabatic effect

$$\text{Temperature} \ll \frac{\Delta}{k_B} \qquad\qquad T \gg \frac{\hbar}{\Delta}$$

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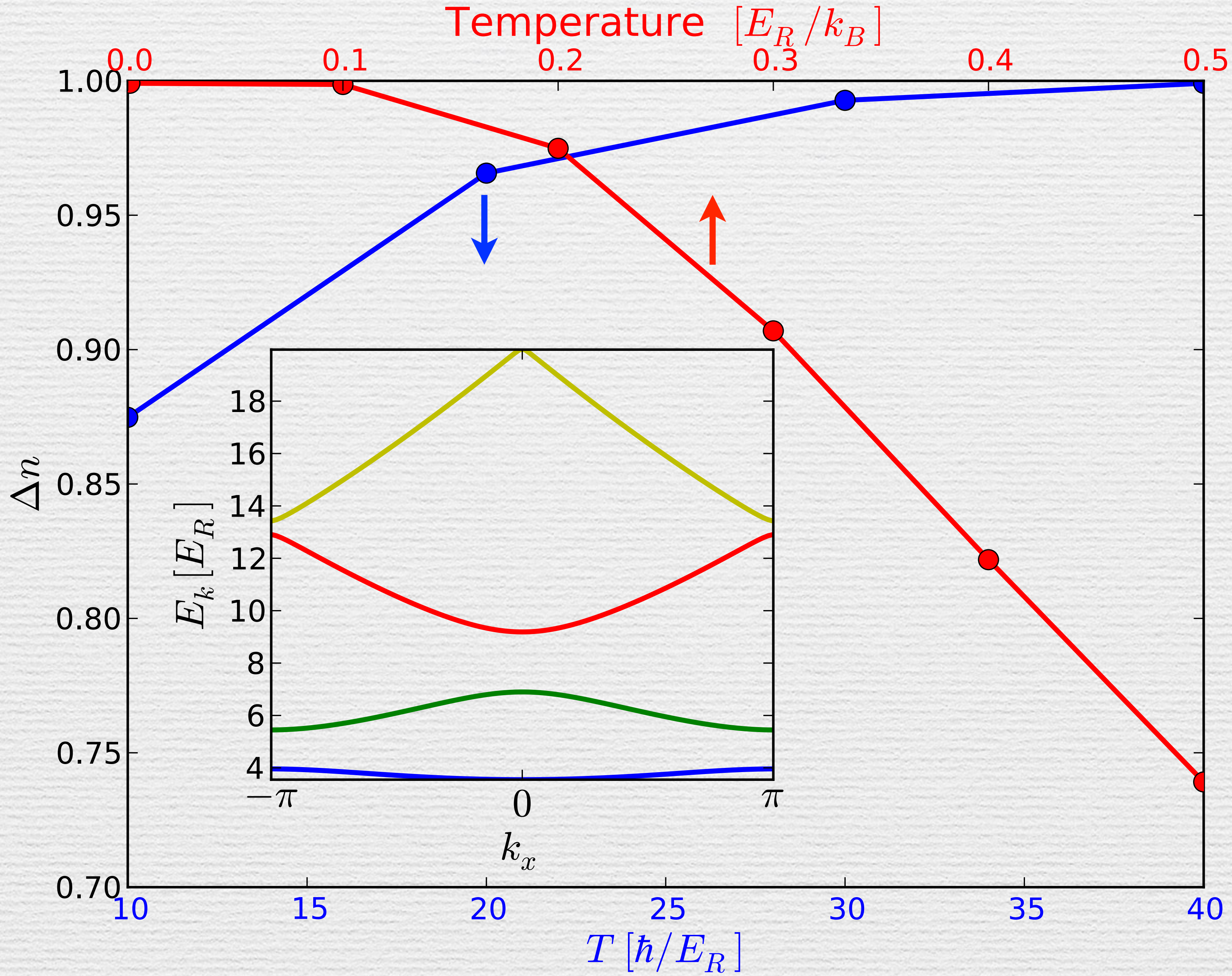
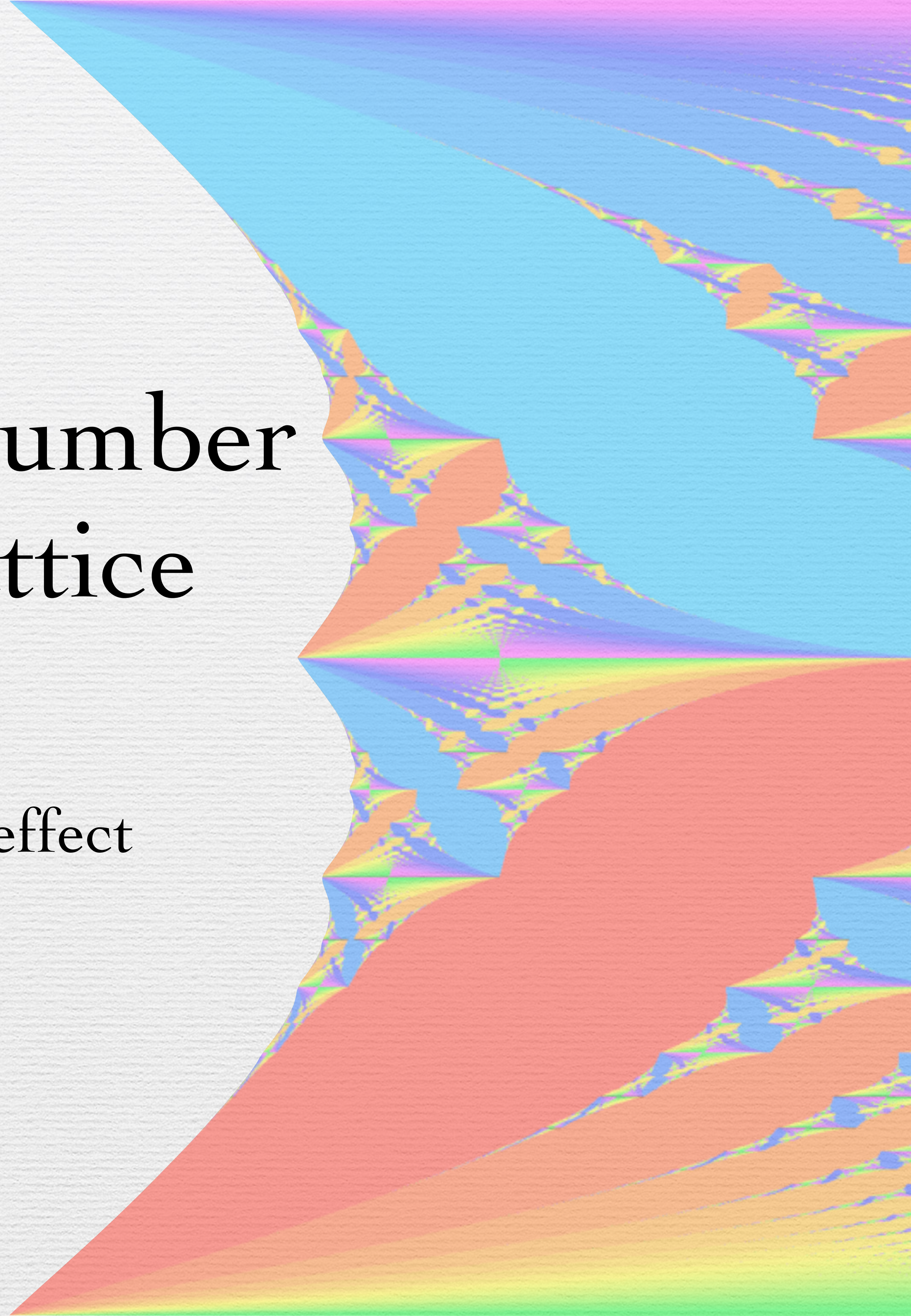


Measure Chern number of 2D optical lattice

with

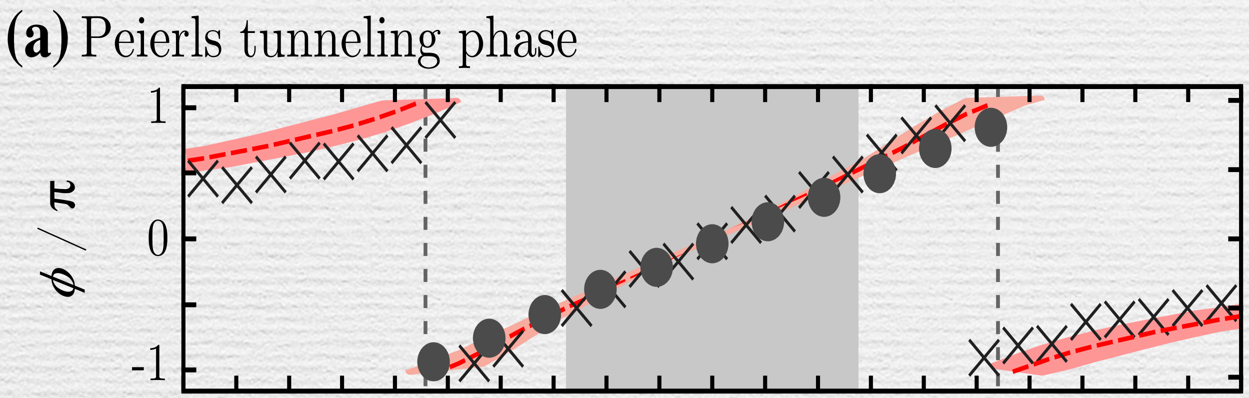
Topological pumping effect



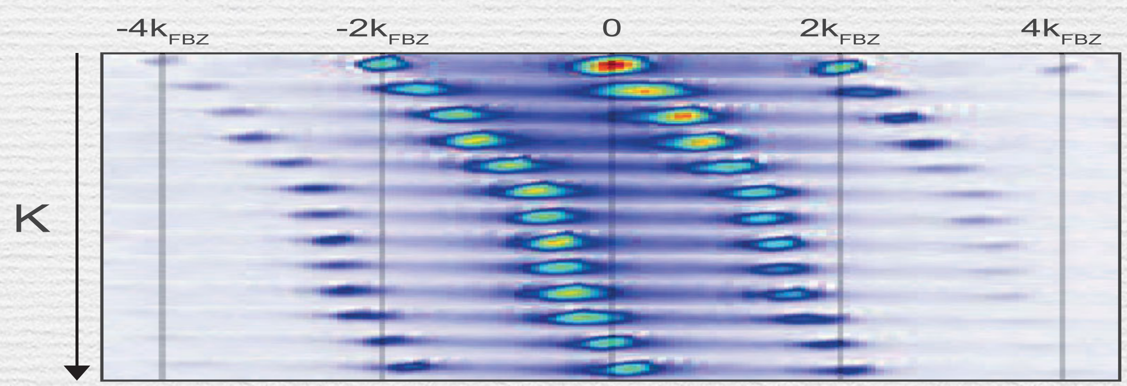
Synthetic gauge-field in optical lattices

- Imprint complex phases to the hopping amplitude

- 1D Peierls lattice NIST, Hamburg $$H = -J \sum_m e^{i2\pi\Phi} c^\dagger_{m+1} c_m + H.c.$$



Jimenez-Garcıa *et al*



Struck *et al*

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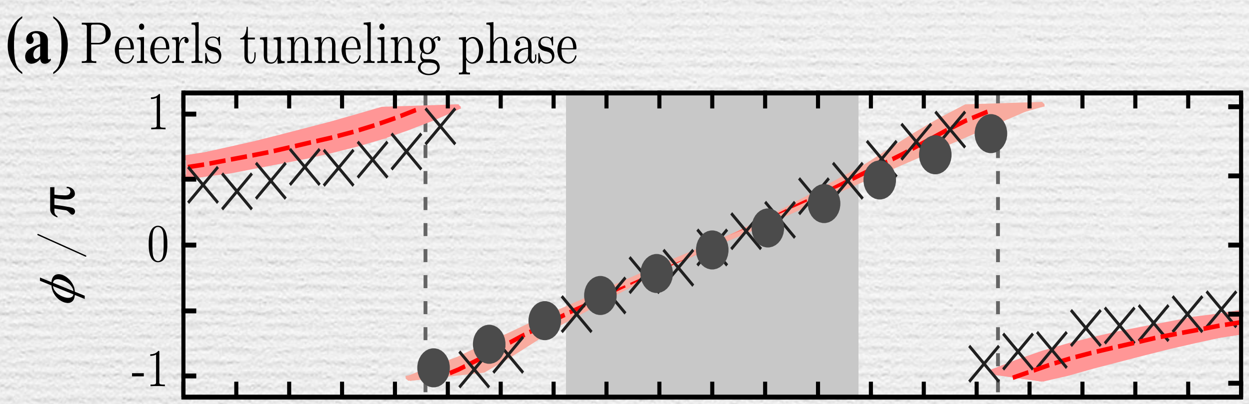
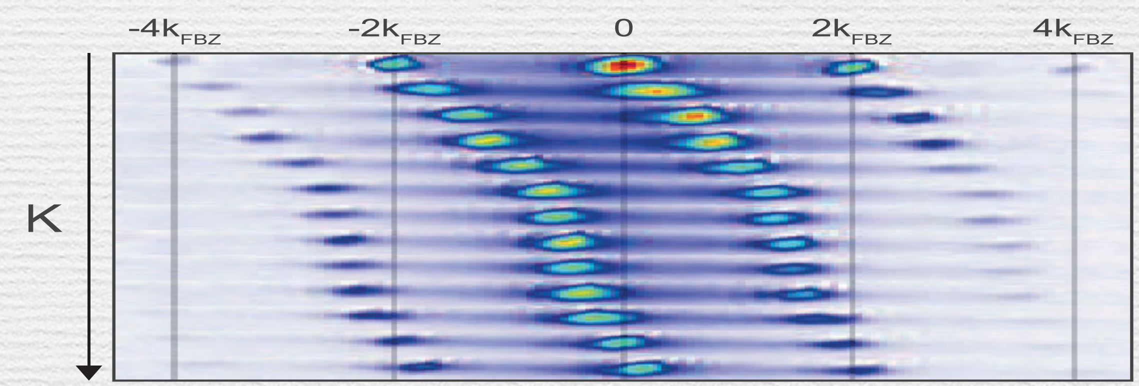
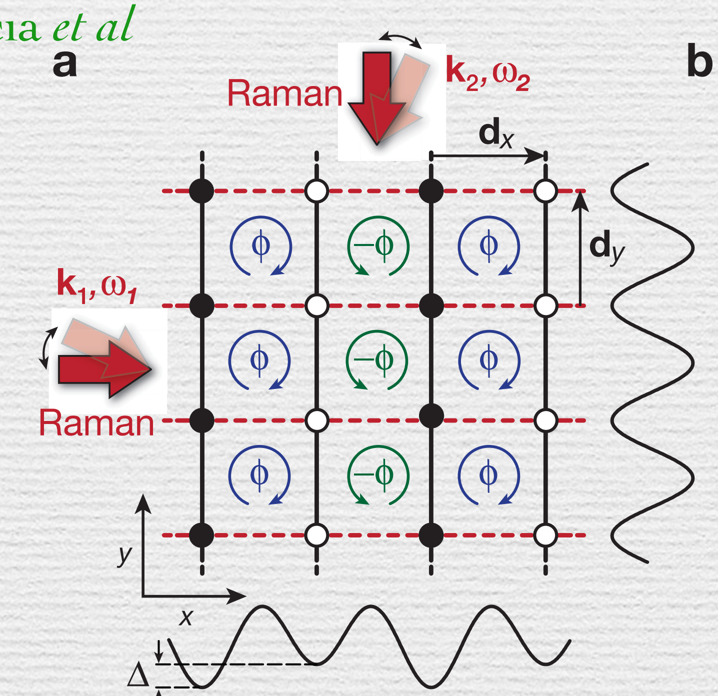




