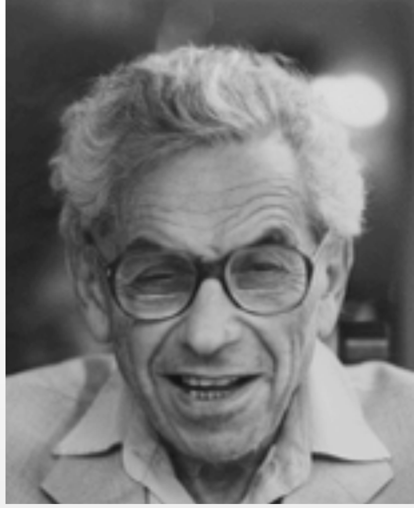
How did I earn an Erdős number of 2 ?

Lei Wang
Institute of Physics

2016.04 @ CSRC

Paul Erdős

1913.3.26–1996.9.20



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- Over 1500 published papers and more than 500 collaborators

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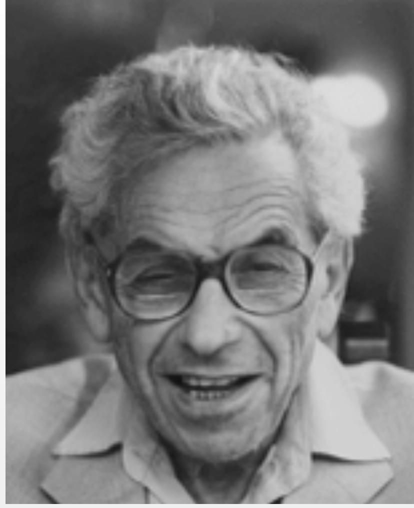

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News & Comment > News > 2015 > September > Article

NATURE | BREAKING NEWS

Maths whizz solves a master's riddle

Terence Tao successfully attacks the Erdős discrepancy problem by building on an online collaboration.

Chris Cesare

25 September 2015

Paul Erdős

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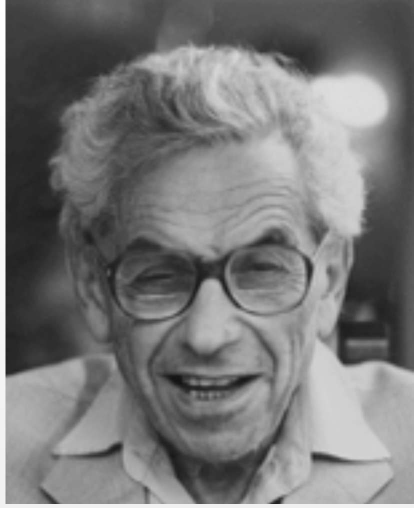

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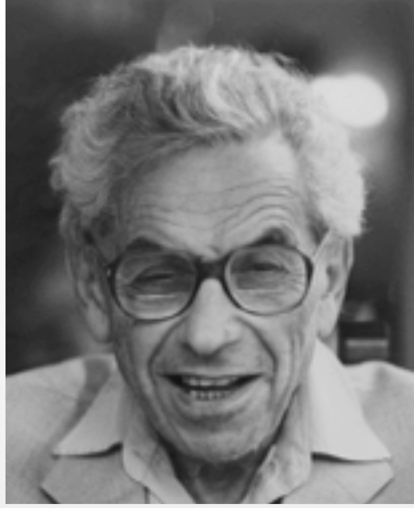

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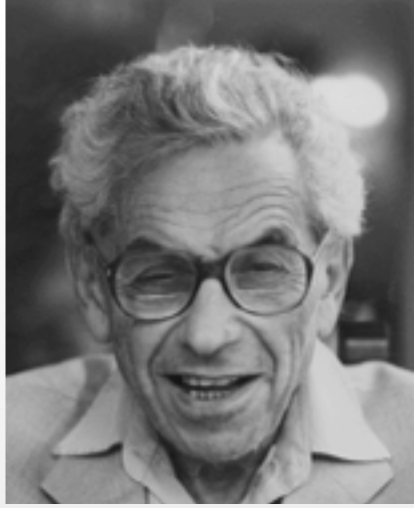

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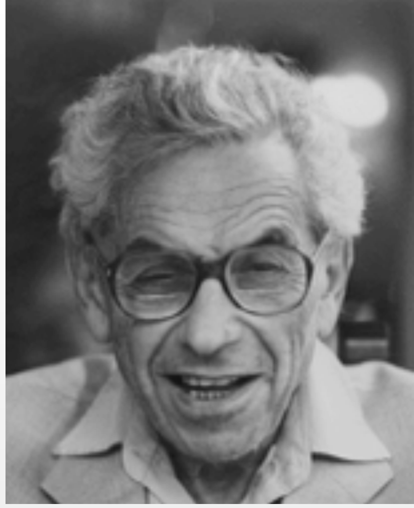

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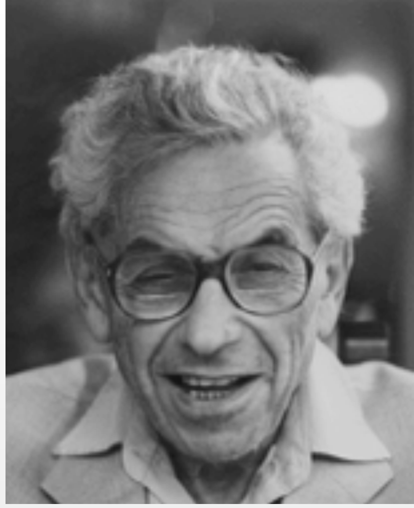

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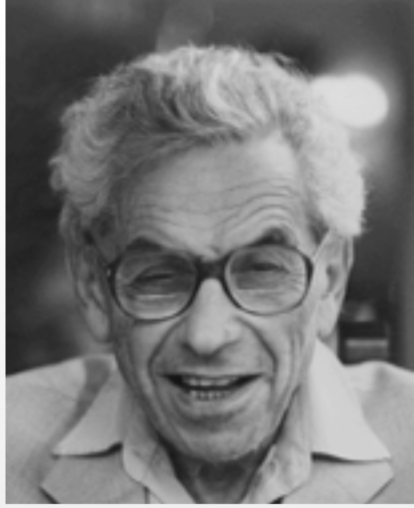

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1	…
2	Einstein, Glashow, S.-S. Chern, Freedman, Knuth, Shor …
3	Bethe, Schrödinger, Fermi, Pauli, Feynman, Wilczek, Witten, Kohn, C.-N. Yang …

How did I earn an Erdős number of 2 ?

—new adventures of quantum Monte Carlo

Lei Wang
Institute of Physics

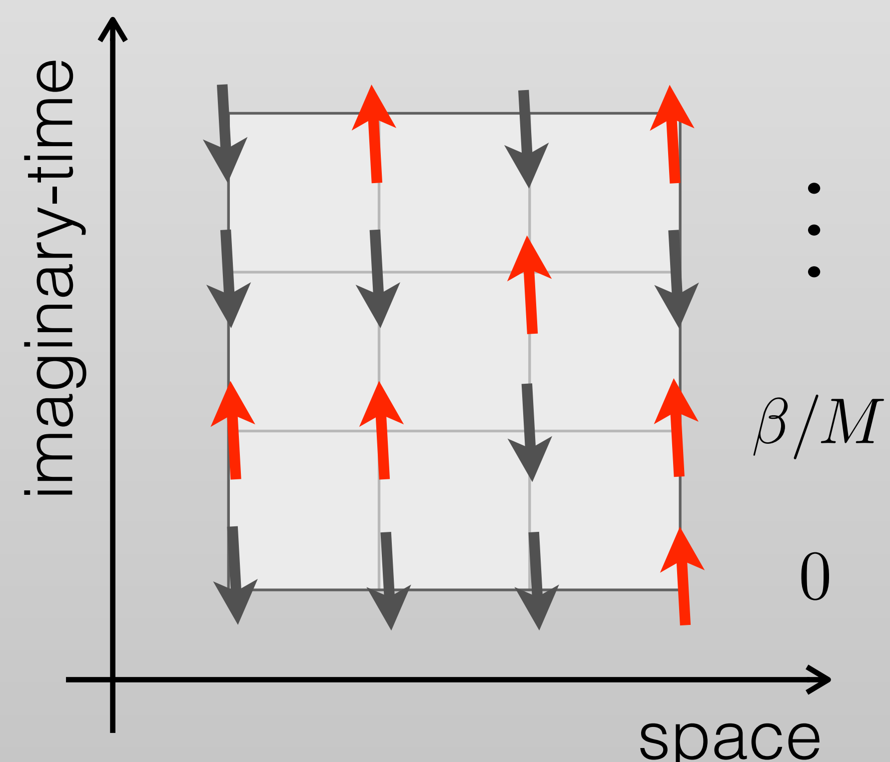
2016.04 @ CSRC

“Quantum” Monte Carlo

$$Z = \mathrm{Tr}\left(e^{-\beta \hat{H}}\right) \qquad \hat{H} = \hat{H}_0 + \lambda \hat{H}_1$$

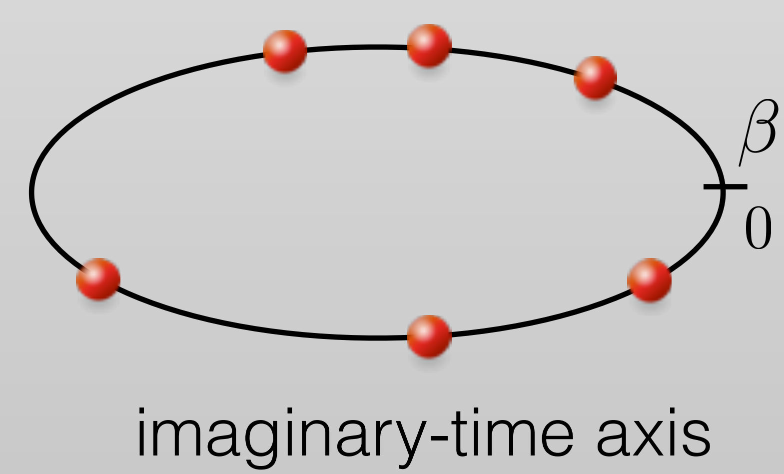
Trotterization

$$Z = \mathrm{Tr}\left(e^{-\frac{\beta}{M}\hat{H}} \dots e^{-\frac{\beta}{M}\hat{H}}\right)$$



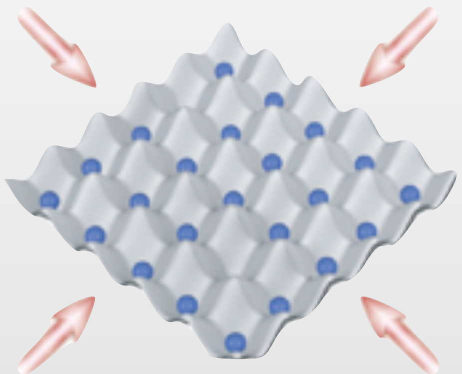
Diagrammatic approach

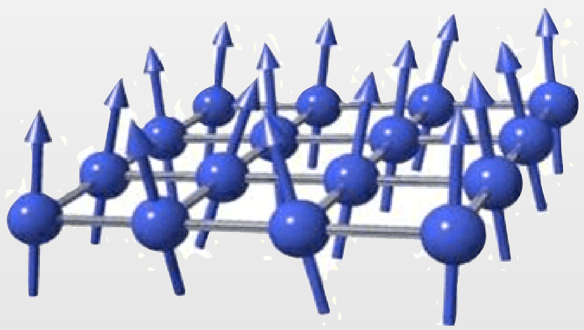
$$Z = \sum_{k=0}^{\infty} \lambda^k \int_0^\beta d\tau_1 \dots \int_{\tau_{k-1}}^{\beta} d\tau_k \times$$
$$\mathrm{Tr}\left[(-1)^k e^{-(\beta-\tau_k)\hat{H}_0} \hat{H}_1 \dots \hat{H}_1 e^{-\tau_1 \hat{H}_0}\right]$$

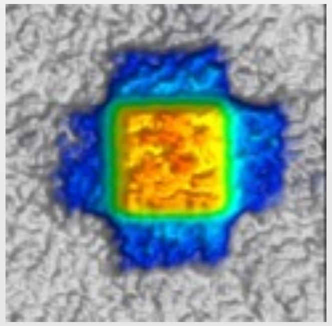


Beard and Wiese, 1996
Prokof’ev, Svistunov, Tupitsyn,1996

Diagrammatic approaches







bosons

World-line Approach

Prokof'ev et al, JETP, **87**, 310 (1998)

quantum spins

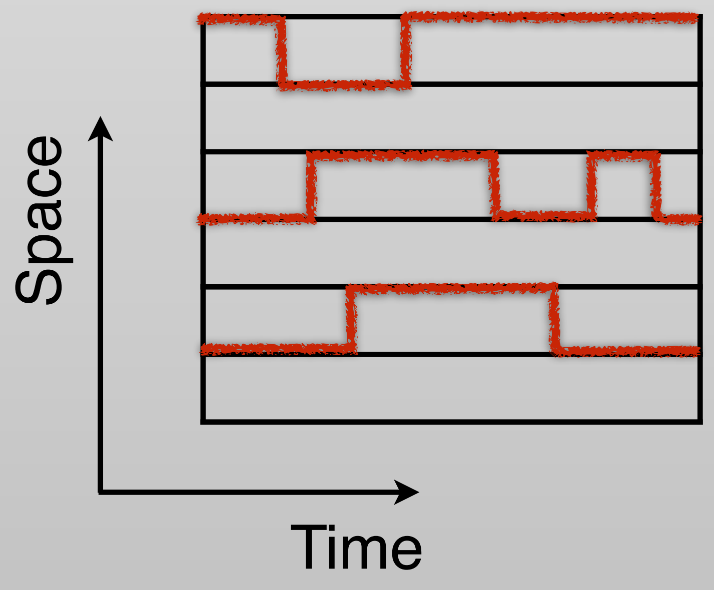
Stochastic Series Expansion

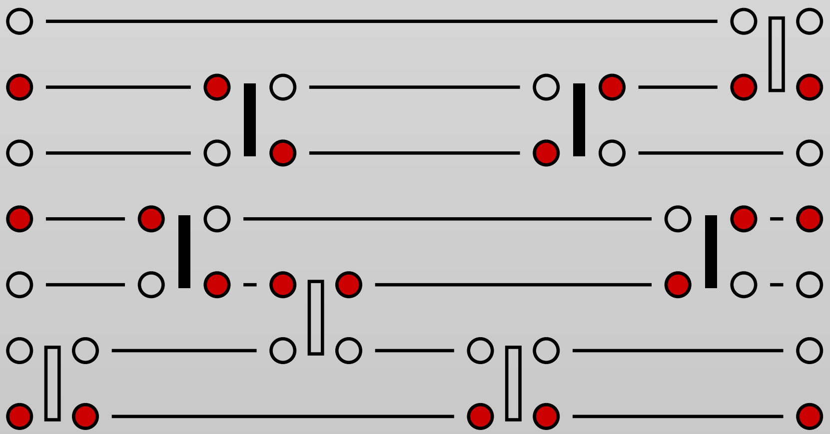
Sandvik et al, PRB, **43**, 5950 (1991)

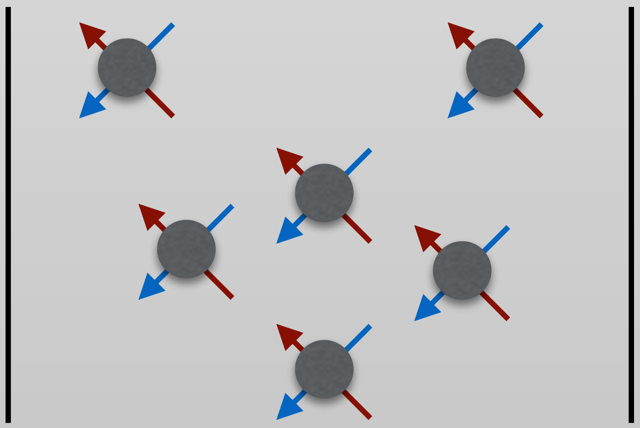
fermions

Determinantal Methods

Gull et al, RMP, **83**, 349 (2011)







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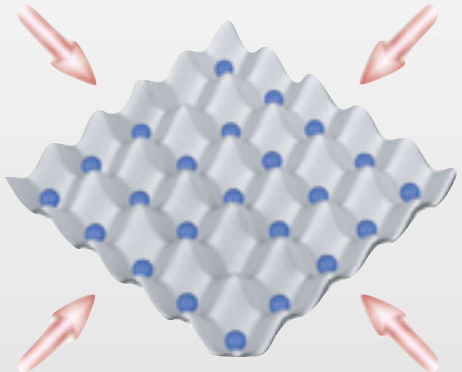

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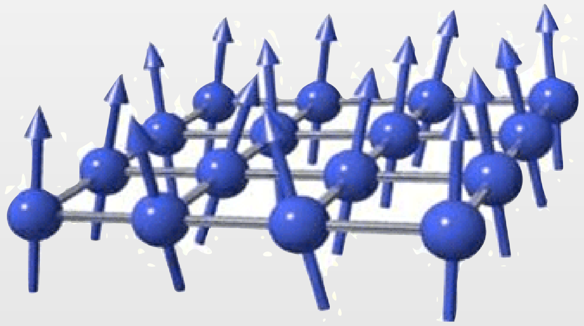

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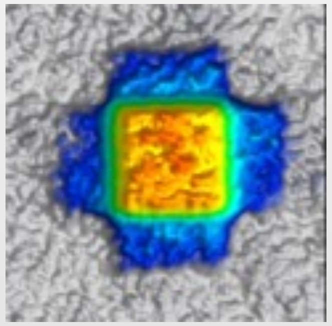

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LW and Troyer, PRL 2014

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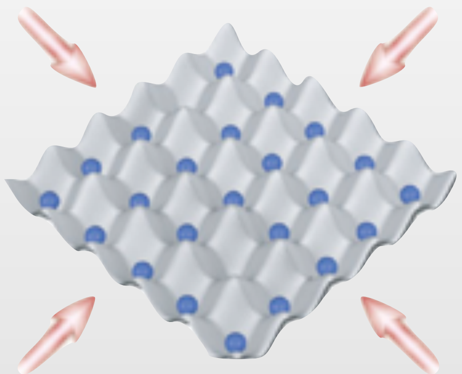

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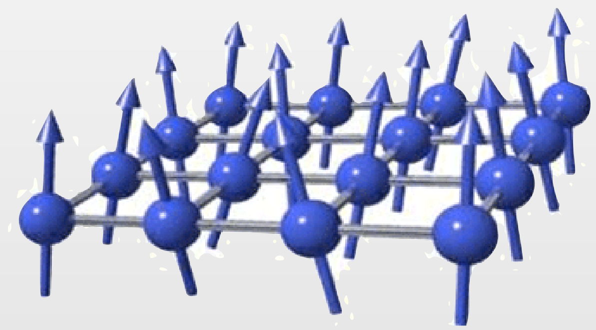

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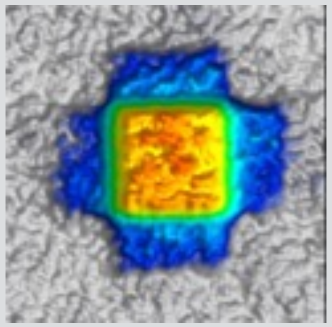

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LCT-QMC methods

Iazzi and Troyer, PRB 2015

LW, Iazzi, Corboz and Troyer, PRB 2015

Liu and LW, PRB 2015

What about the sign problem ?

Sign problem free: Kramers pairs due to the time-reversal symmetry

$$\det M \geq 0 \quad \text{if} \quad \begin{aligned} \Theta M \Theta^{-1} &= M \\ \Theta^2 &= -1 \end{aligned}$$

Lang et al, Phys. Rev. C, 1993
Koonin et al, Phys. Rep, 1997
Hands et al, EPJC, 2000
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- Attractive interaction at any filling on any lattice
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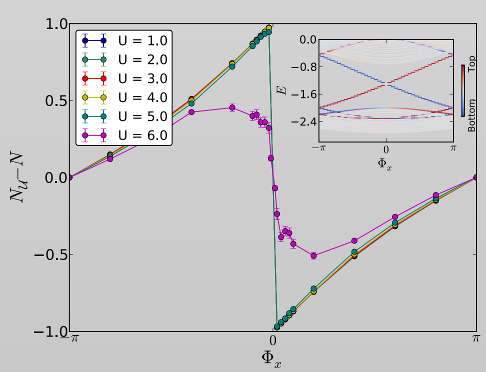
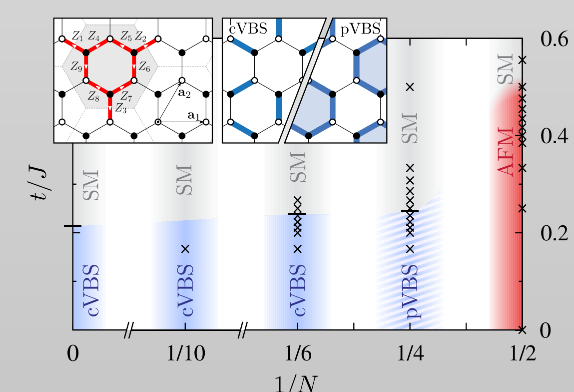
- And more ...