Jimenez-Garcia *et al*

Struck *et al*

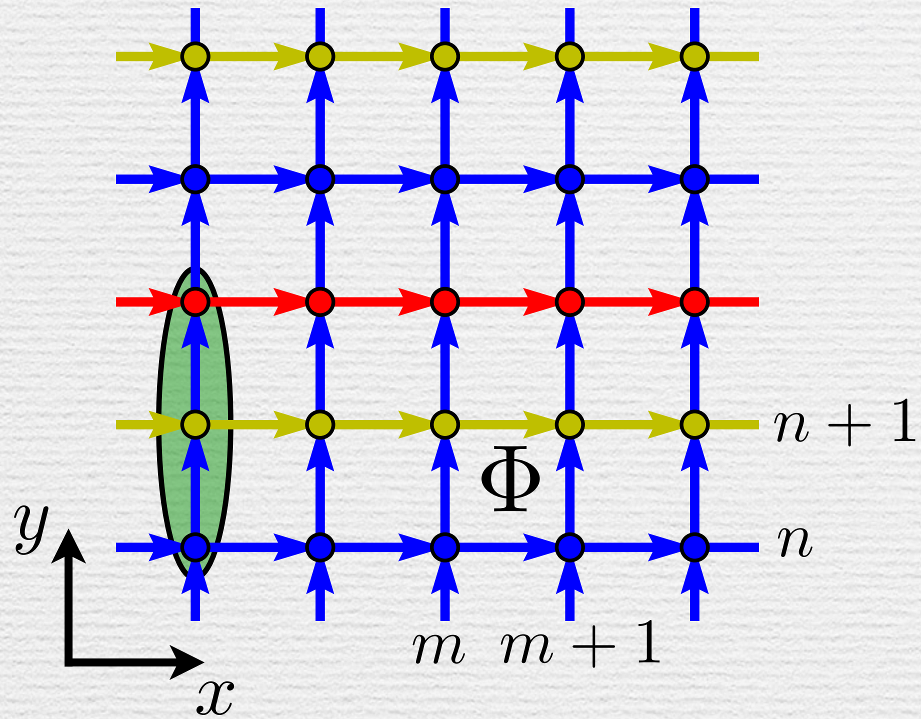
- Staggered flux lattice Munich



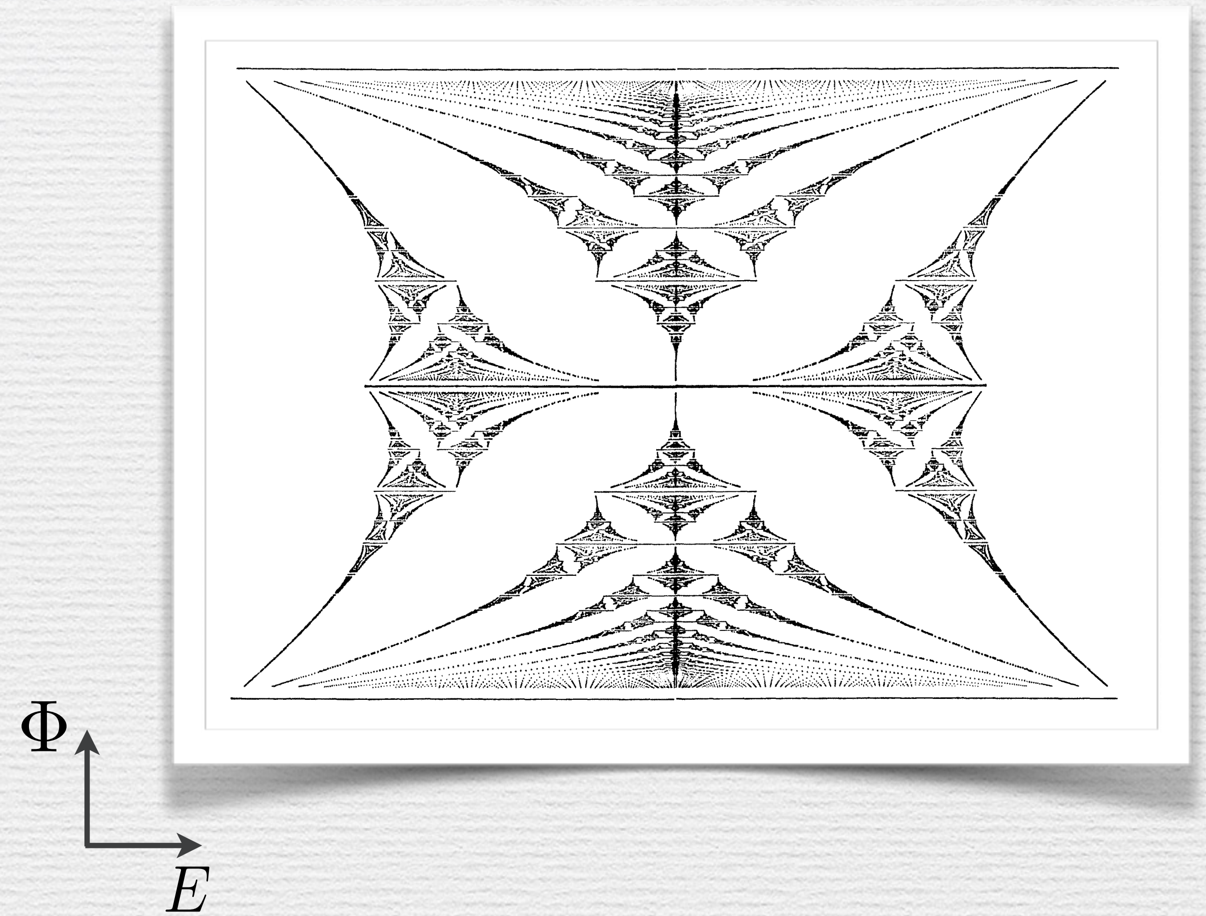
Aidelsburger *et al*

Hofstadter optical lattice

$$H = -J \sum_{m,n} e^{i2\pi n\Phi} c^\dagger_{m+1,n} c_{m,n} + c^\dagger_{m,n+1} c_{m,n} + H.c. \quad \Phi = p/q$$



Hofstadter, 1976



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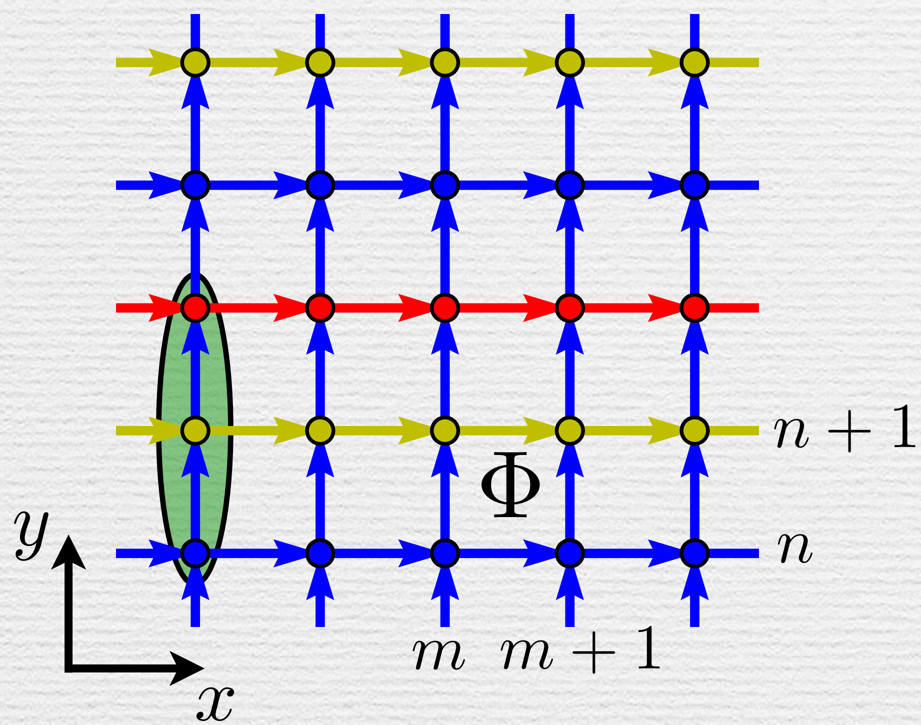
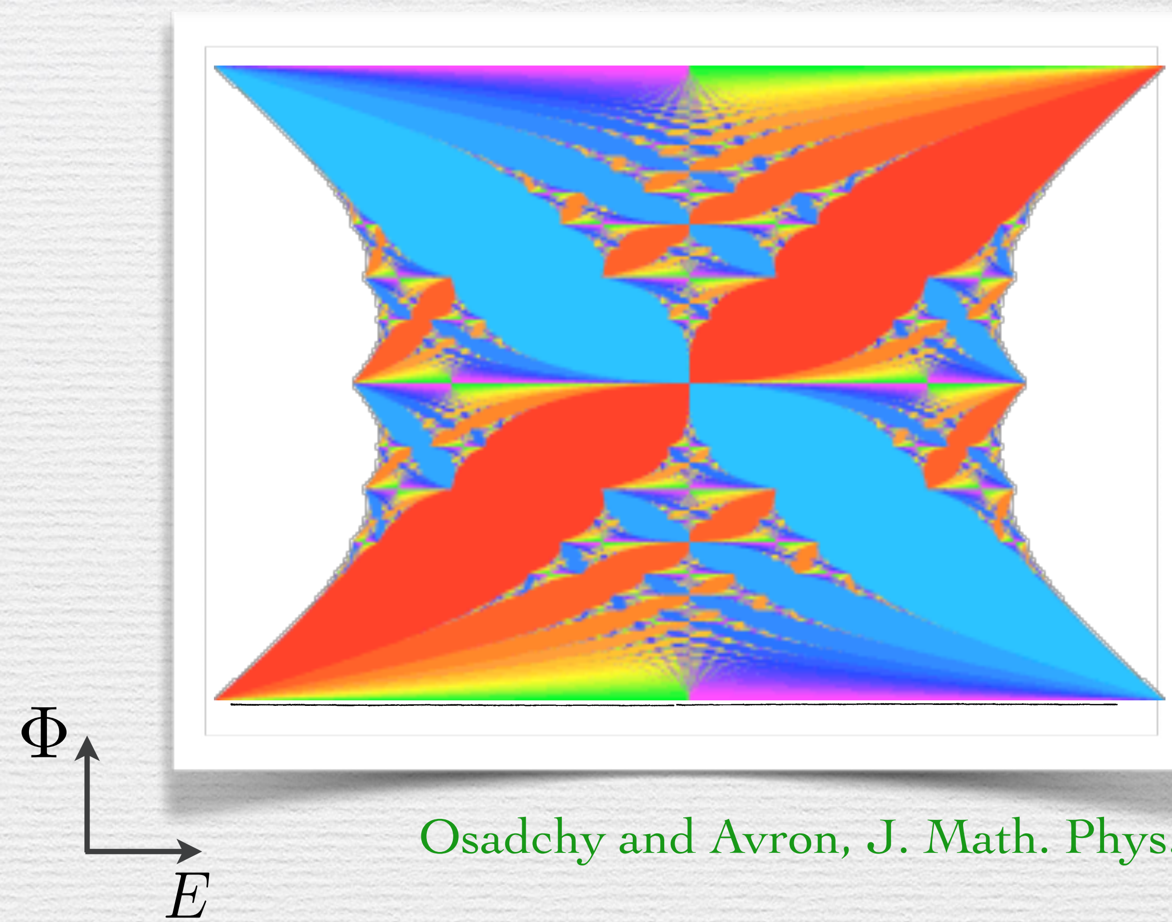
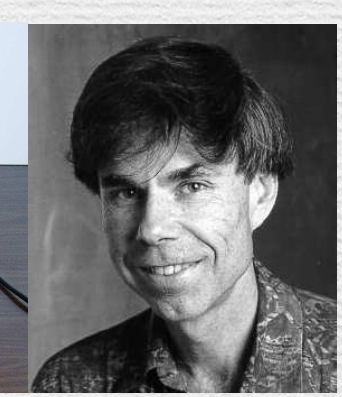