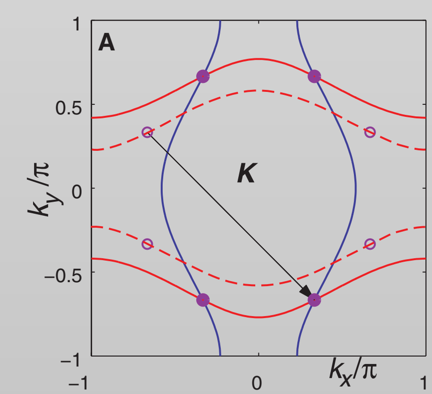
Hofstadter model

LW, Hung and Troyer, PRB 2014



SU(2N) models

Lang, Meng, Muramatsu, Wessel and Assaad, PRL 2013



Spin-fermion models

Berg, Metliski and Sachdev, Science 2012

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Hands et al, EPJC, 2000
Wu et al, PRB, 2005

But, how about this ?

Spinless fermions $\hat{H} = \sum_{\langle i,j \rangle} -t \left(\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i \right) + V \left(\hat{n}_i - \frac{1}{2} \right) \left(\hat{n}_j - \frac{1}{2} \right)$

Scalapino et al, PRB 1984 Gubernatis et al, PRB 1985

up to 8*8 square lattice and T≥0.3t

Meron cluster approach, Chandrasekharan and Wiese, PRL 1999

solves sign problem for V ≥ 2t

Solutions !

PHYSICAL REVIEW B **89**, 111101(R) (2014)

Solution to sign problems in half-filled spin-polarized electronic systems

Emilie Fulton Huffman and Shailesh Chandrasekharan
Department of Physics, Duke University, Durham, North Carolina 27708, USA

(Received 19 December 2013; revised manuscript received 14 February 2014; published 12 March 2014)

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Solving the fermion sign problem in quantum Monte Carlo simulations by Majorana representation

Zi-Xiang Li,[1] Yi-Fan Jiang,[1,2] and Hong Yao[1,3,*]
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[2]*Department of Physics, Stanford University, Stanford, California 94305, USA*
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PHYSICAL REVIEW B **91**, 235151 (2015)

Efficient continuous-time quantum Monte Carlo method for the ground state of correlated fermions

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PRL **115,** 250601 (2015) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2015

Split Orthogonal Group: A Guiding Principle for Sign-Problem-Free Fermionic Simulations

Lei Wang,[1] Ye-Hua Liu,[1] Mauro Iazzi,[1] Matthias Troyer,[1] and Gergely Harcos[2]
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Latest update

Wei, Wu, Li, Zhang, Xiang, arXiv:1601.01994

Li, Jiang Yao, arXiv:1601.05780

A tale of open science

$$w(\mathcal{C}) \sim \det\left(I + \mathcal{T}e^{-\int_0^\beta d\tau H_\mathcal{C}(\tau)}\right)$$

Free fermions with an effective imaginary-time dependent Hamiltonian

A tale of open science

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Free fermions with an effective imaginary-time dependent Hamiltonian

Let real matrices $A_i = \begin{pmatrix} 0 & B_i \\ B_i^T & 0 \end{pmatrix}$

then $\det\left(I + e^{A_1} e^{A_2} \ldots e^{A_N}\right) \geq 0$

http://mathoverflow.net/questions/204460/how-to-prove-this-determinant-is-positive

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Tao and Paul Erdős in 1985

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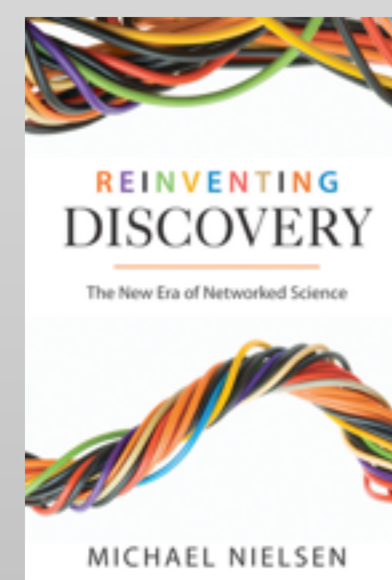

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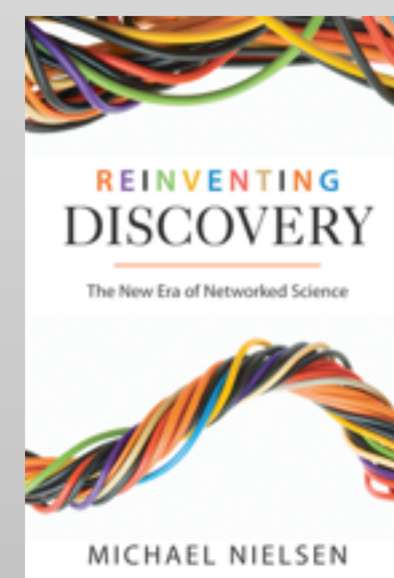

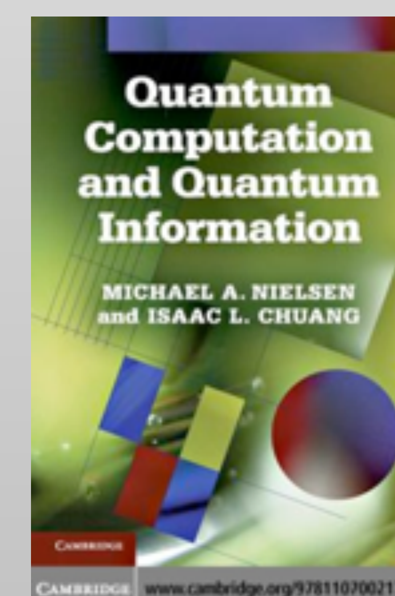

A new "de-sign" principle

LW, Liu, Iazzi, Troyer and Harcos, PRL 2015

If $M^T \eta M = \eta$ where $\eta = \mathrm{diag}(I, -I)$

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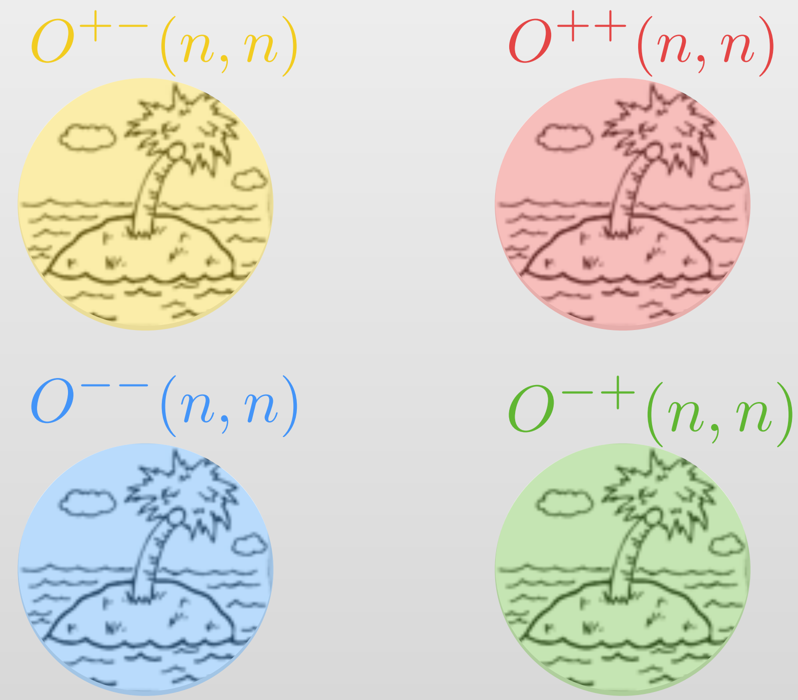
LW, Liu, Iazzi, Troyer and Harcos, PRL 2015

If $M^T \eta M = \eta$ where $\eta = \mathrm{diag}(I, -I)$

Then $M \in O(n,n)$

split orthogonal group



A new "de-sign" principle

LW, Liu, Iazzi, Troyer and Harcos, PRL 2015

If $M^T \eta M = \eta$ where $\eta = \mathrm{diag}(I, -I)$

Then $\det(I + M)$

has a definite sign

for each component !

$O^{+-}(n,n)$

$\equiv 0$

$O^{++}(n,n)$

≥ 0

$O^{--}(n,n)$

≤ 0

$O^{-+}(n,n)$

$\equiv 0$

A new "de-sign" principle

LW, Liu, Iazzi, Troyer and Harcos, PRL 2015

If $M^T \eta M = \eta$ where $\eta = \mathrm{diag}(I, -I)$

$\mathcal{T} e^{-\int_0^\beta d\tau H_{\mathcal{C}}(\tau)}$

Then $\det(I + M)$

has a definite sign

for each component !

$O^{+-}(n,n)$

$\equiv 0$

$O^{++}(n,n)$

≥ 0

$O^{--}(n,n)$

≤ 0

$O^{-+}(n,n)$

$\equiv 0$

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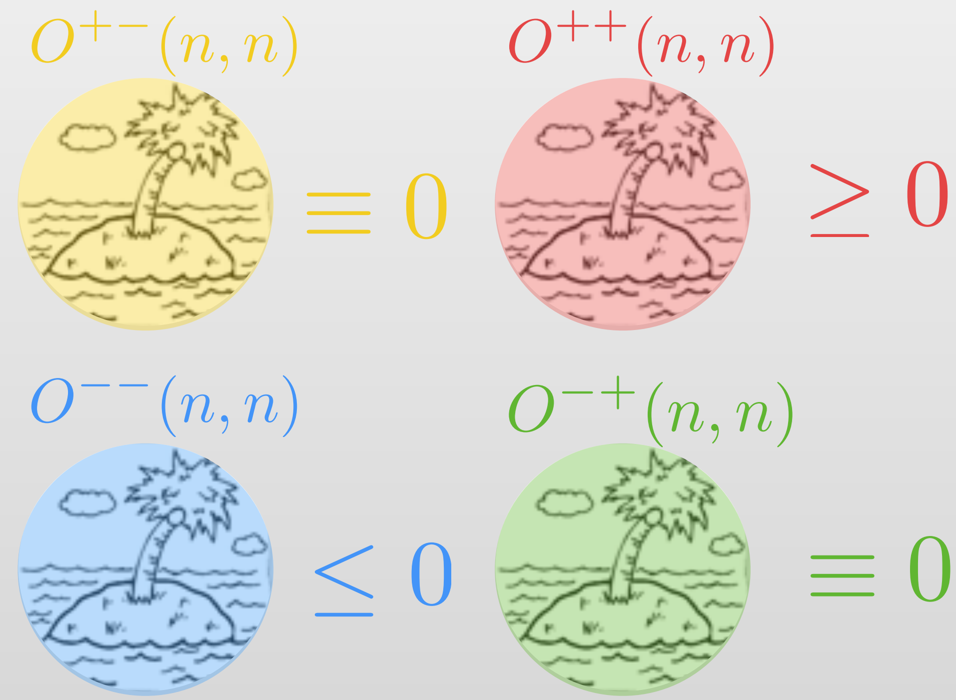

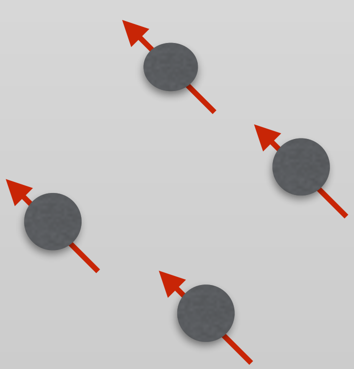

spinless fermions

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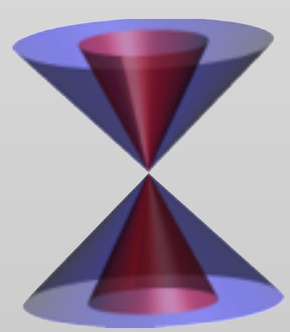
LW, Corboz, Troyer, NJP 2014

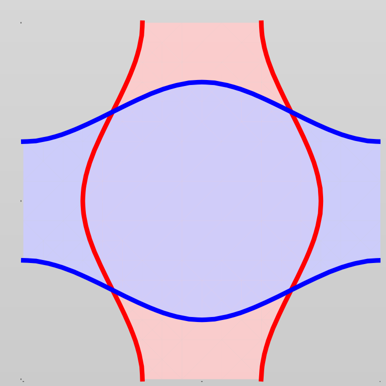
LW, Iazzi, Corboz, Troyer, PRB, 2015

LW, Liu and Troyer, PRB 2016

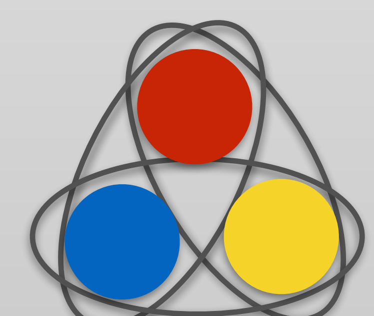


split Dirac cone

Liu and LW, PRB 2015



spin nematicity



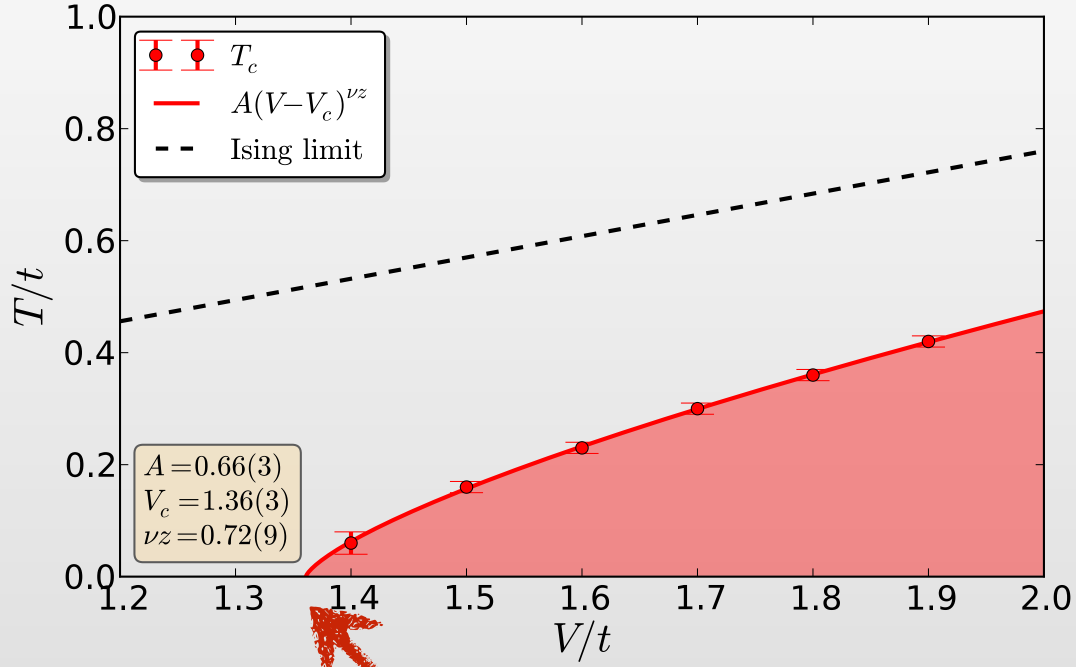
SU(3)

Solutions to the spinless t-V model

$$\hat{H}_0 = -t \sum_{\langle i,j \rangle} \left(\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i \right)$$

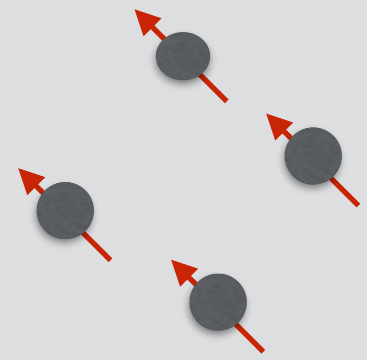
$$\hat{H}_1 = V \sum_{\langle i,j \rangle} \left(\hat{n}_i - \frac{1}{2} \right) \left(\hat{n}_j - \frac{1}{2} \right)$$

$$w(\mathcal{C}) \sim \mathrm{Tr}\left[(-1)^k e^{-(\beta-\tau_k)\hat{H}_0} \hat{H}_1 \ldots \hat{H}_1 e^{-\tau_1 \hat{H}_0} \right]$$



Novel fermionic quantum critical point

cf Li, Jiang, Yao 2015 Hesselmann and Wessel 2016



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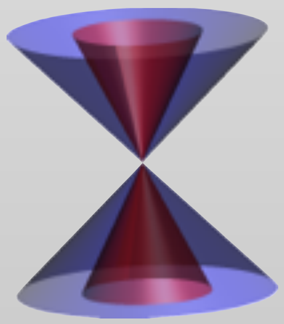

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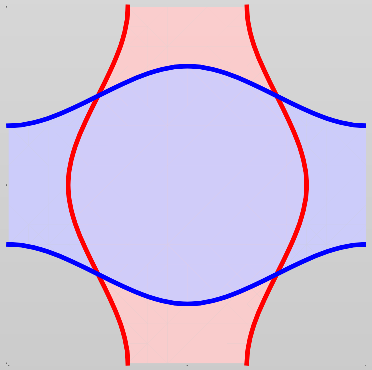

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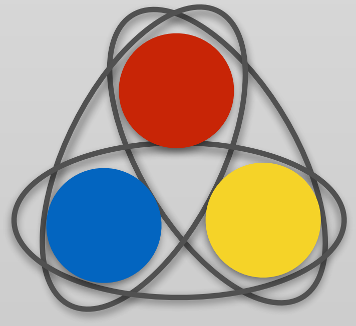

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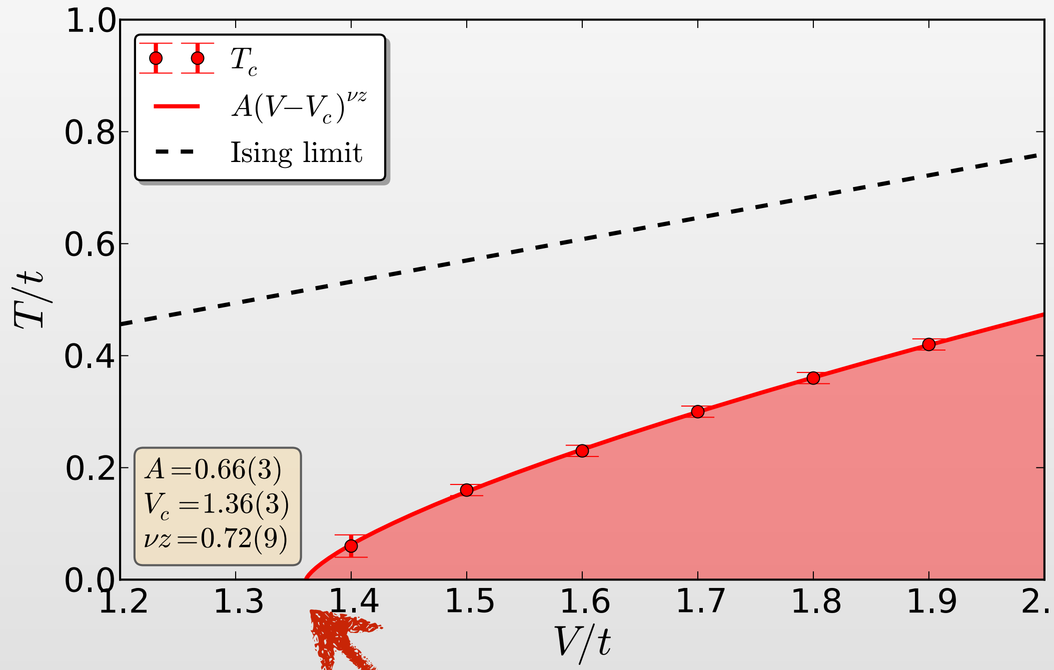

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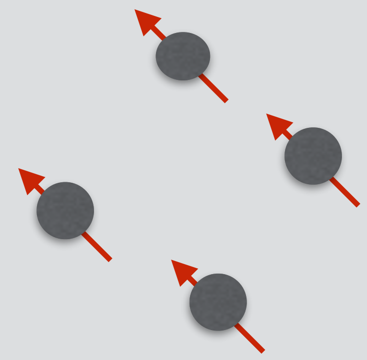

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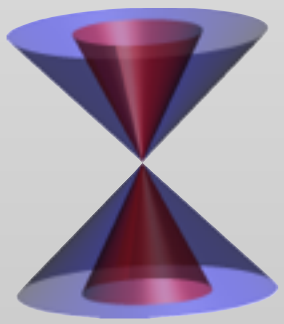

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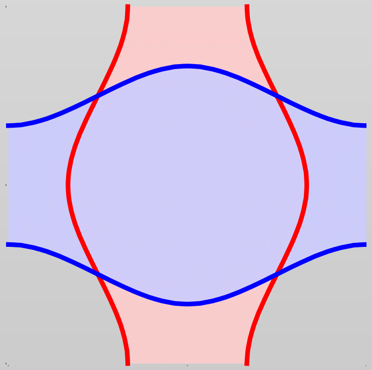

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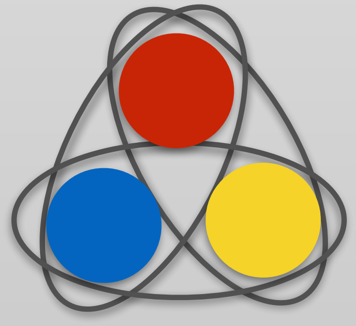