Hofstadter, 1976

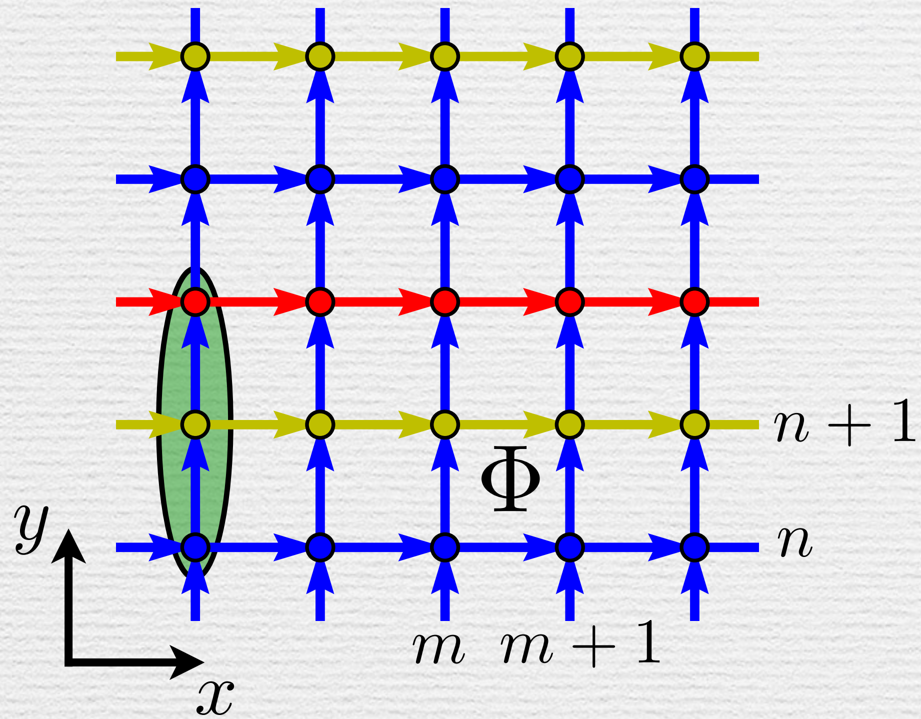


Osadchy and Avron, J. Math. Phys. 2001

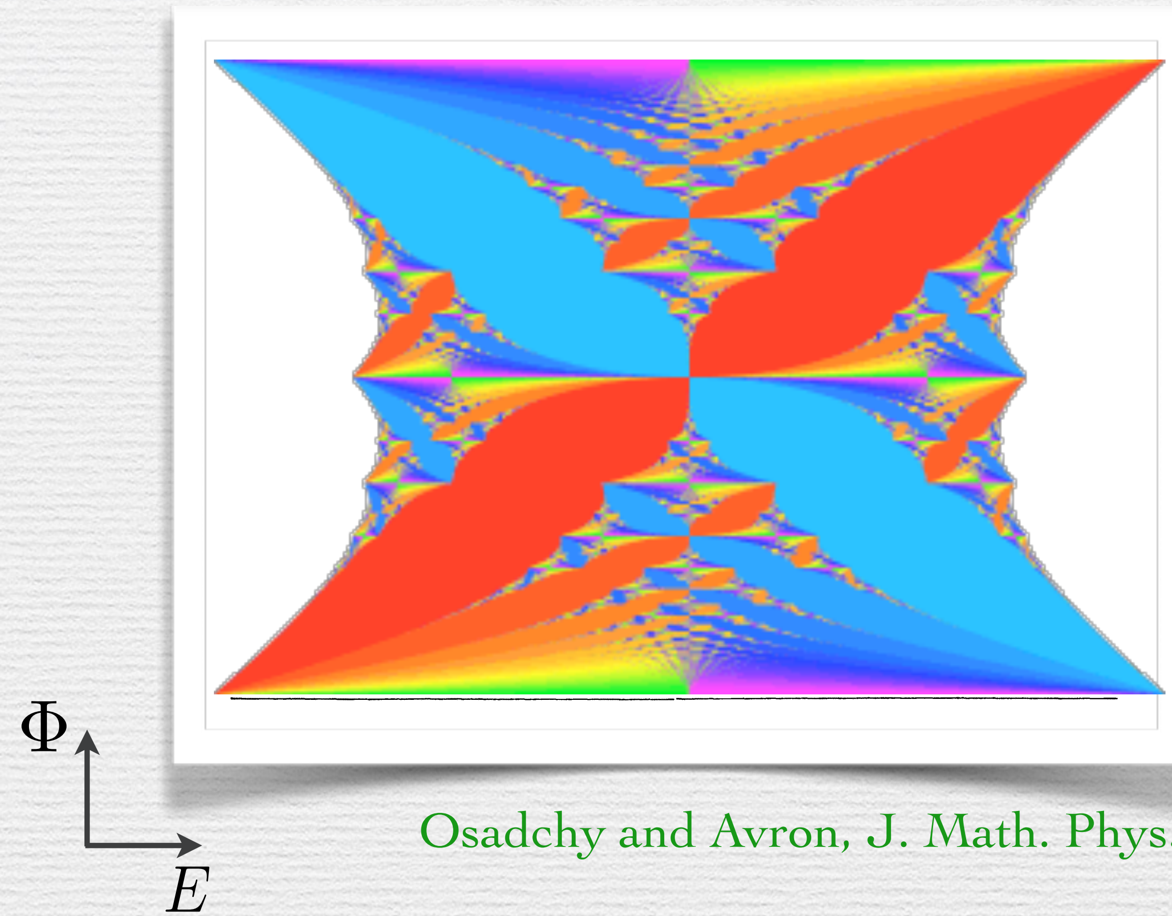


Hofstadter optical lattice

$$H = -J \sum_{m,n} e^{i2\pi n\Phi} c^\dagger_{m+1,n} c_{m,n} + c^\dagger_{m,n+1} c_{m,n} + H.c. \qquad \Phi = p/q$$



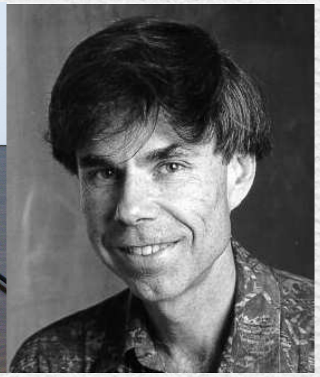
Hofstadter, 1976



Osadchy and Avron, J. Math. Phys. 2001

NO sharp edge states in harmonic trapping potential

Buchhold *et al*

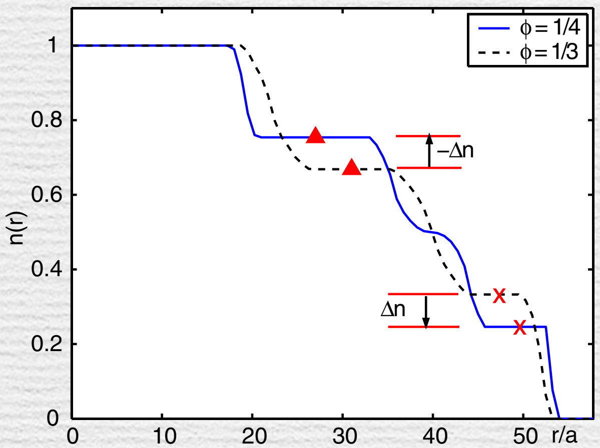


How to measure Chern # ?

How to measure Chern # ?

Density profile

Umucalilar *et al*

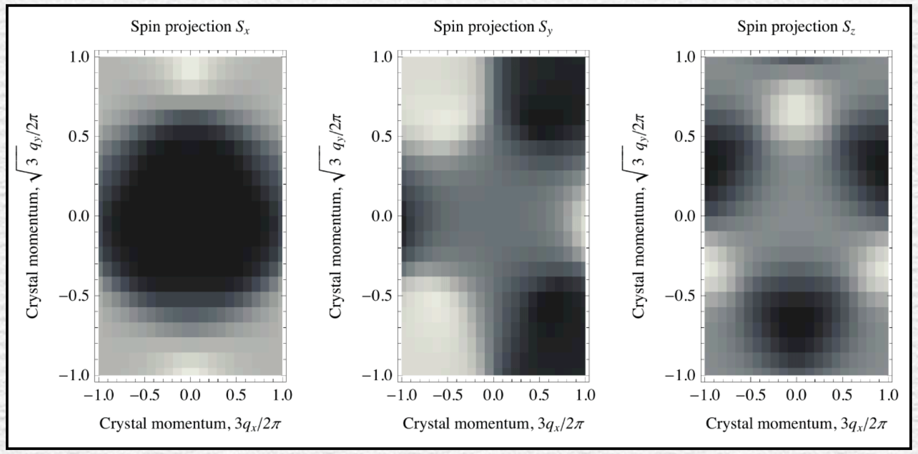


(a)

How to measure Chern # ?

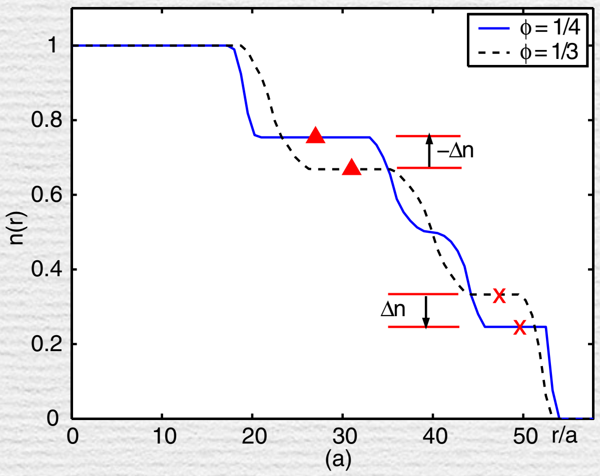
Time-of-flight

Alba *et al*, Zhao *et al*



Density profile

Umucalilar *et al*



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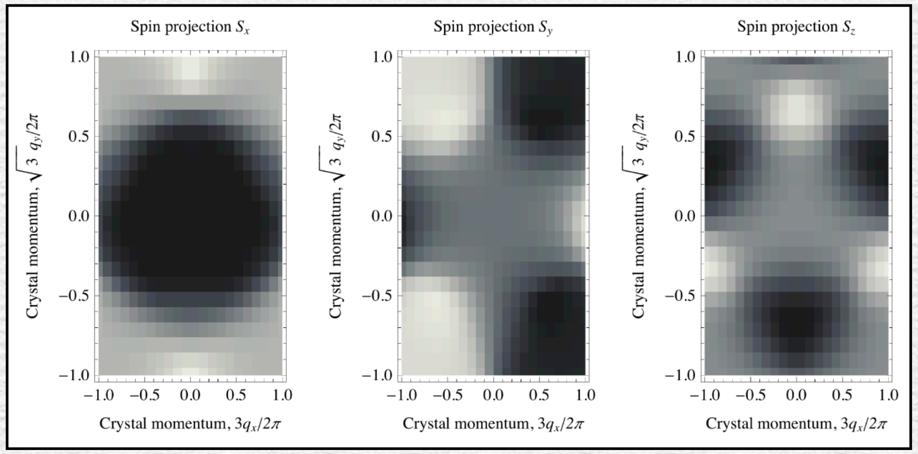
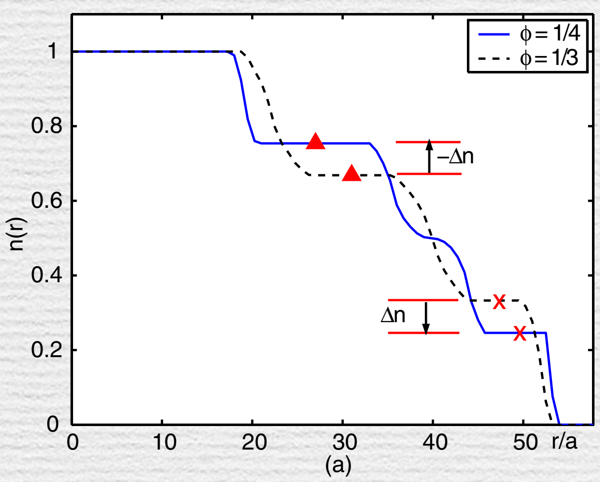


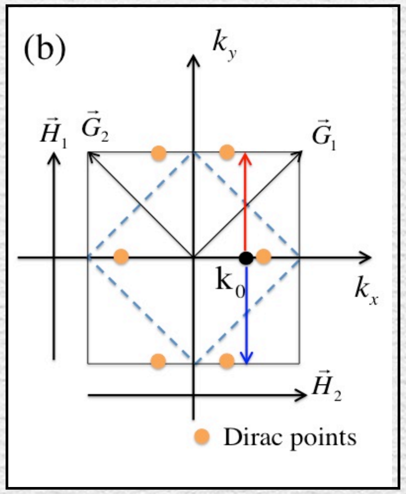
Density profile

Umucalilar *et al*



Zak phases

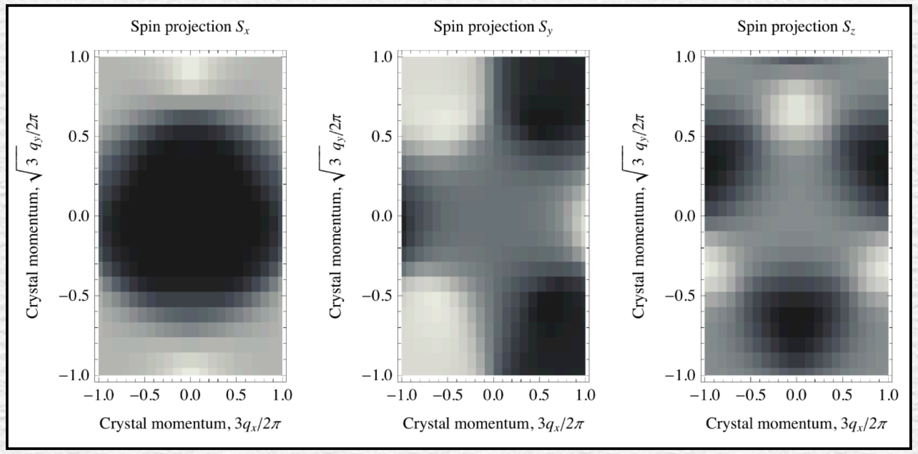
Abanin *et al*



How to measure Chern # ?

Time-of-flight

Alba *et al*, Zhao *et al*



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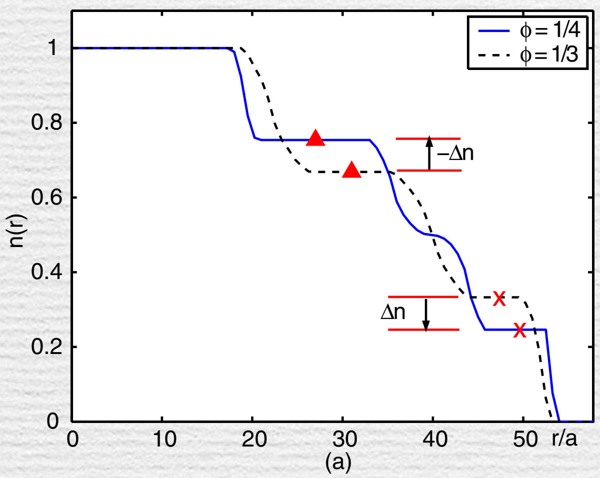

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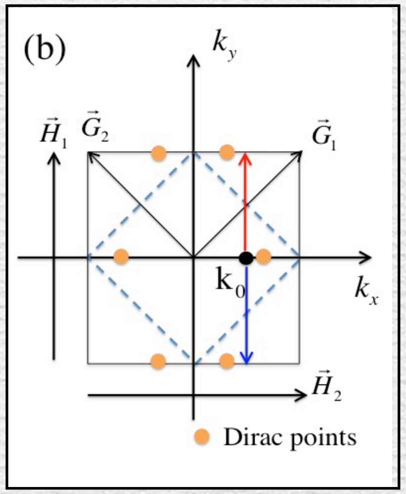
Abanin *et al*



Semi-classical dynamics

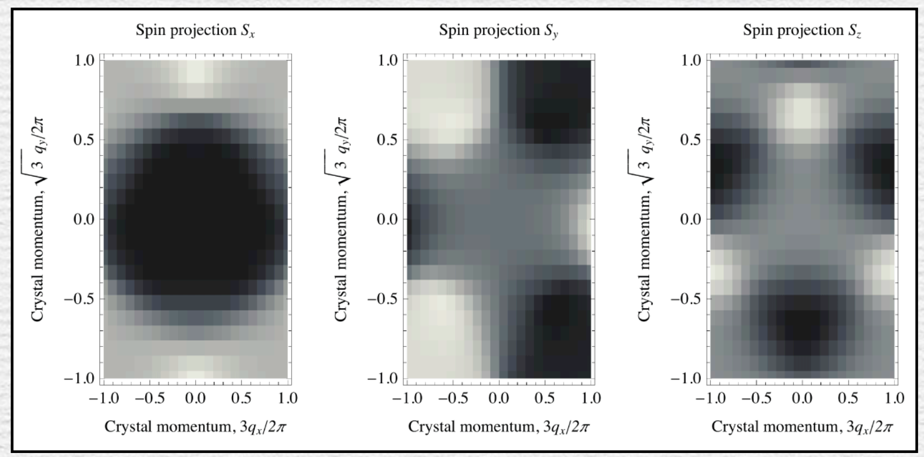
Price *et al*

$$\boxed{\dot{\mathbf{r}} = \frac{1}{\hbar}\frac{\partial \varepsilon(\mathbf{k})}{\partial \mathbf{k}} - \frac{\mathbf{F}}{\hbar} \times \hat{\mathbf{z}}\Omega(\mathbf{k})}$$

How to measure Chern # ?