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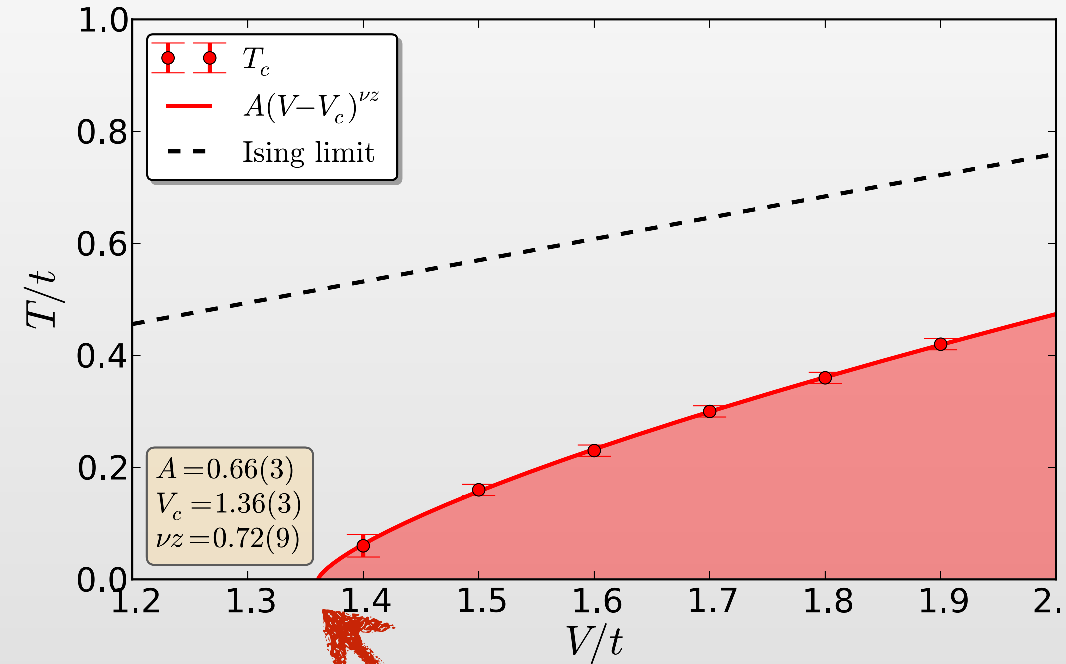

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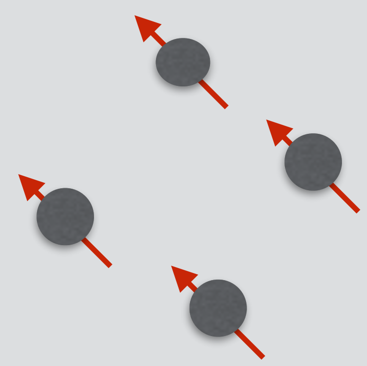

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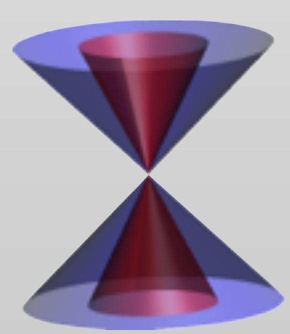

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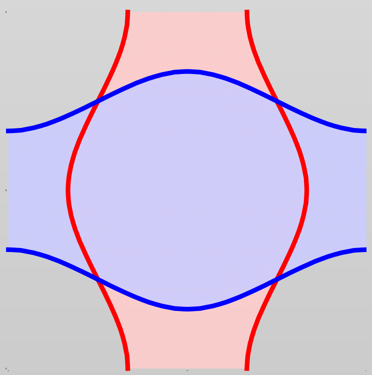

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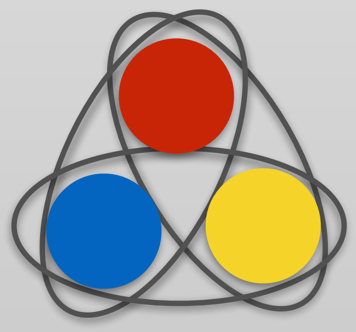

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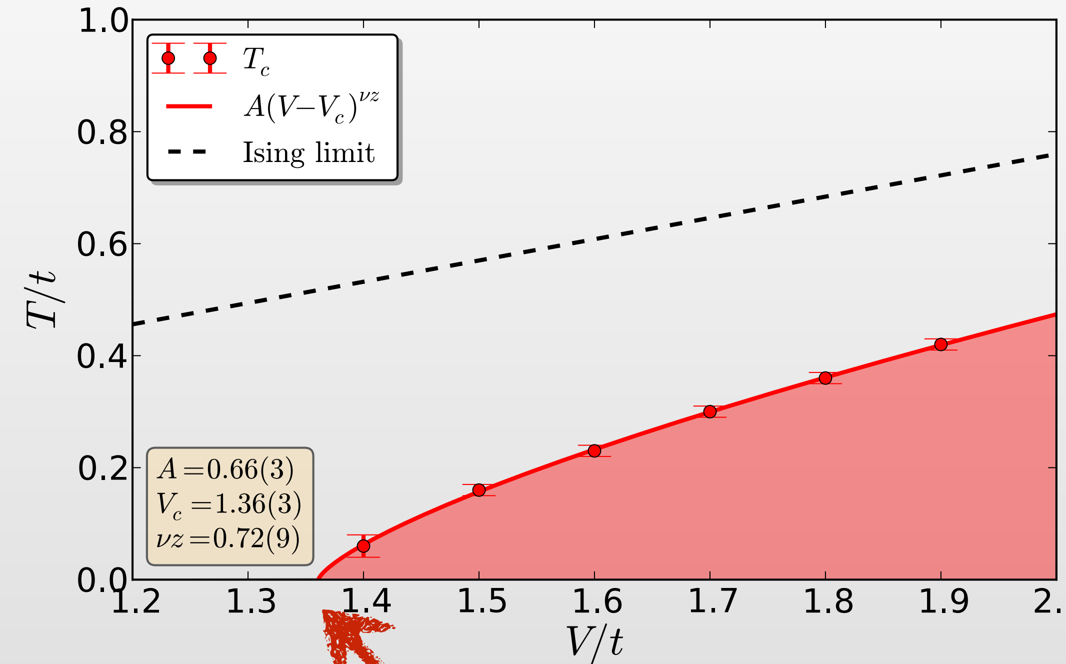

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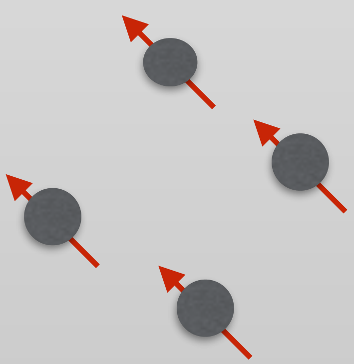

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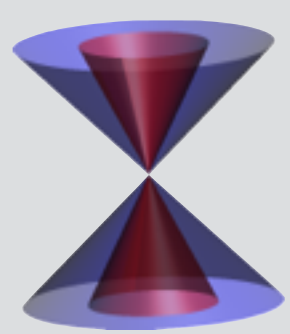

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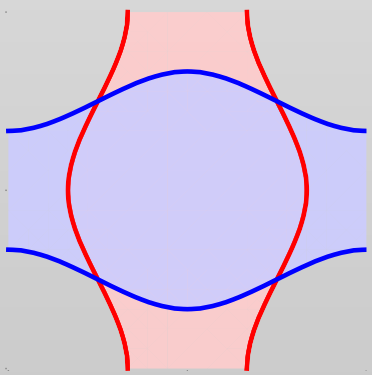

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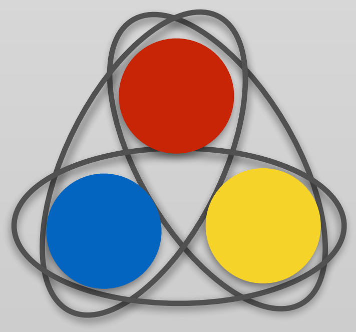
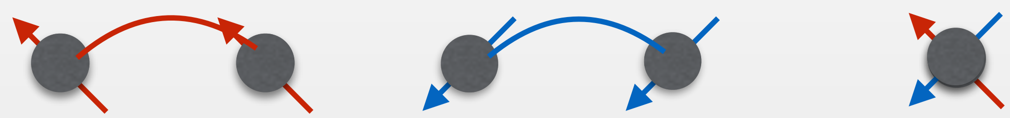


SU(3)

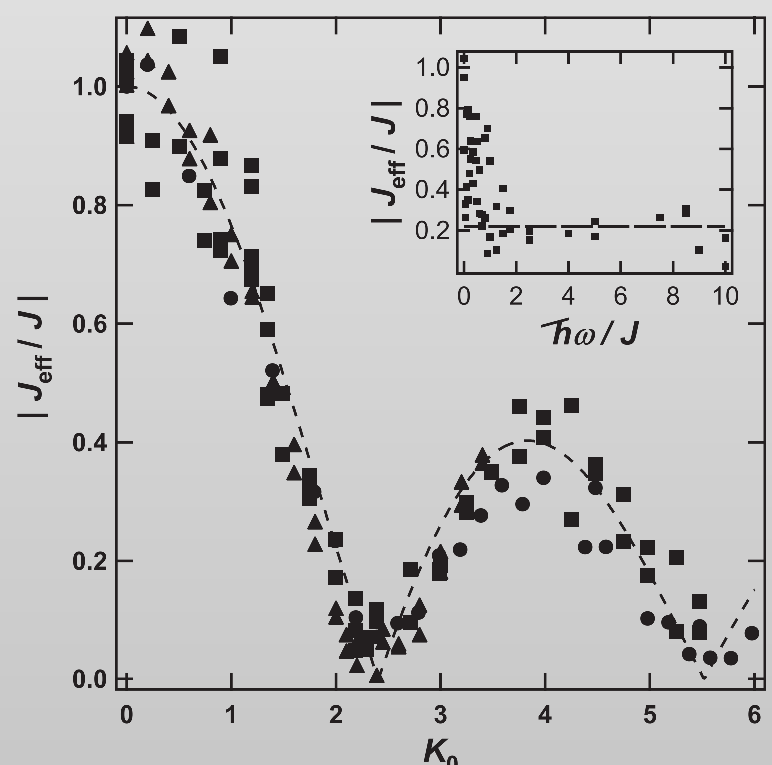
Asymmetric Hubbard model

$$t_{\uparrow} \neq t_{\downarrow} \qquad U$$

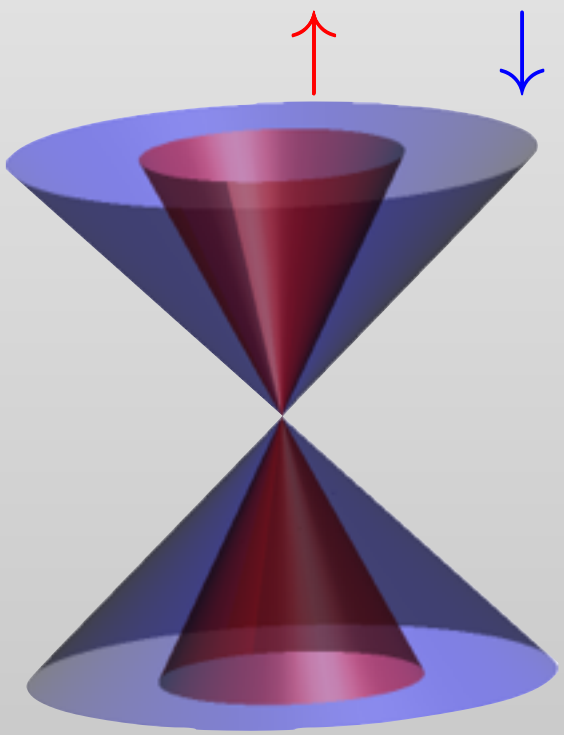


- Realization: mixture of ultracold fermions (e.g. ^{6}Li and ^{40}K)
- Now, continuously tunable by spin-dependent modulations Jotzu et al, PRL 2015

$$t_{\downarrow}/t_{\uparrow} \in (-\infty, \infty)$$



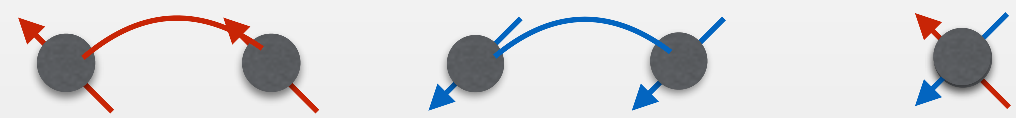
Lignier et al, PRL 2007 and many others



Dirac fermions with unequal Fermi velocities

Asymmetric Hubbard model

$$t_{\uparrow} \neq t_{\downarrow} \qquad U$$



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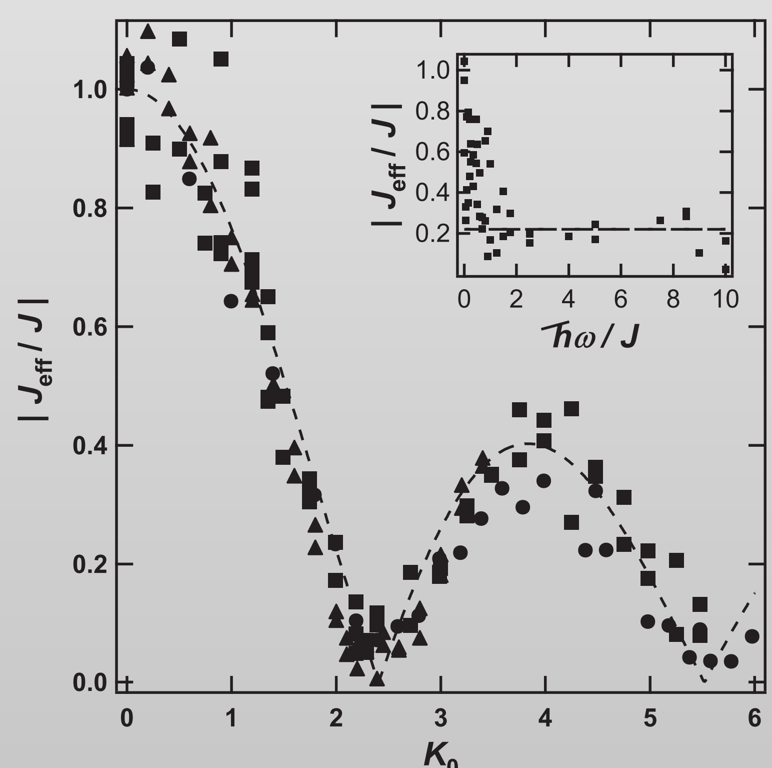

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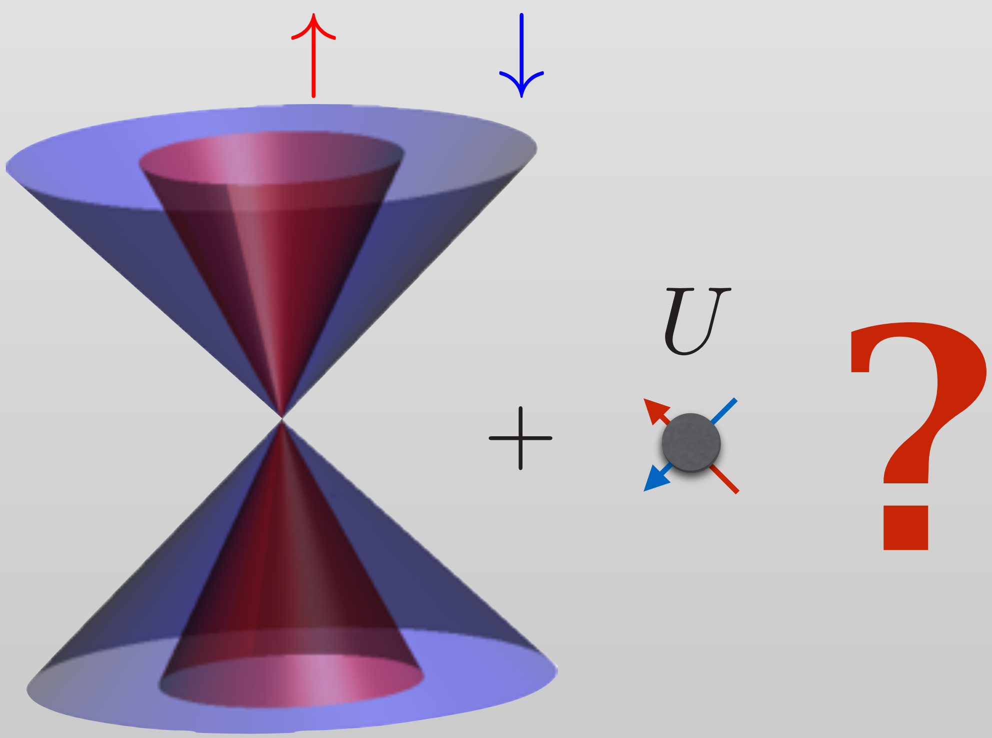
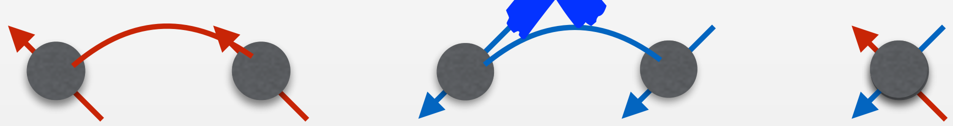


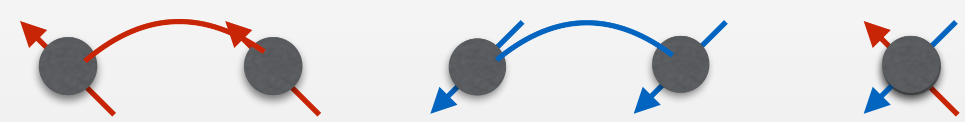
Dirac fermions with unequal Fermi velocities

Two limiting cases

$t_\uparrow$ ~~$t_\downarrow$~~ U



$t_\uparrow$ $t_\downarrow$ $\ll$ U



Falicov-Kamball Limit

SIMPLE MODEL FOR SEMICONDUCTOR-METAL TRANSITIONS: SmB_6 AND TRANSITION-METAL OXIDES

L. M. Falicov*
Department of Physics, University of California, Berkeley, California 94720

and

J. C. Kimball†
Department of Physics, and The James Franck Institute, University of Chicago, Chicago, Illinois 60637

(Received 12 March 1969)

We propose a simple model for a semiconductor-metal transition, based on the existence of both localized (ionic) and band (Bloch) states. It differs from other theories in that we assume the one-electron states to be essentially unchanged by the transition. The electron-hole interaction is responsible for the anomalous temperature dependence of the number of conduction electrons. For interactions larger than a critical value, a first-order semiconductor-metal phase transition takes place.

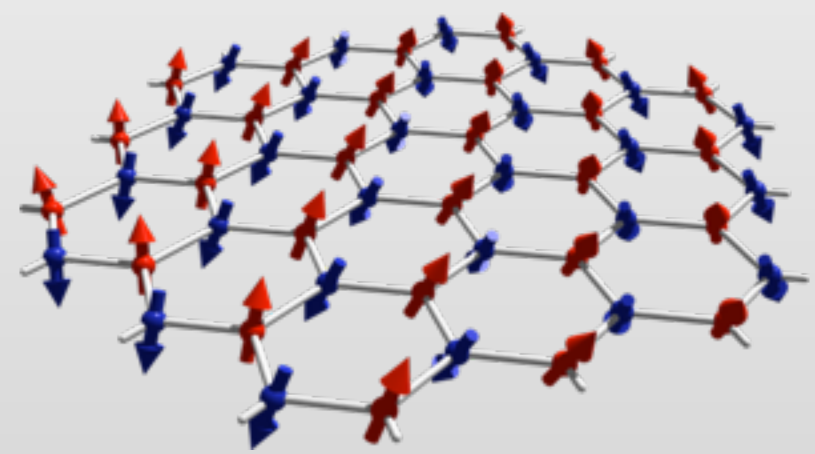
Long-range spin order on bipartite lattices with infinitesimal repulsion