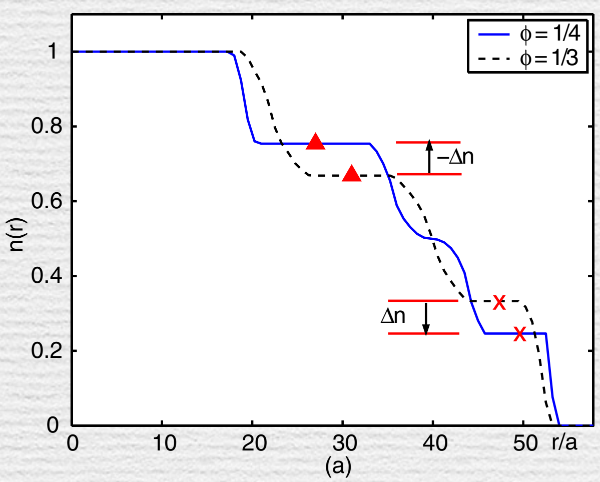
Time-of-flight

Alba *et al*, Zhao *et al*



Density profile

Umucalilar *et al*



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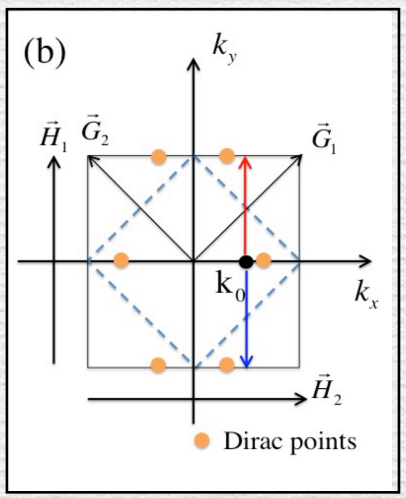

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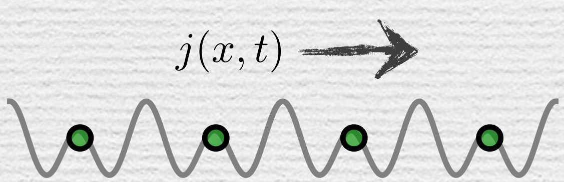


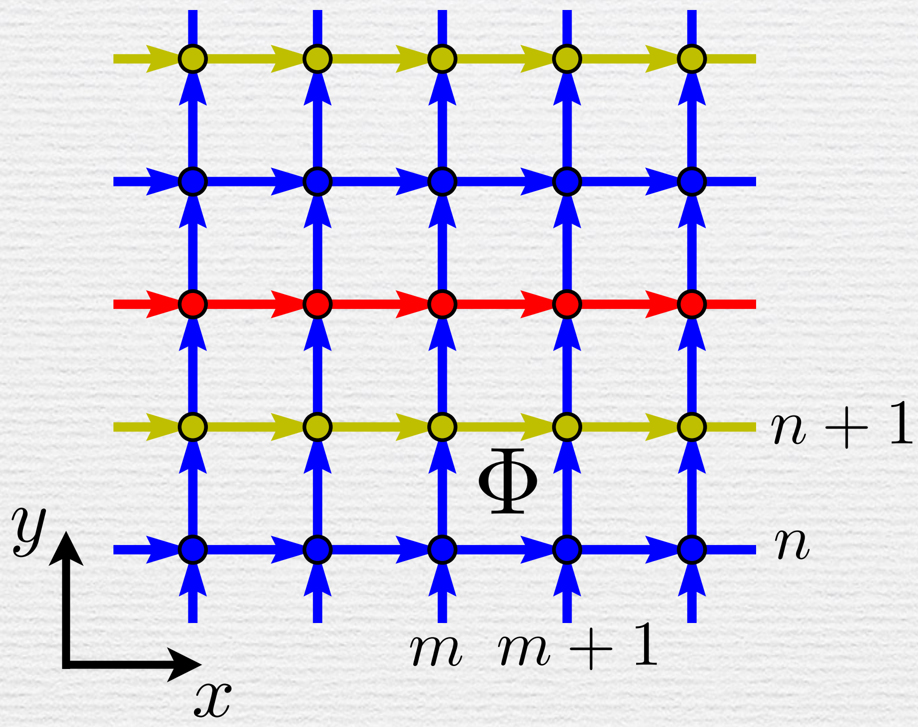
We propose a new probe based on

Topological Pumping Effect

$\rho(k_x, y)$

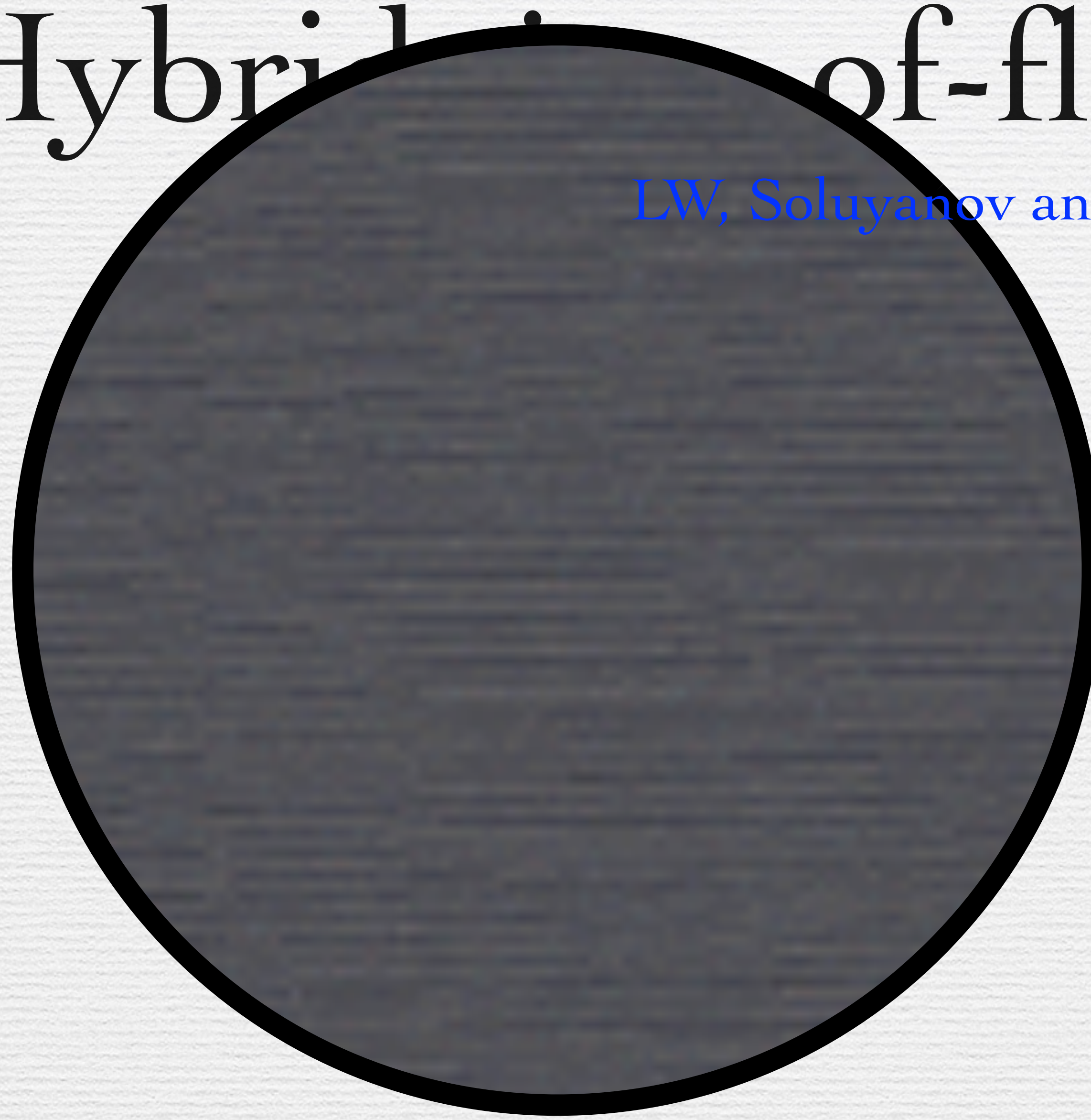
Hybrid time-of-flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



Hybri ... of-flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



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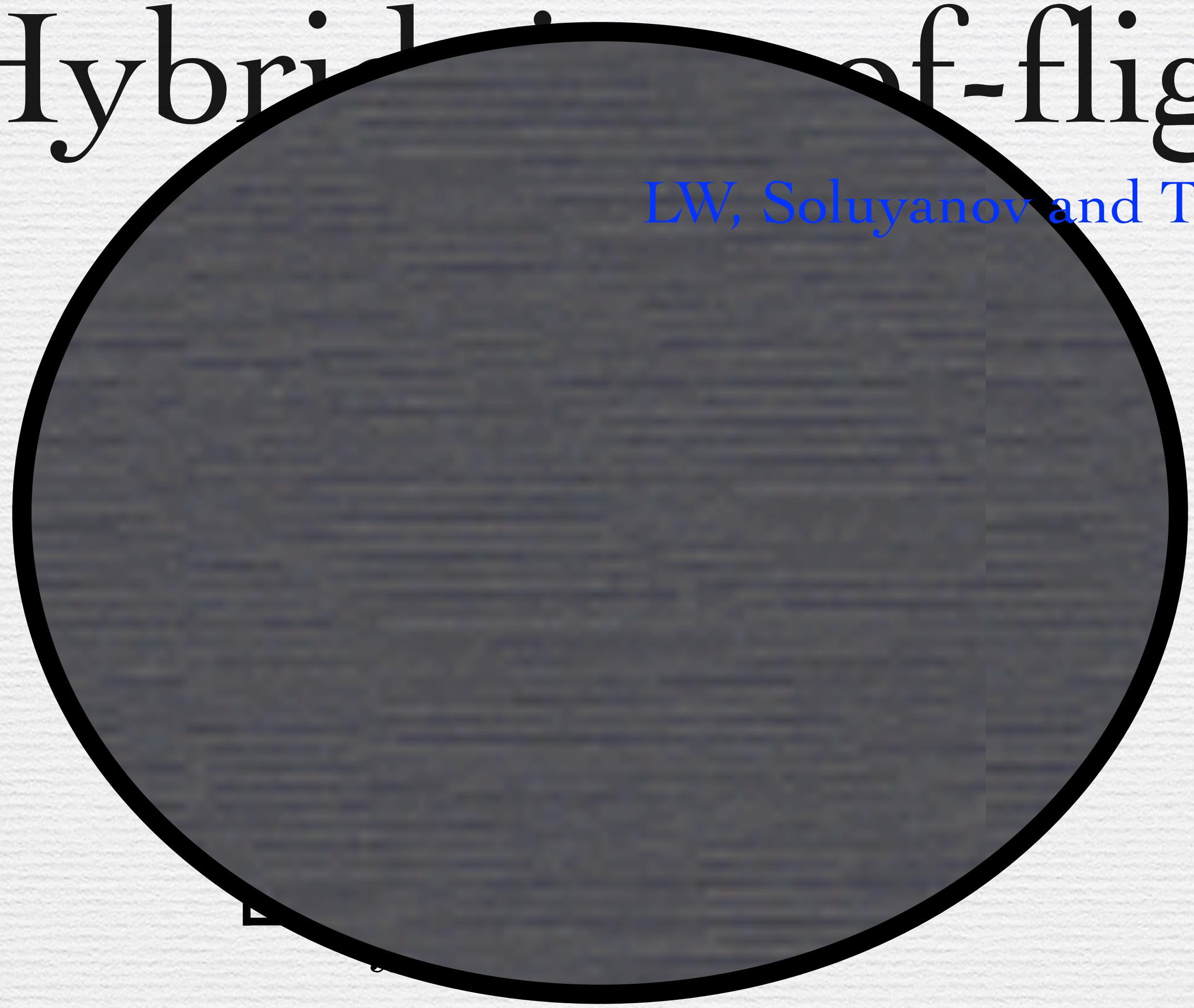
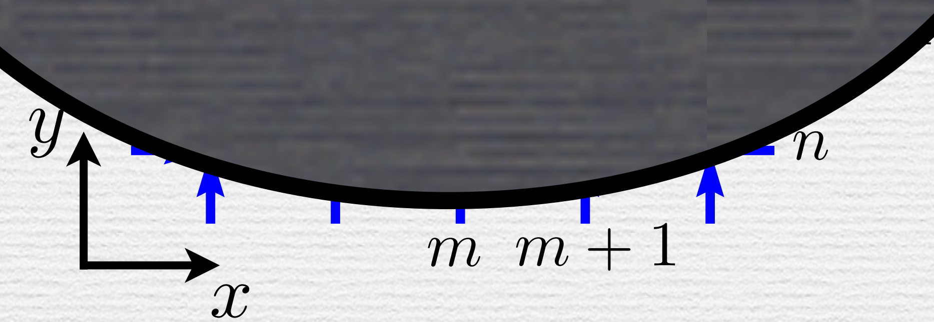
LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



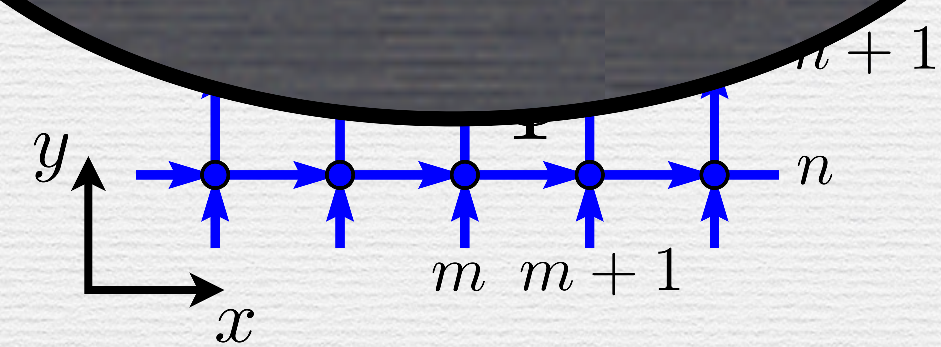
Hybr[redacted]f-flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



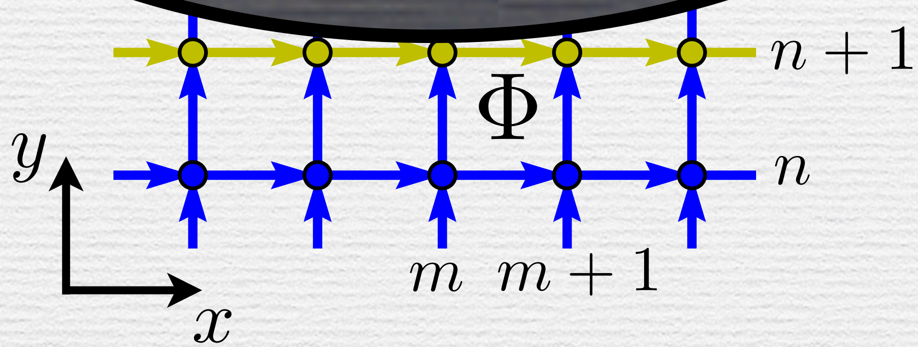
Hyb[redacted]f-flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



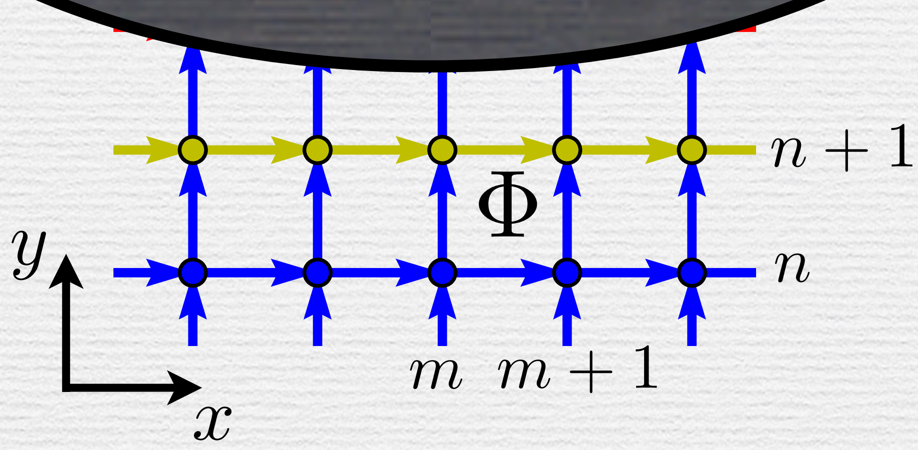
Hyb[redacted]f-flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



Hyb[redacted]f-flight

LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



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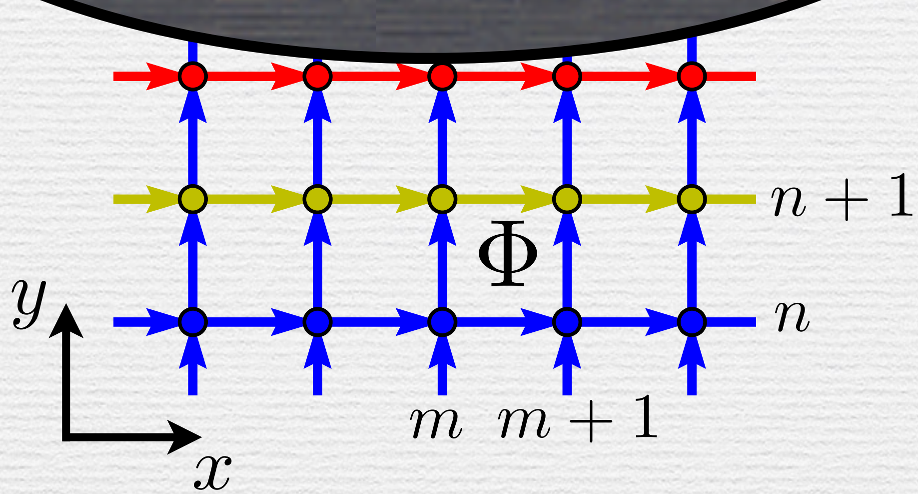
LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



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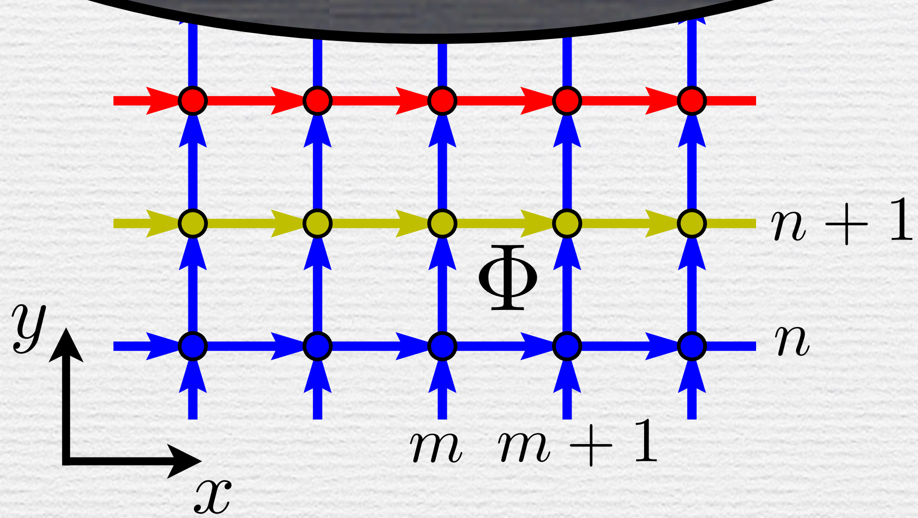

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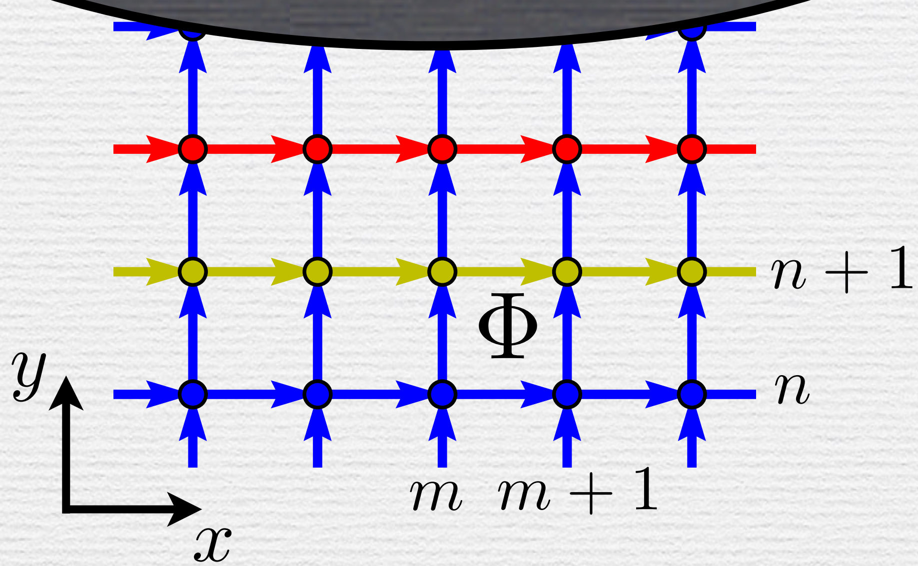
LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



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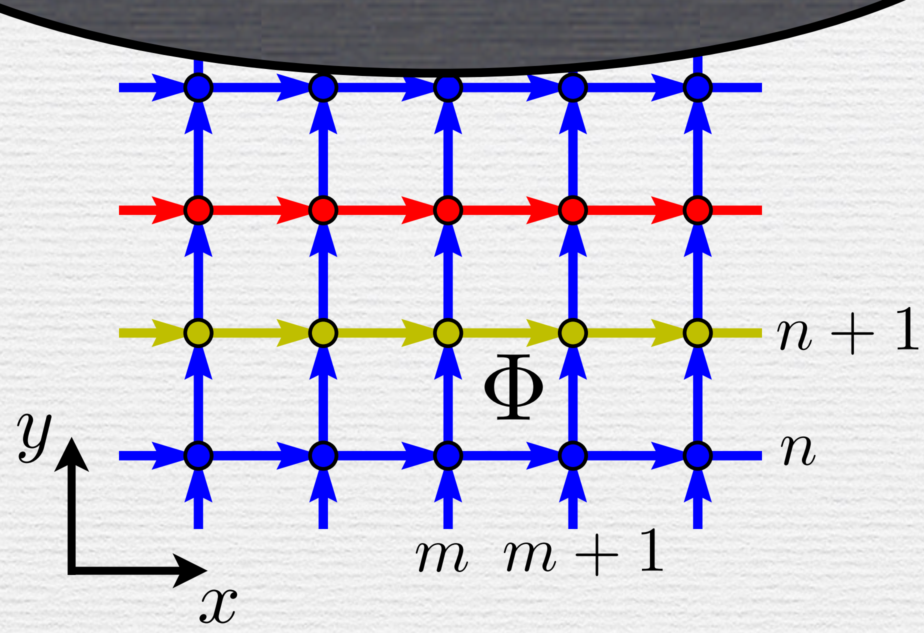
LW, Soluyanov and Troyer, PRL 110, 166802 (2013)



Hy [redacted] f flight

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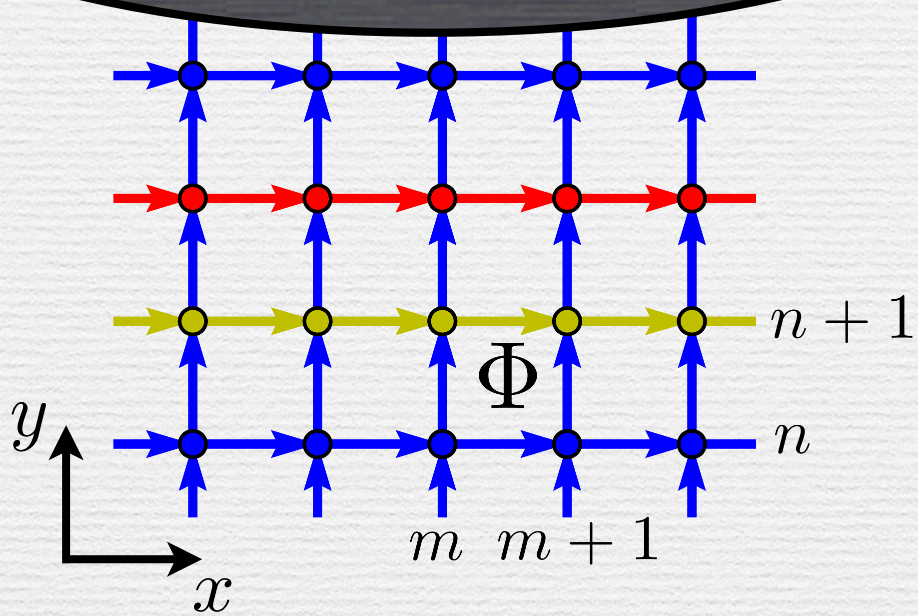

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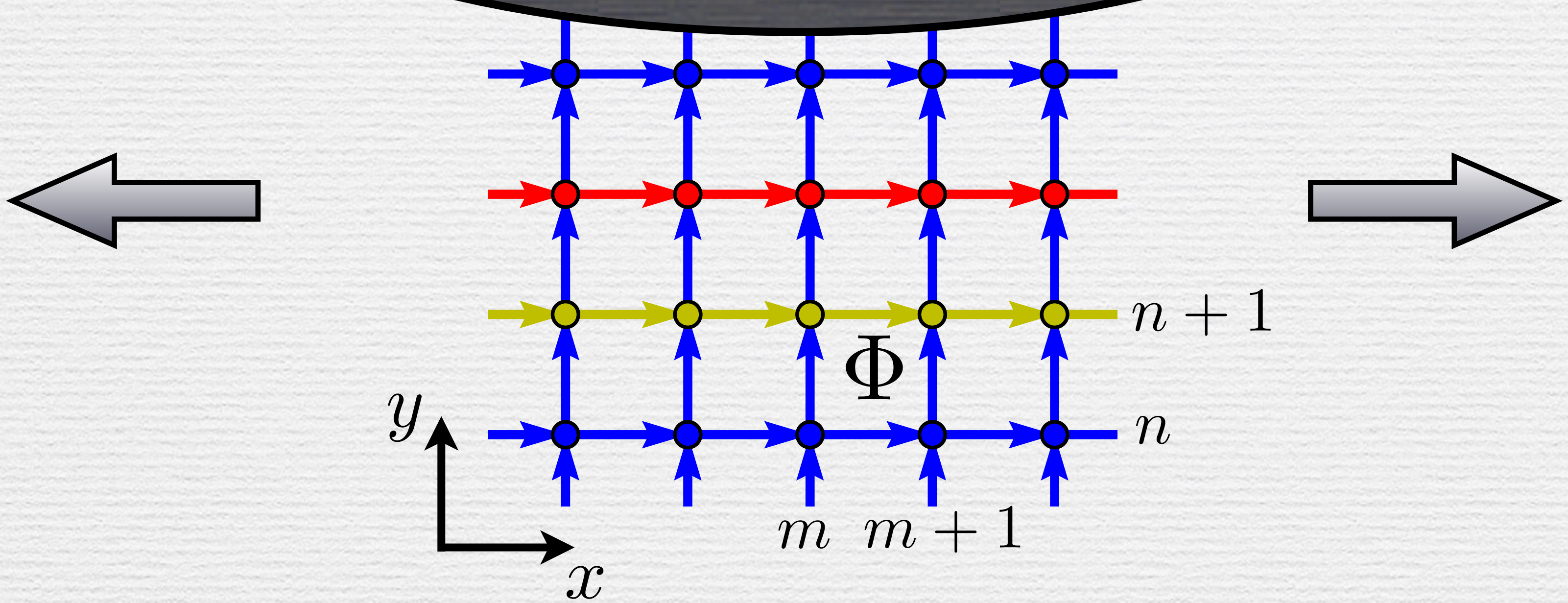

Hy… f flight

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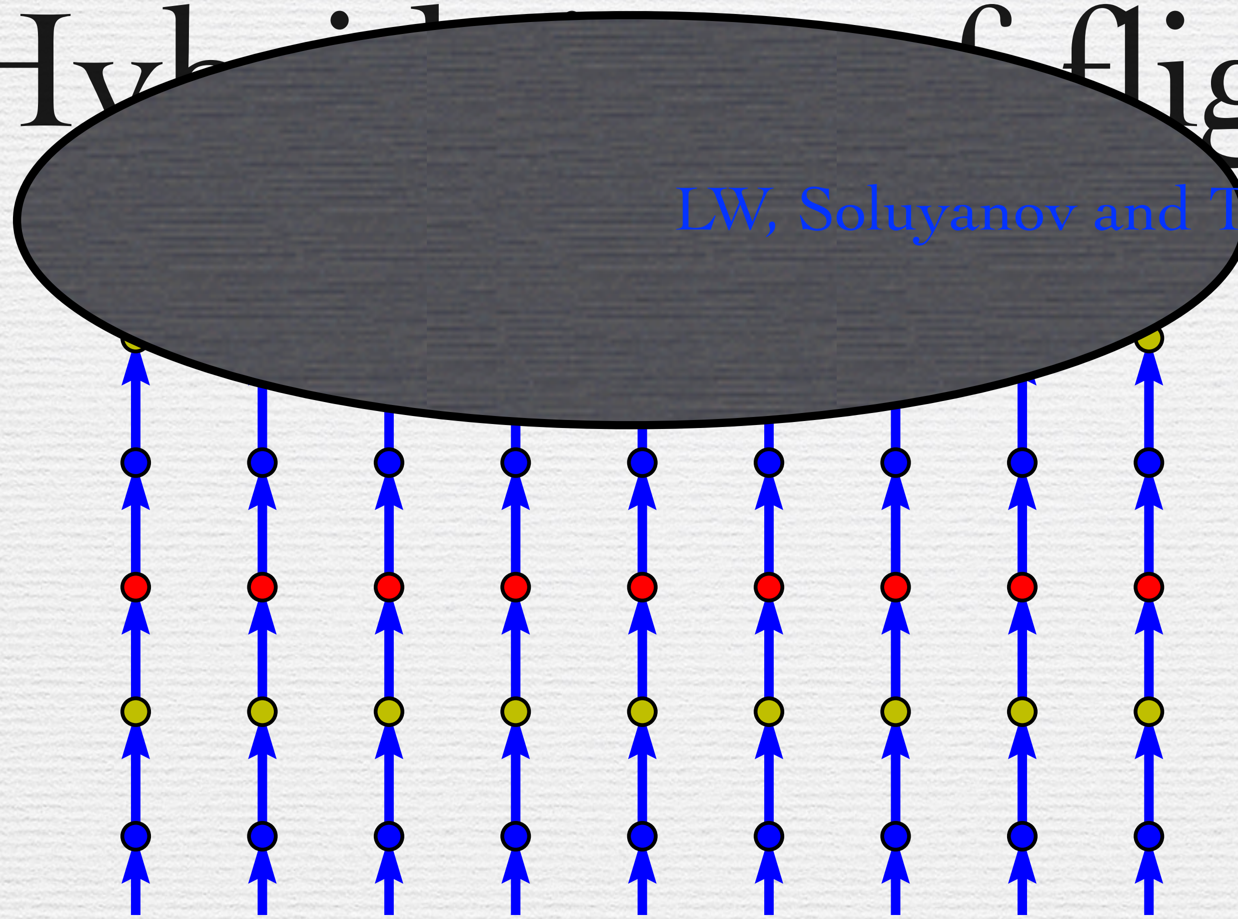