Kennedy and Lieb 1986

"Fruit fly" of DMFT

Freericks and Zlatić, RMP, 2003

Strong Coupling Limit

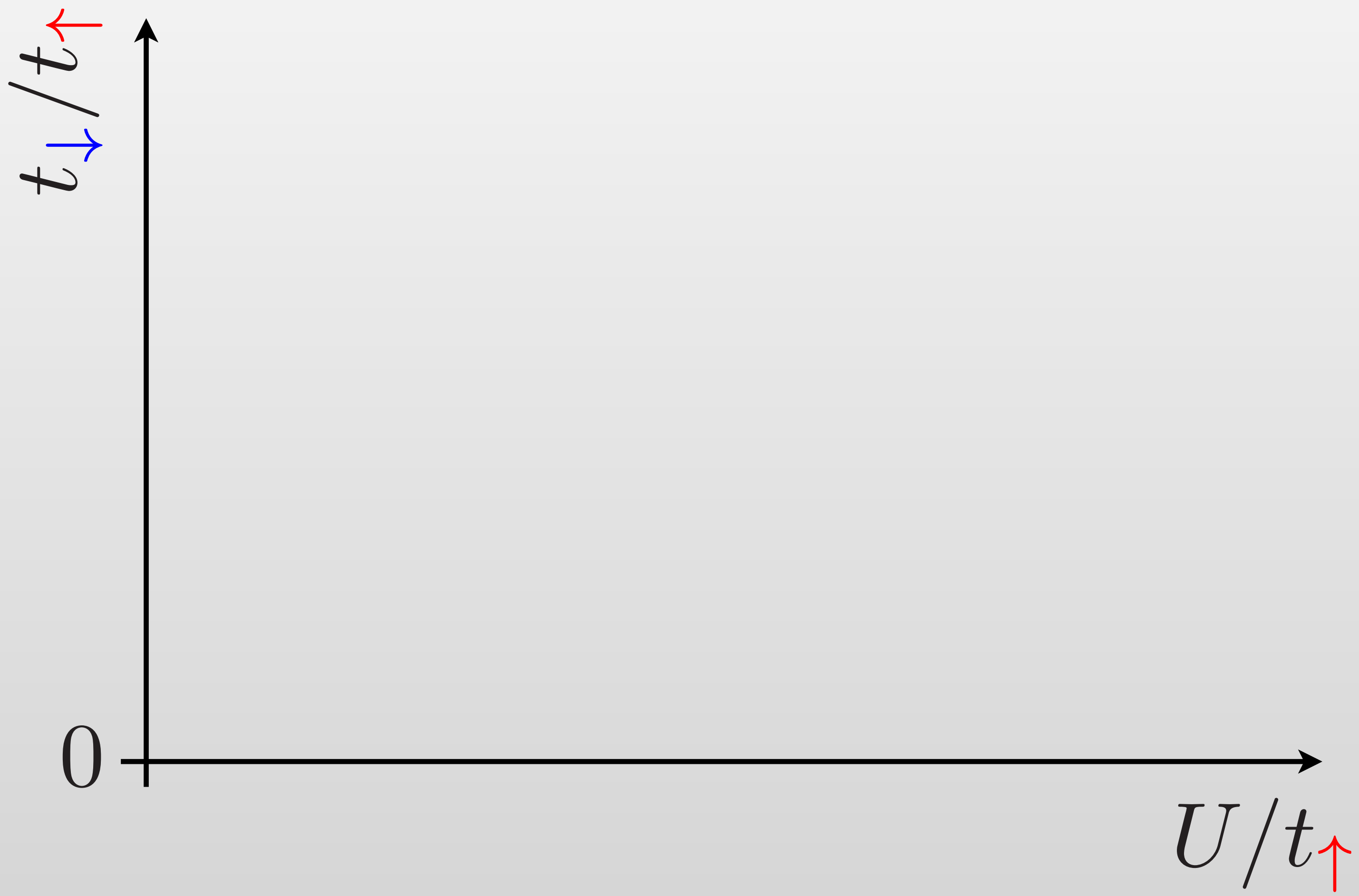


$$J_{xy}\left(\hat{S}_i^x\hat{S}_j^x + \hat{S}_i^y\hat{S}_j^y\right) + J_z\hat{S}_i^z\hat{S}_j^z$$

$$\frac{4t_\uparrow t_\downarrow}{U} \le \frac{2(t_\uparrow^2 + t_\downarrow^2)}{U}$$

XXZ model with Ising anisotropy

Phase diagram



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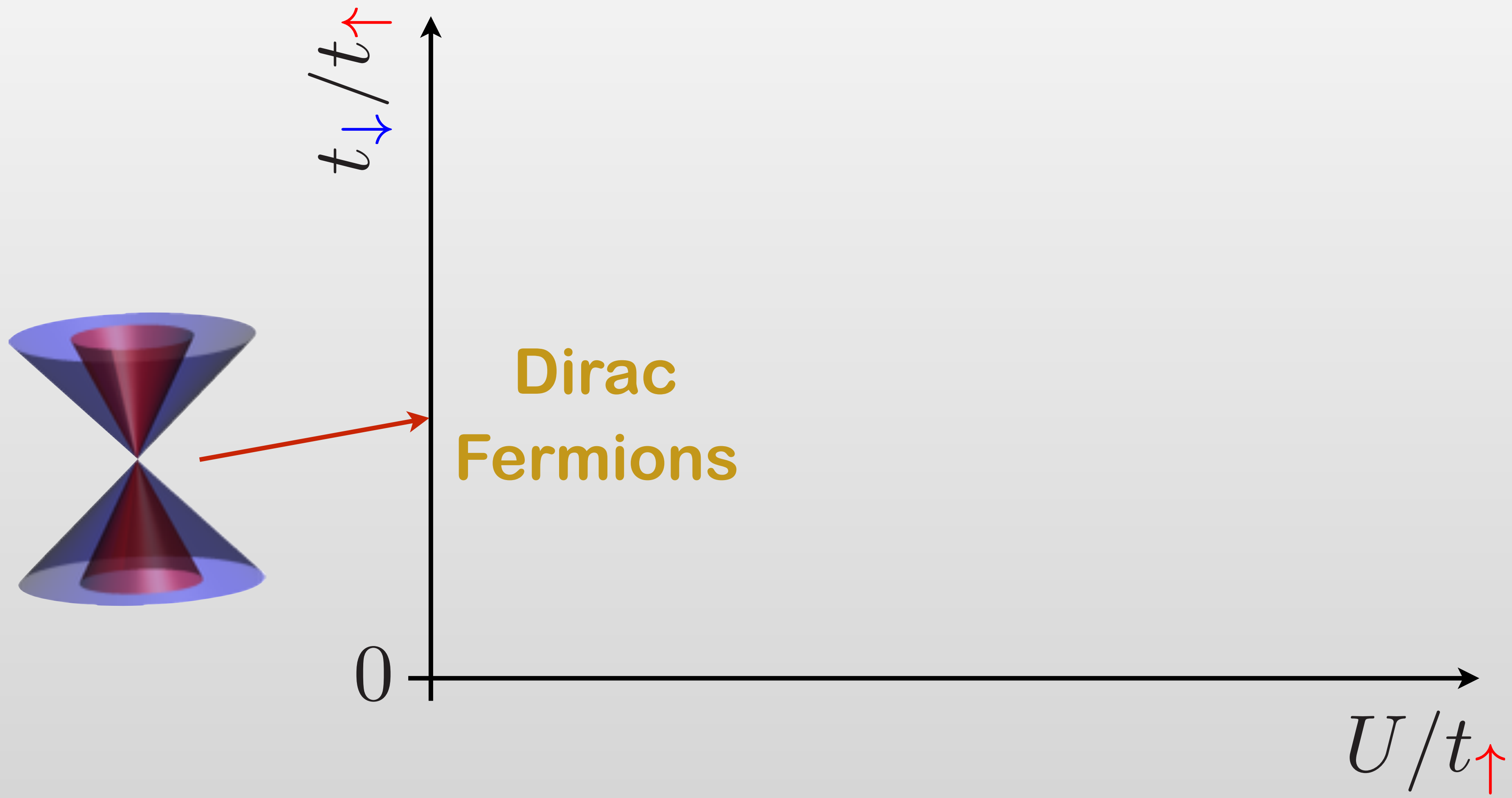

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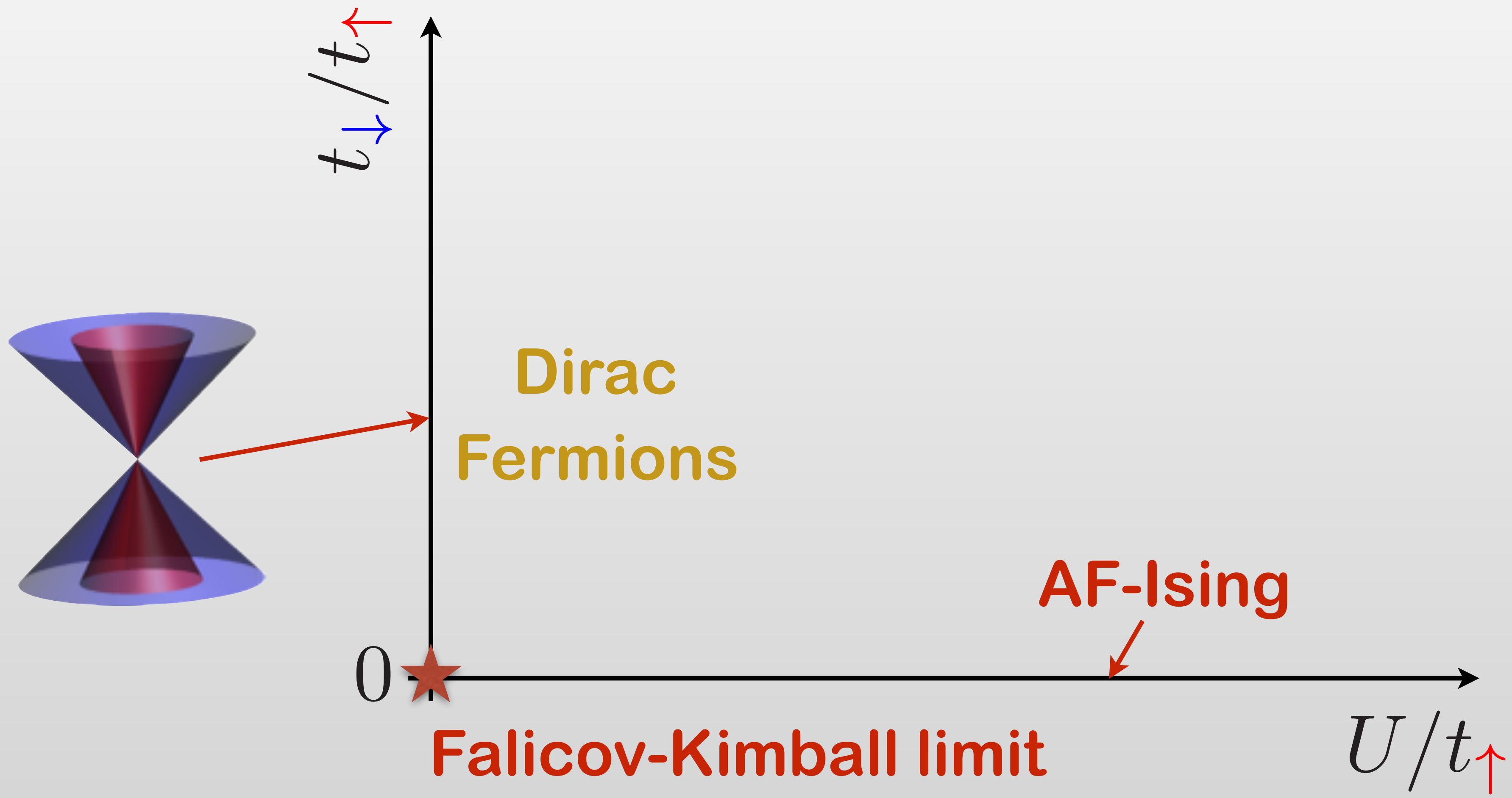