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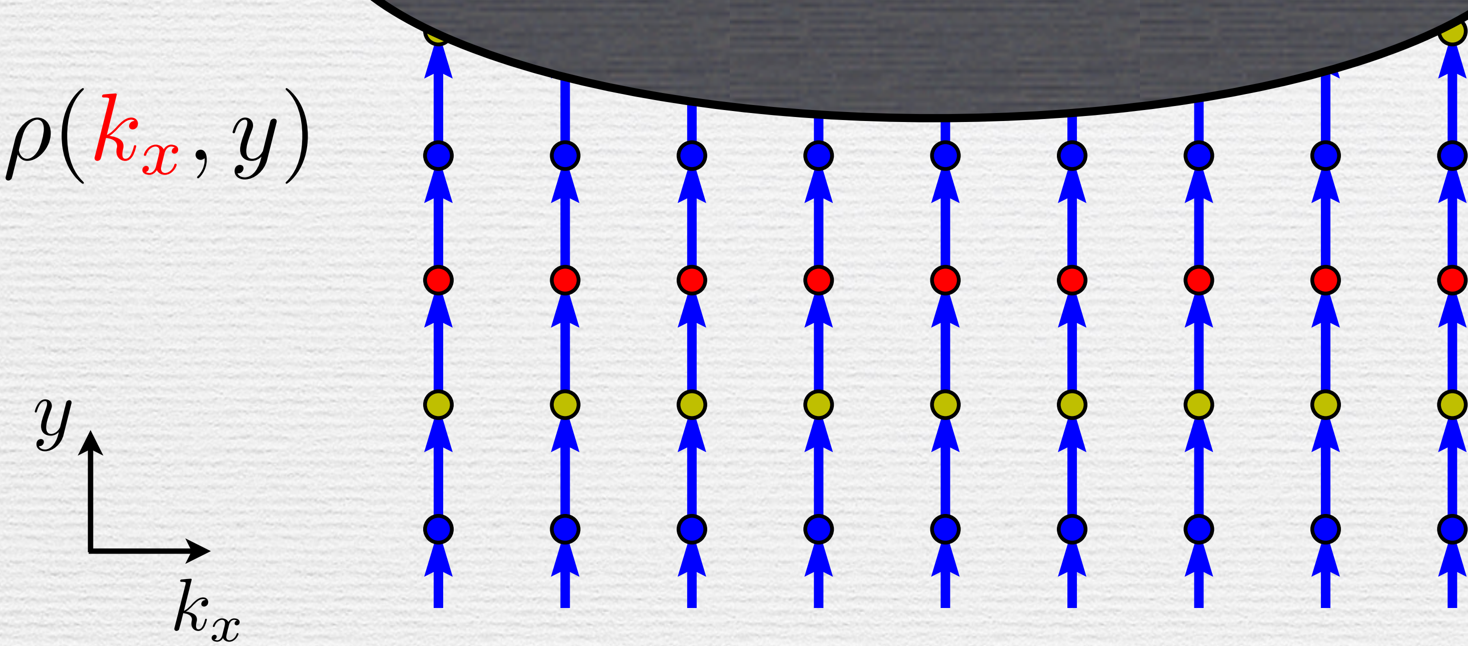
LW, Soluyanov and Troyer, PRL 110, 166802 (2013)

$\rho(k_x, y)$



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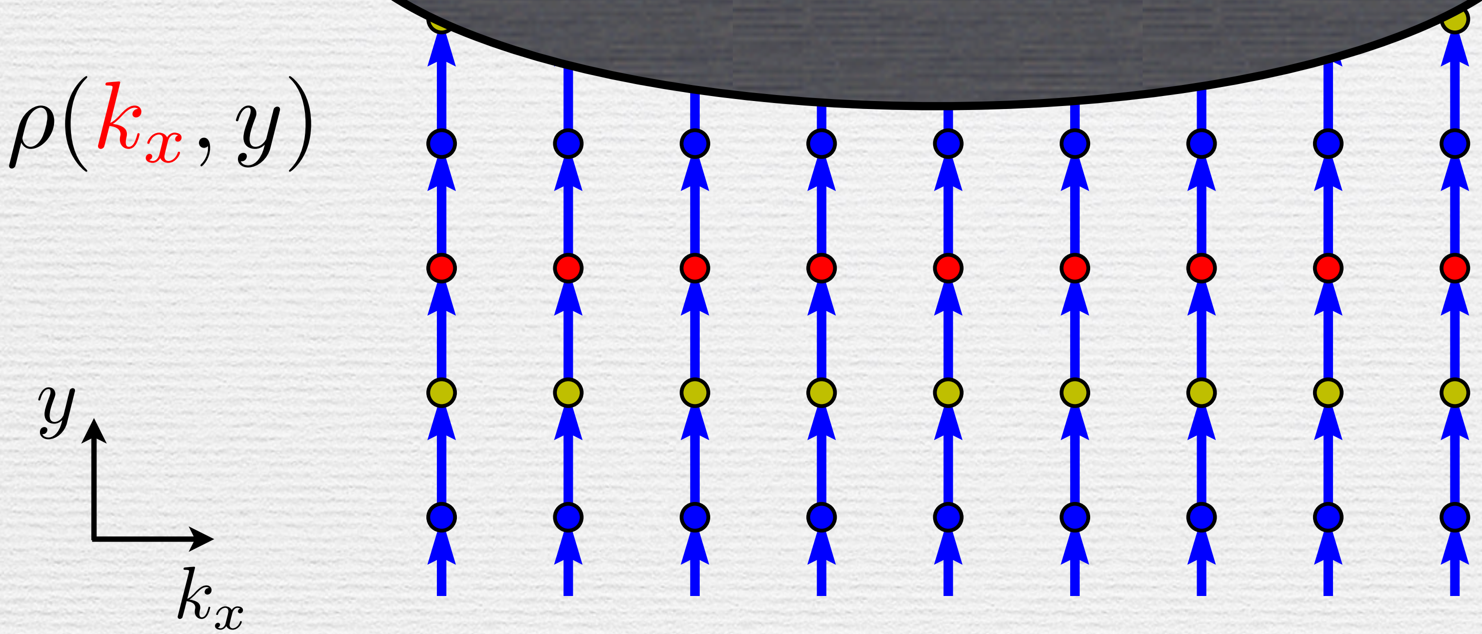
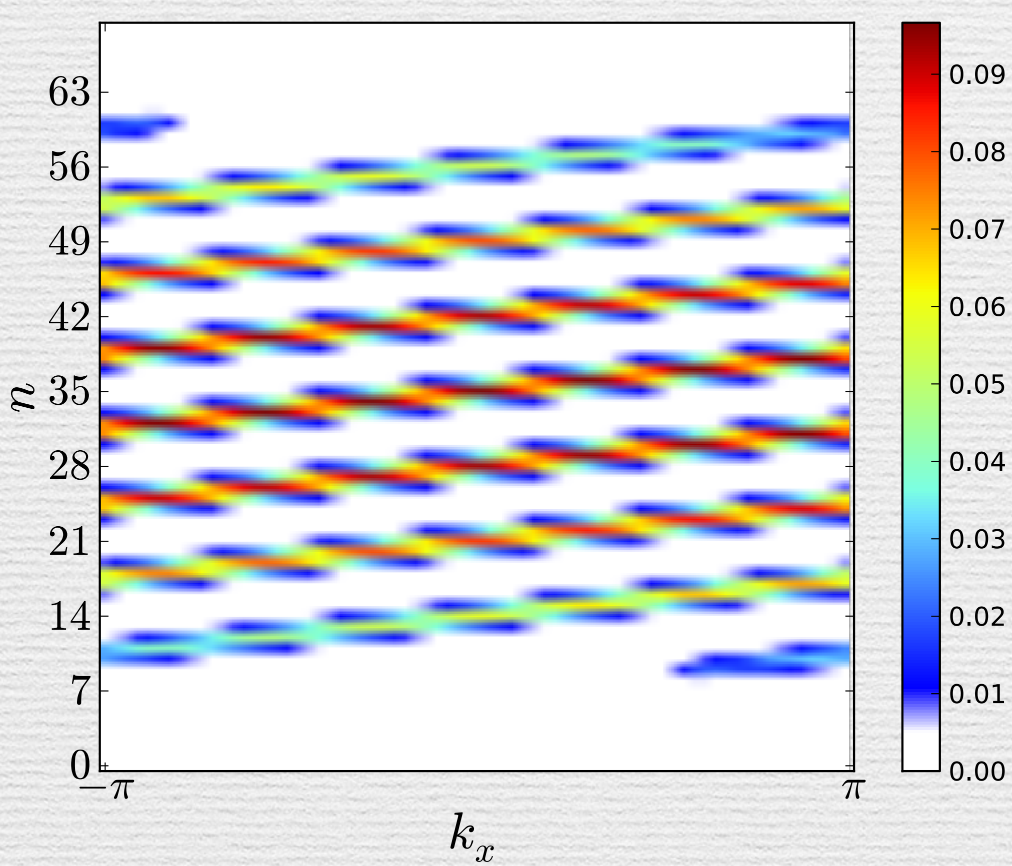


$\Phi = 1/7 \quad C = 1$



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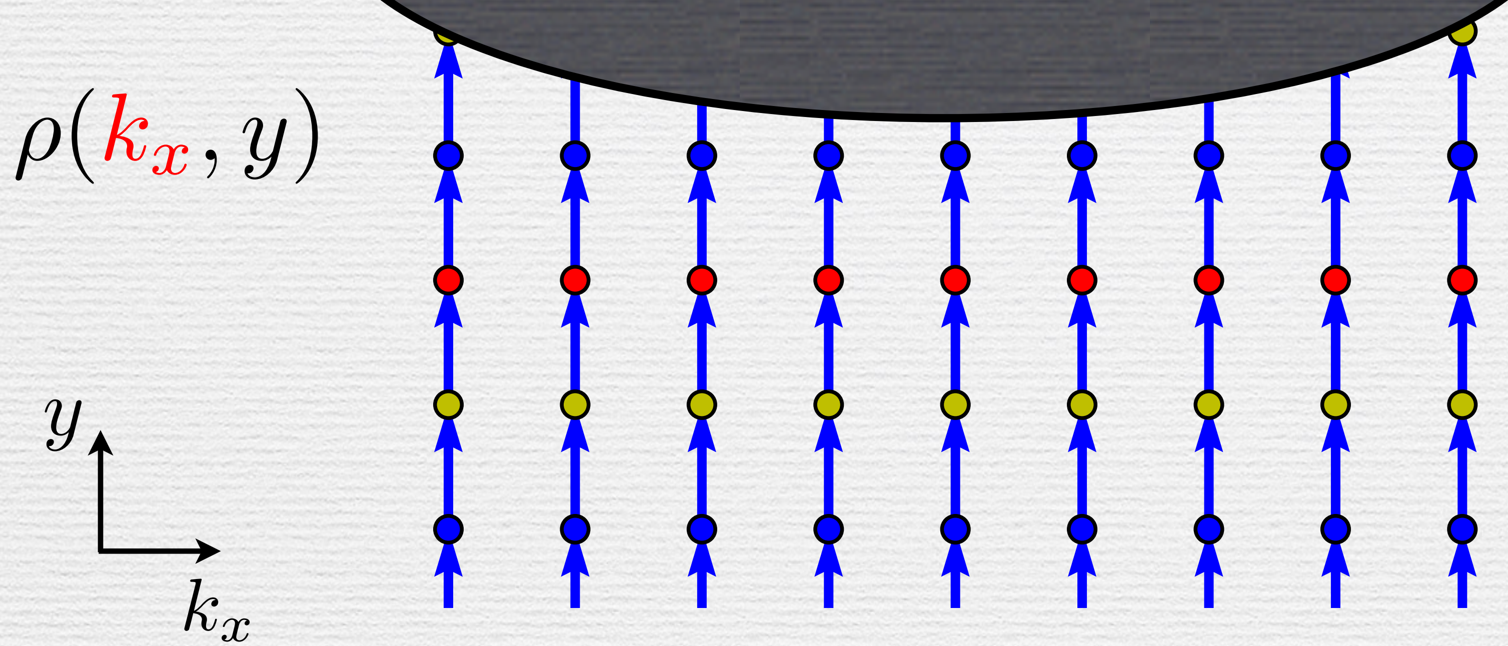

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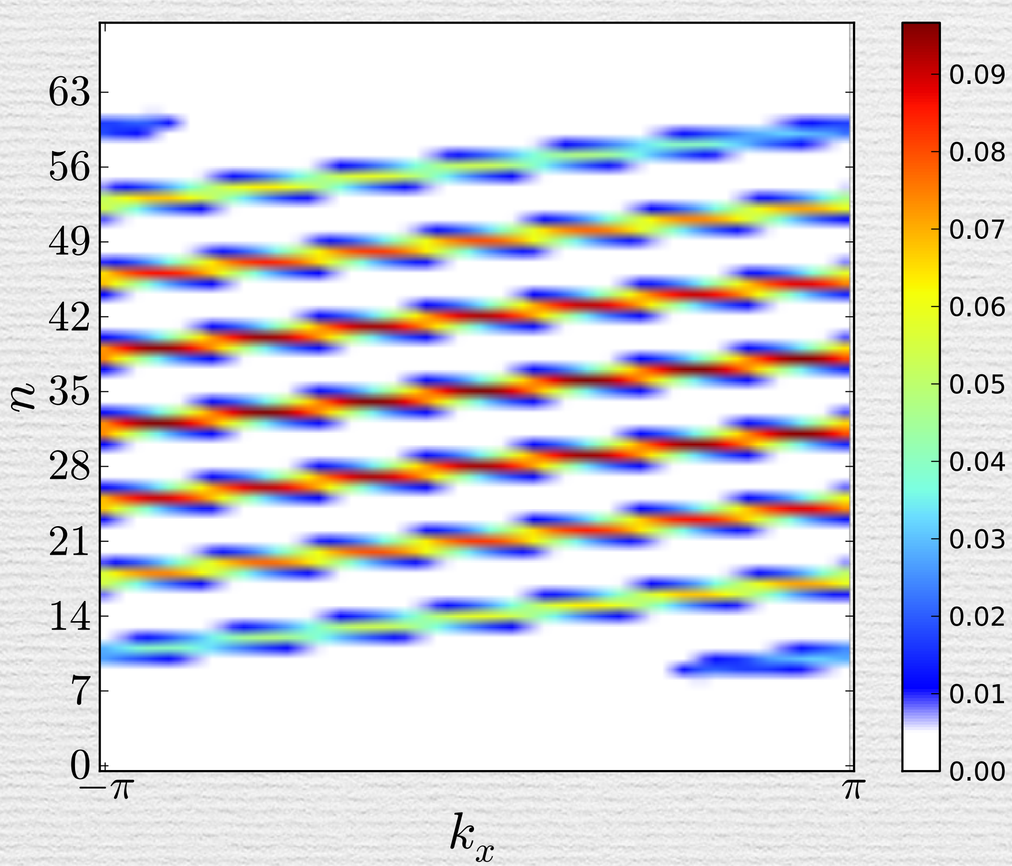
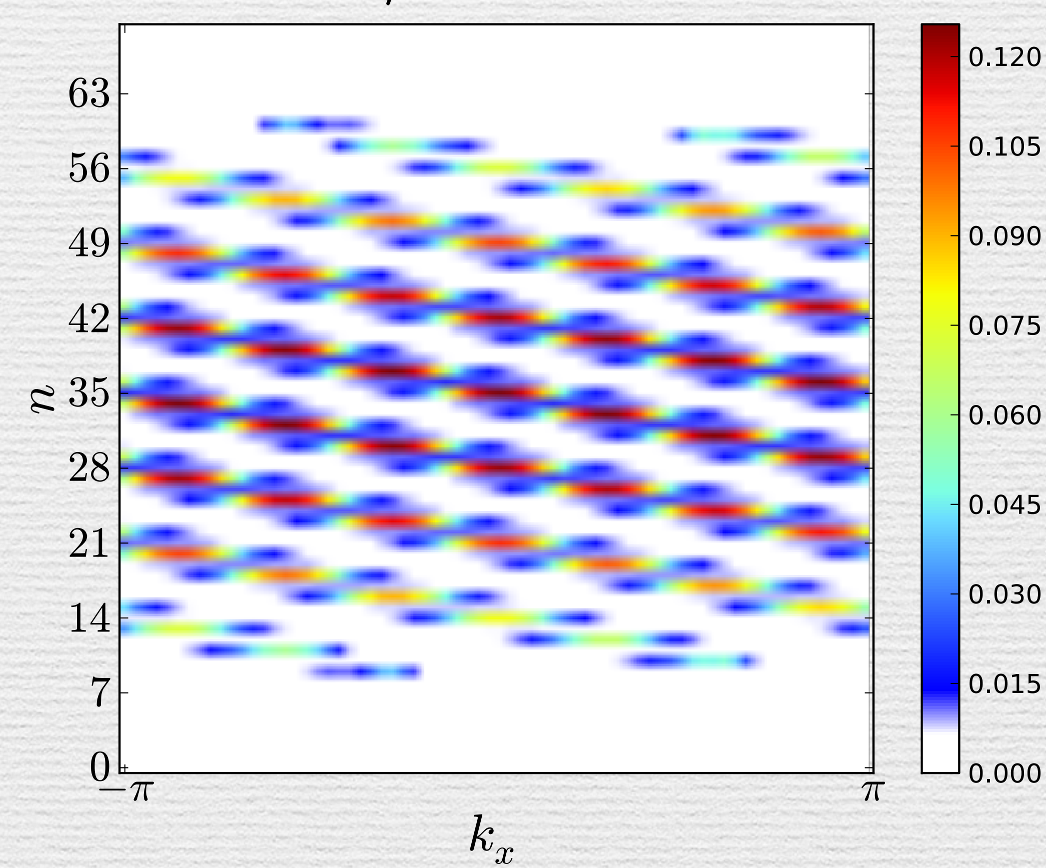


$\Phi = 3/7 \quad C = -2$



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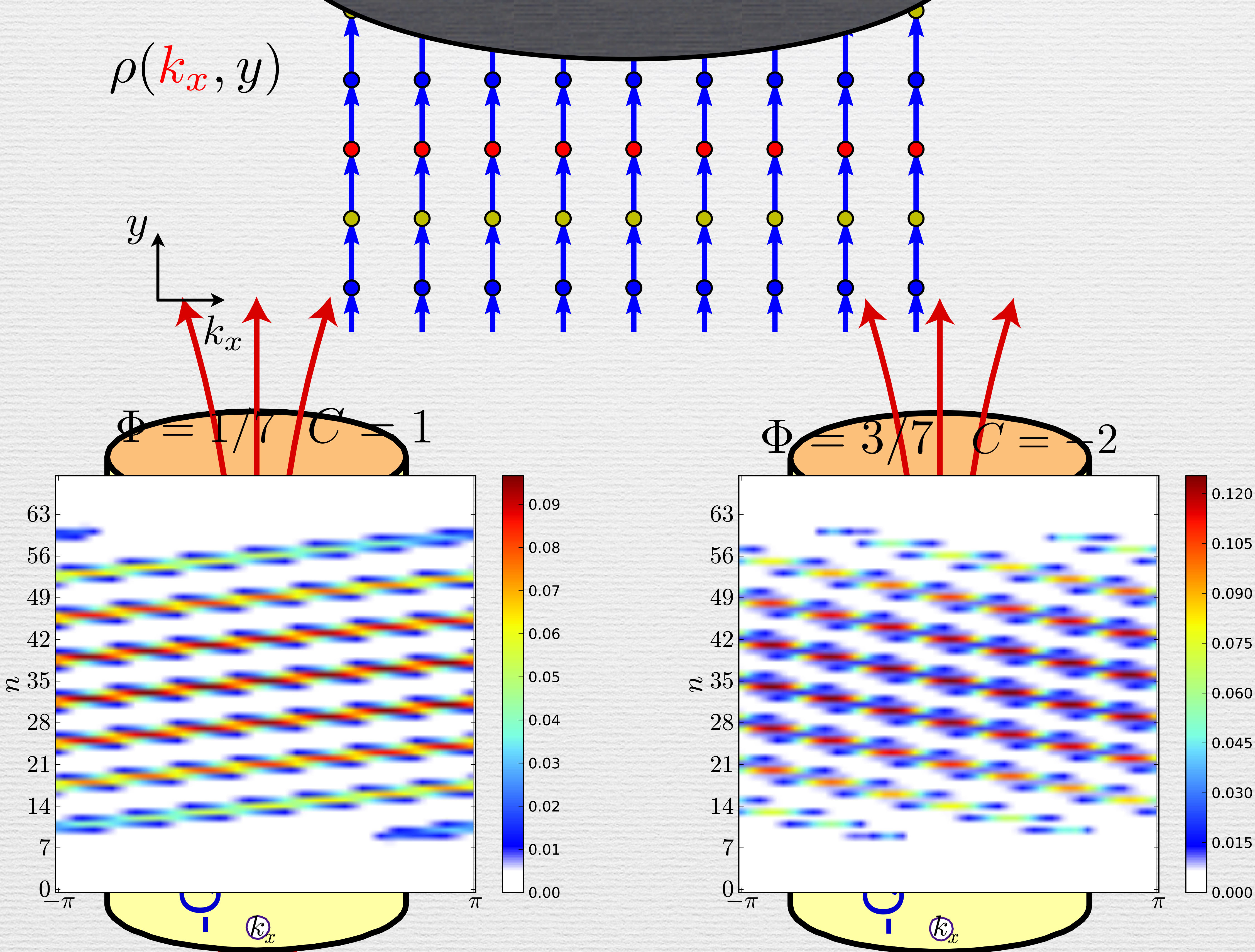
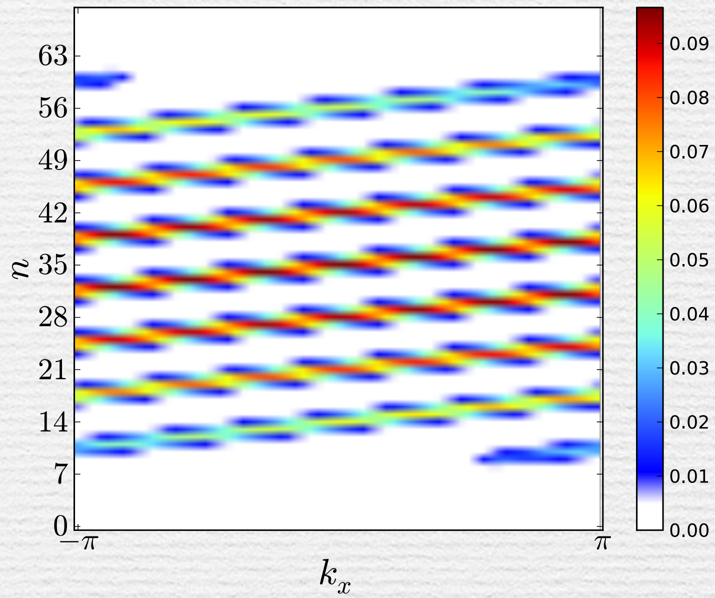


Quantitative Characterizations

- Slope
- # of cuts (edge modes)
- COM along y-direction
- Bipartition particle number (trace index)



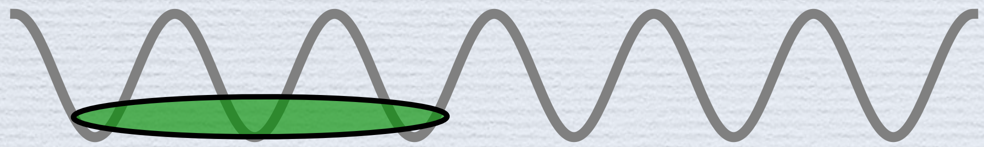
Alexandradinata *et al*

Salient features

- Bulk detection, does not require edge states
- $\rho(k_x, y)$ is almost impossible to measure in solids, but is natural to cold atom toolbox
- Can be extended to interacting case

Fractional charge pumping

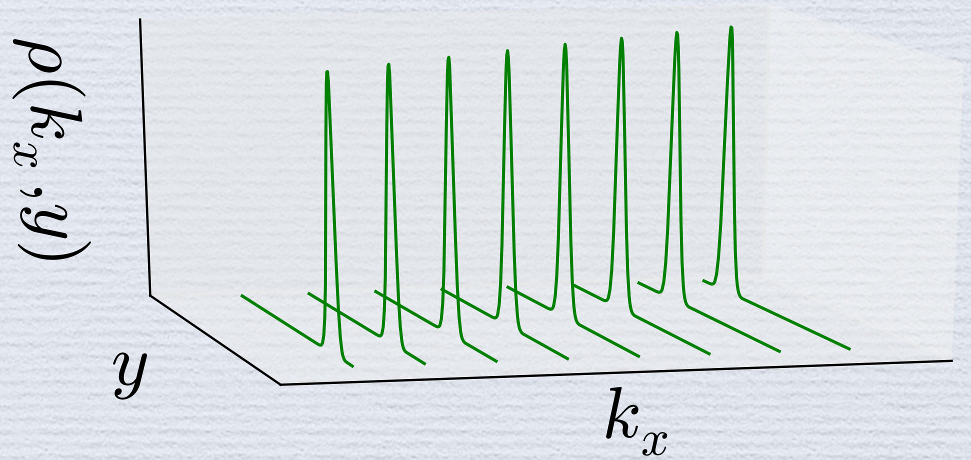
1D lattice



Interaction is crucial
for opening an energy gap

2D Laughlin state

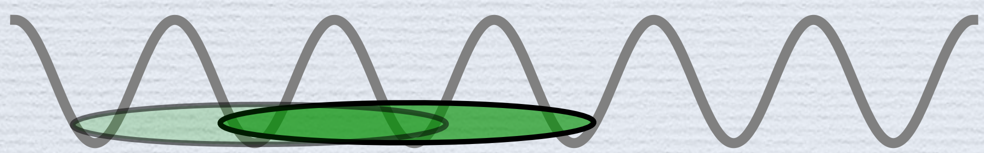
$$\rho(k_x, y) = \frac{\nu}{\sqrt{\pi}} e^{-(y-k_x)^2}$$



Use hybrid ToF to detect FQHE and fractional Chern insulators realized in optical lattices

Fractional charge pumping

1D lattice



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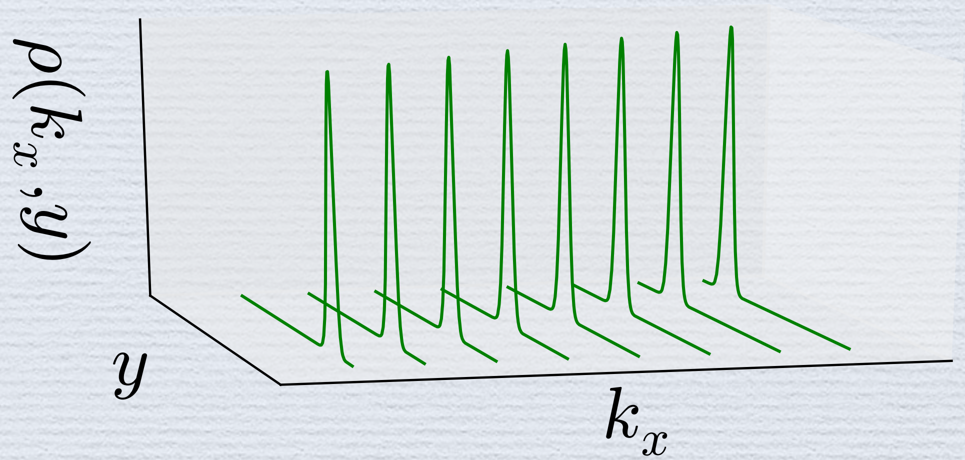
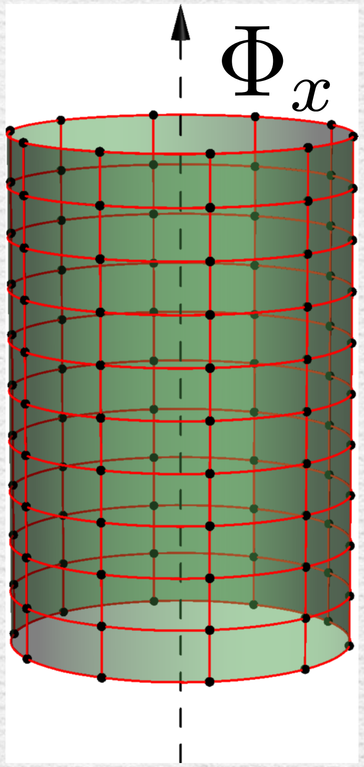


Use hybrid ToF to detect FQHE and fractional Chern insulators realized in optical lattices

Numerical diagnosis of fractional Hall conductance

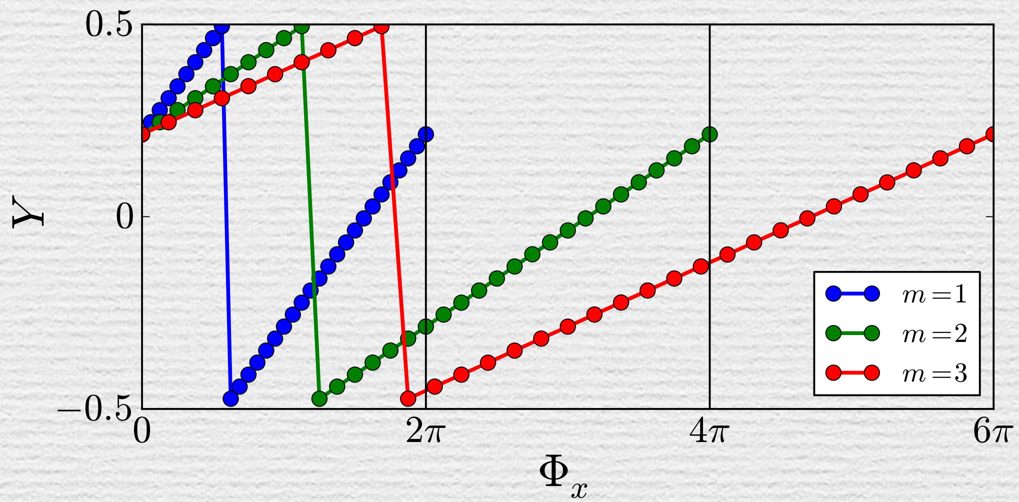
1. Center-of-mass shift

$$Y(\Phi_x) = \frac{1}{L_y}\sum_{\mathbf{r}} y_{\mathbf{r}} n_{\mathbf{r}}(\Phi_x)$$



2. Particle number flow

$$N_{\mathrm{A}}(\Phi_x) = \sum_{\mathbf{r}\in A} n_{\mathbf{r}}(\Phi_x)$$



Avoid calculating overlap between wavefunctions TKNN

LW, Soluyanov and Troyer, to appear

Numerical diagnosis of fractional Hall conductance

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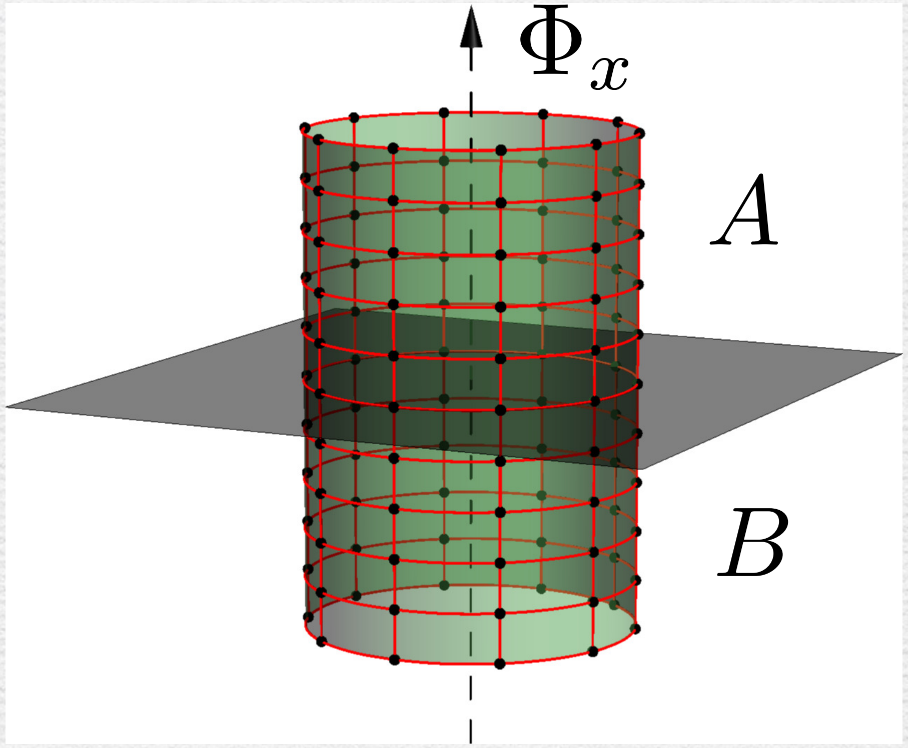

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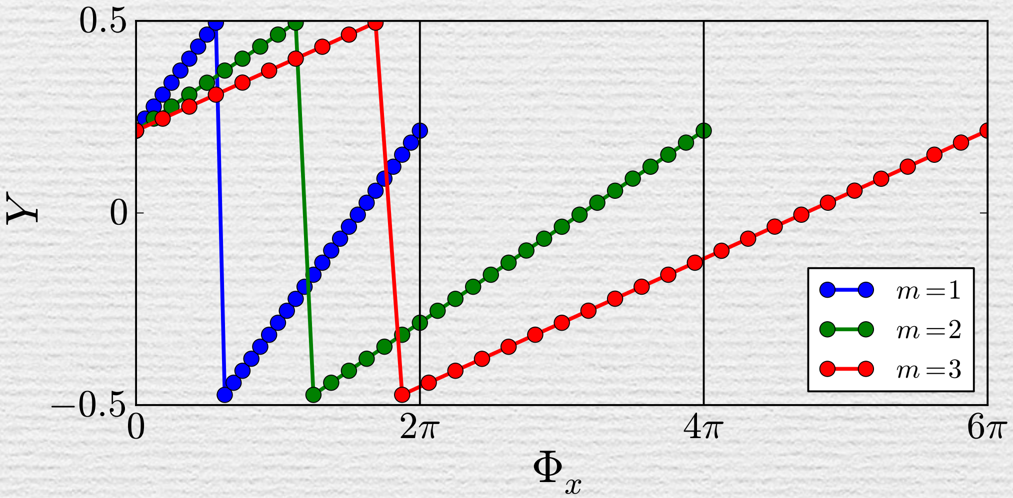

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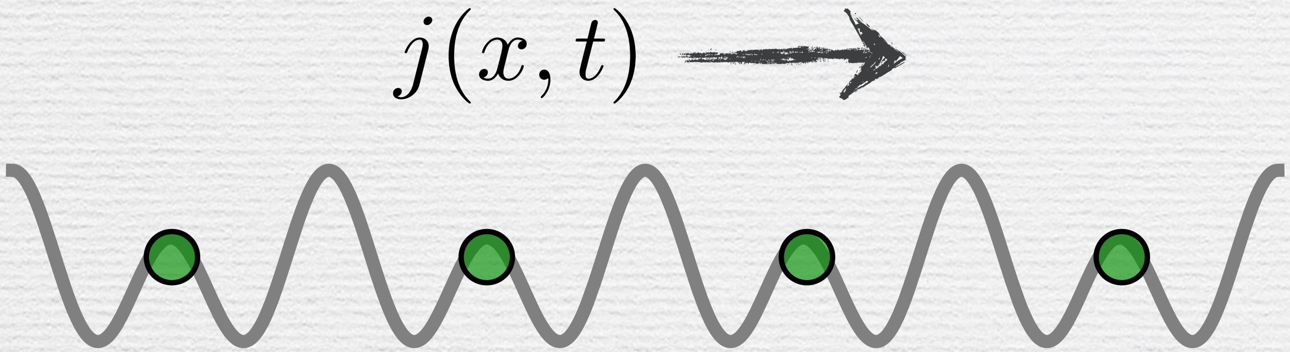
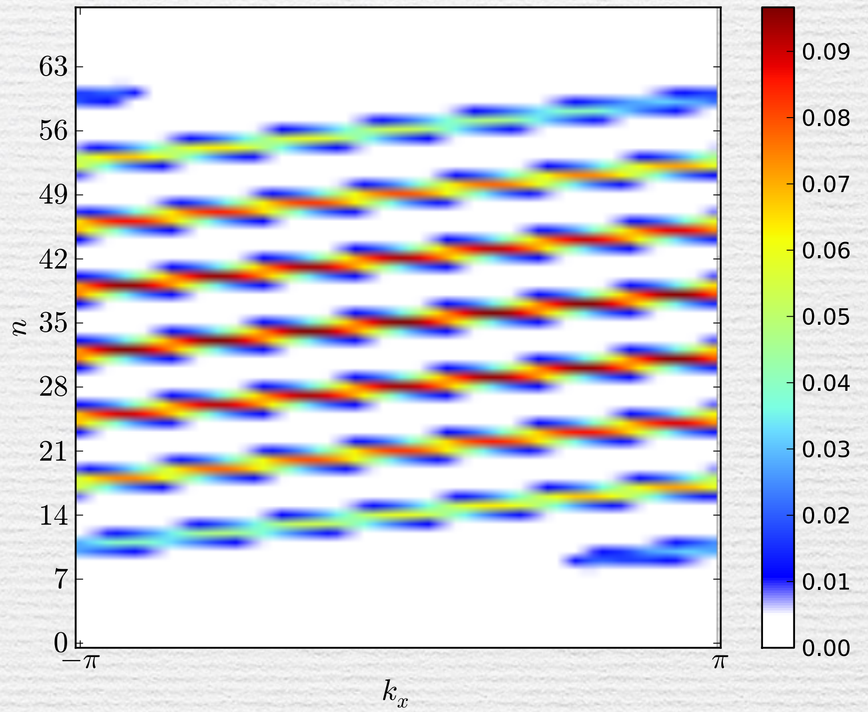
Summary

arXiv:1301.7435, in press

PRL 110, 166802 (2013)

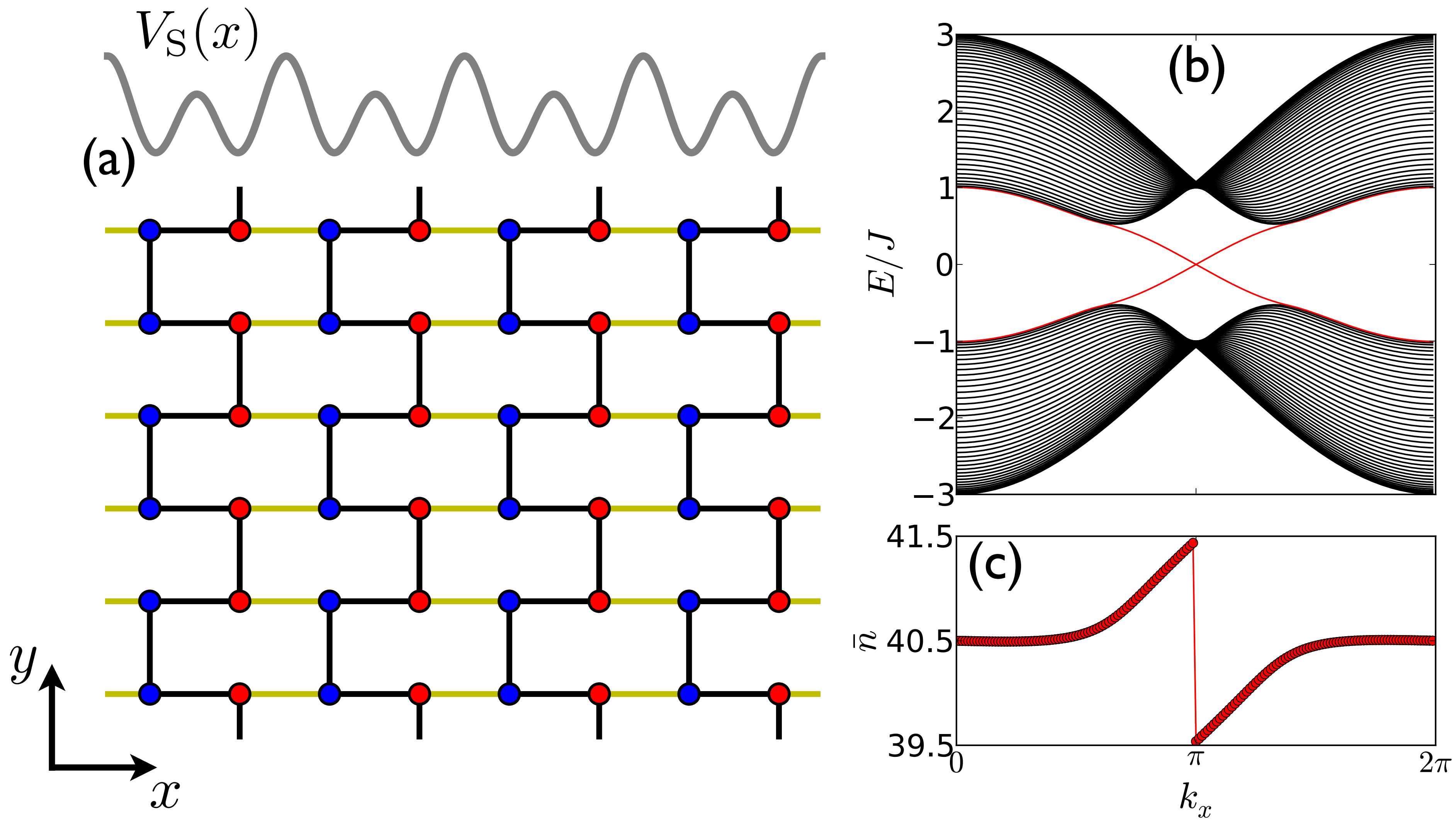




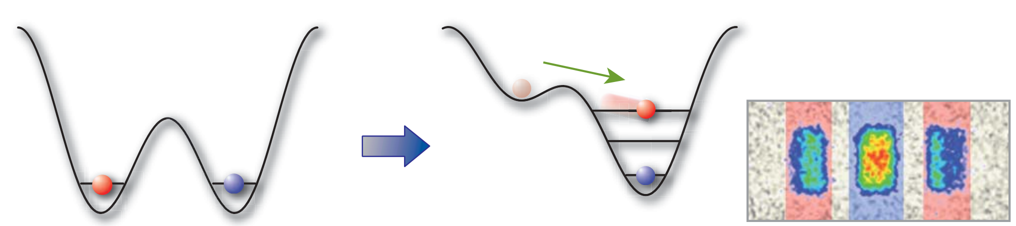
Topological charge pumping is a common thread unifies many features of topological states

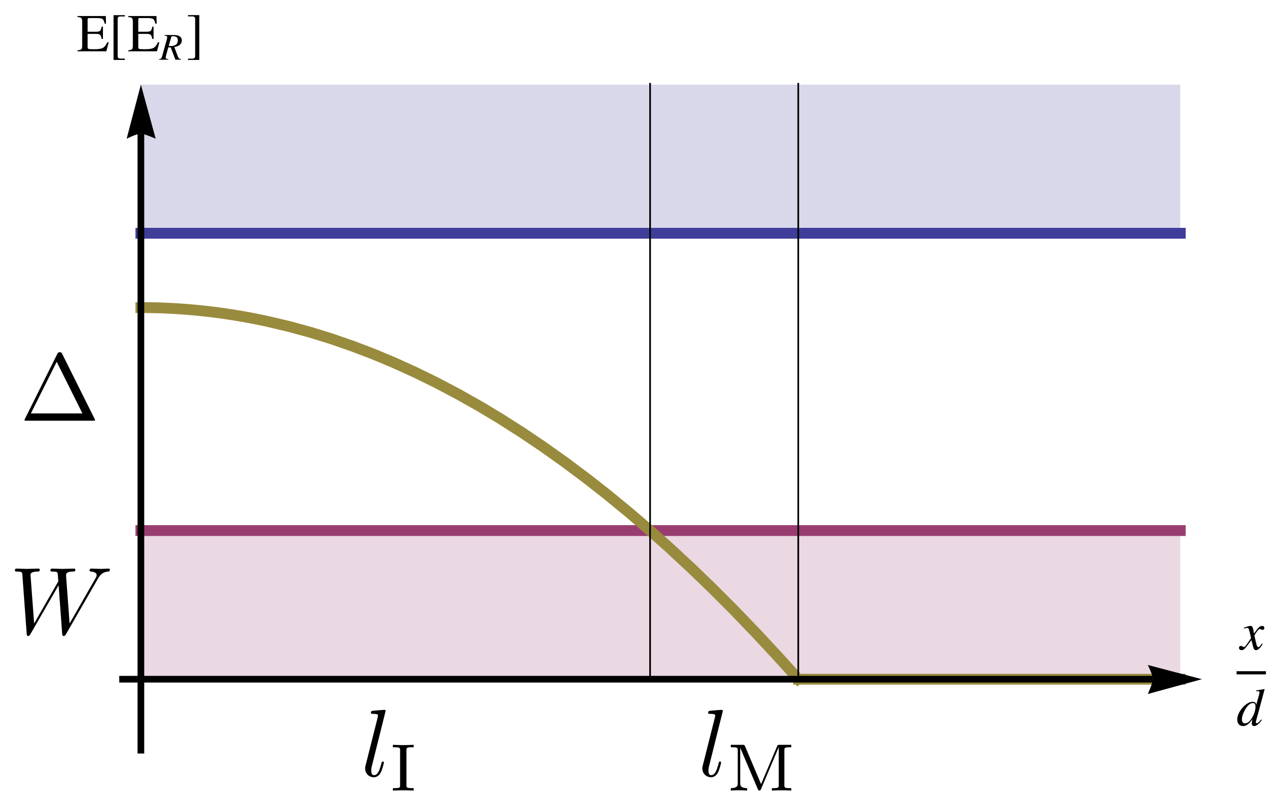
Guideline for design and detection of topological phases in cold atom systems

Thank you!



$V_S(x)$
(a)
y
x
(b)
E/J
3
2
1
0
−1
−2
−3
(c)
$\bar{n}$
41.5
40.5
39.5
0
π
2π
k_x




E[E$_R$]
Δ
W
$\frac{x}{d}$
l_{I}
l_{M}