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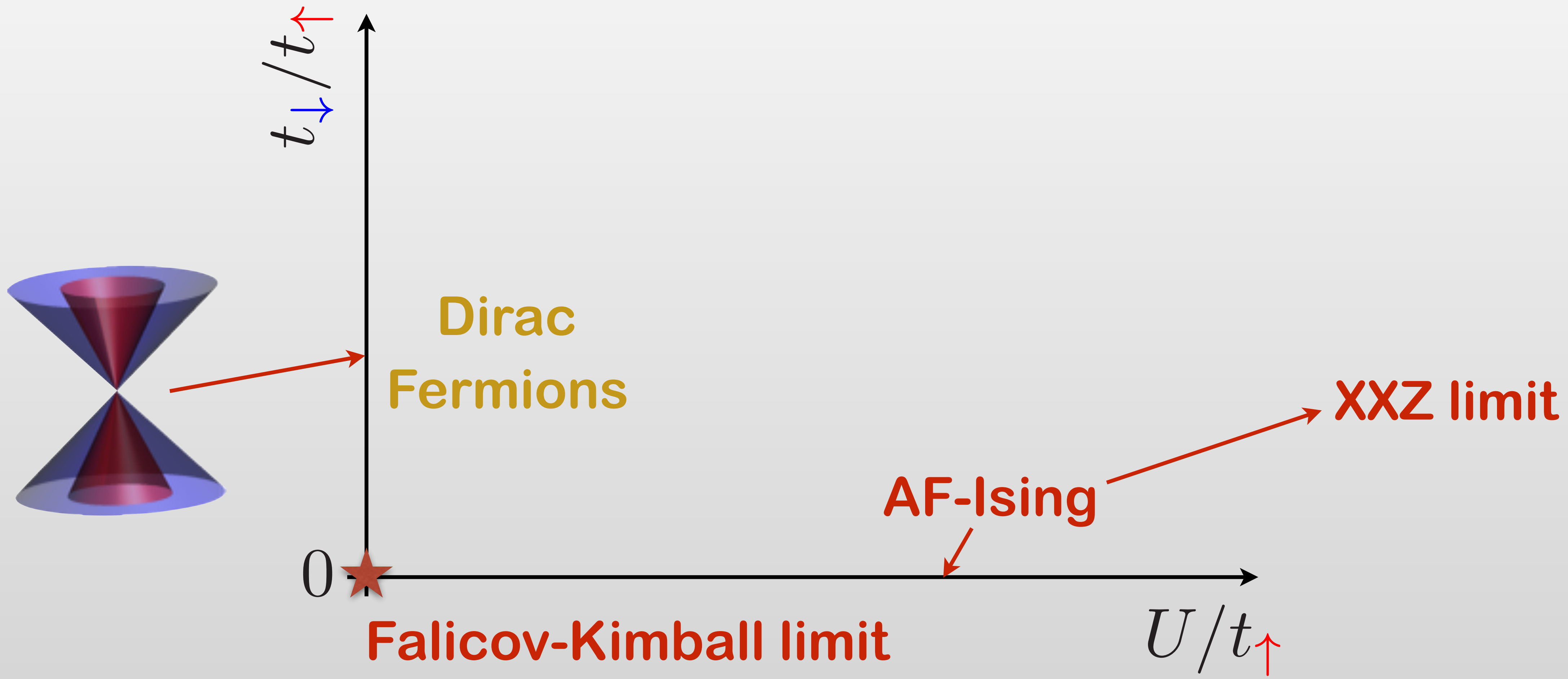

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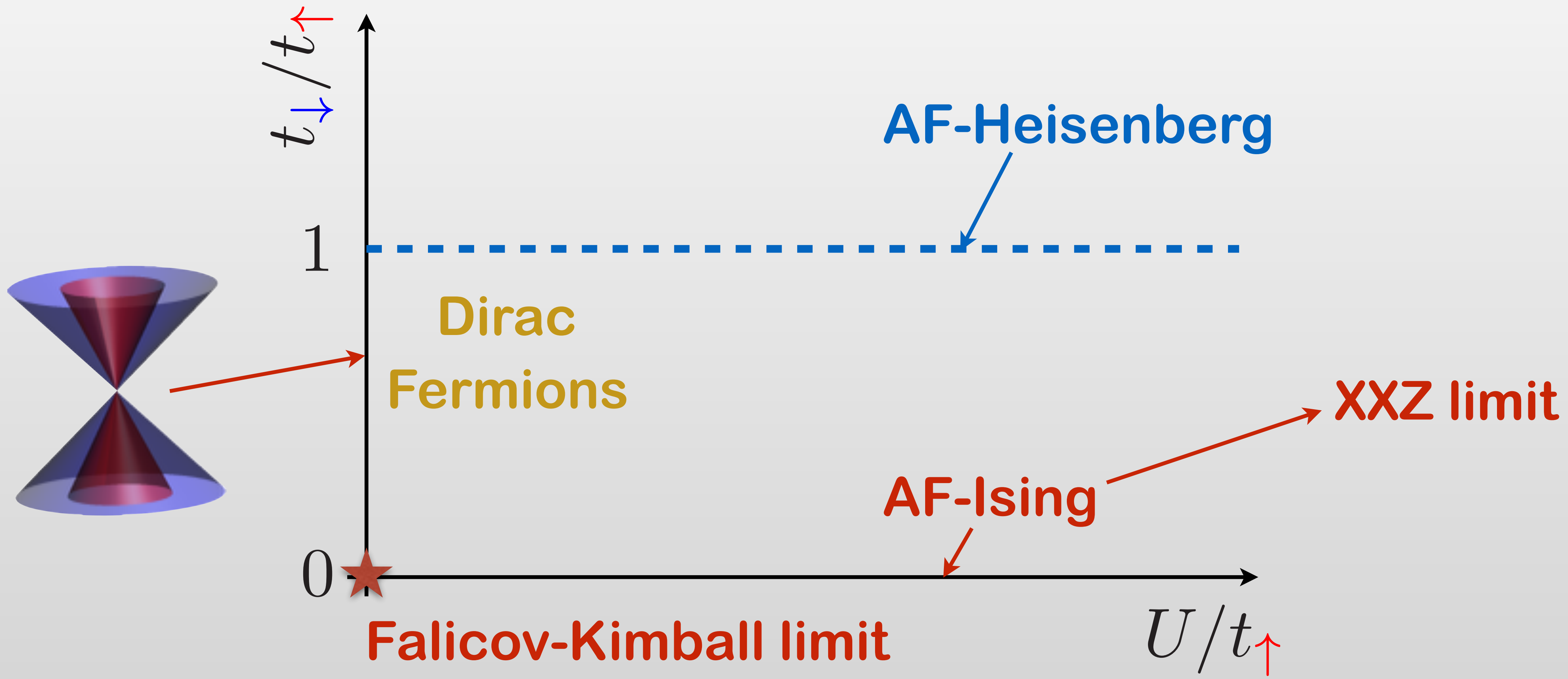

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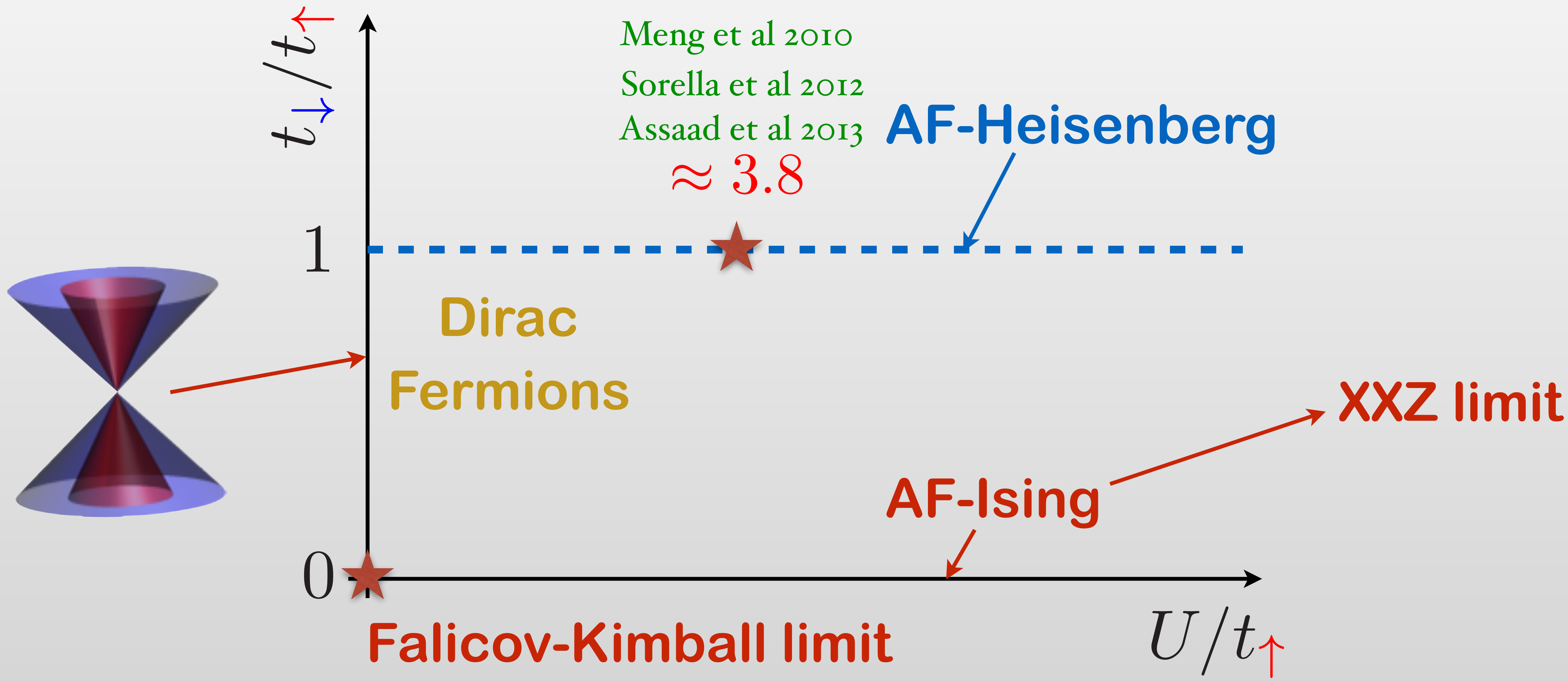

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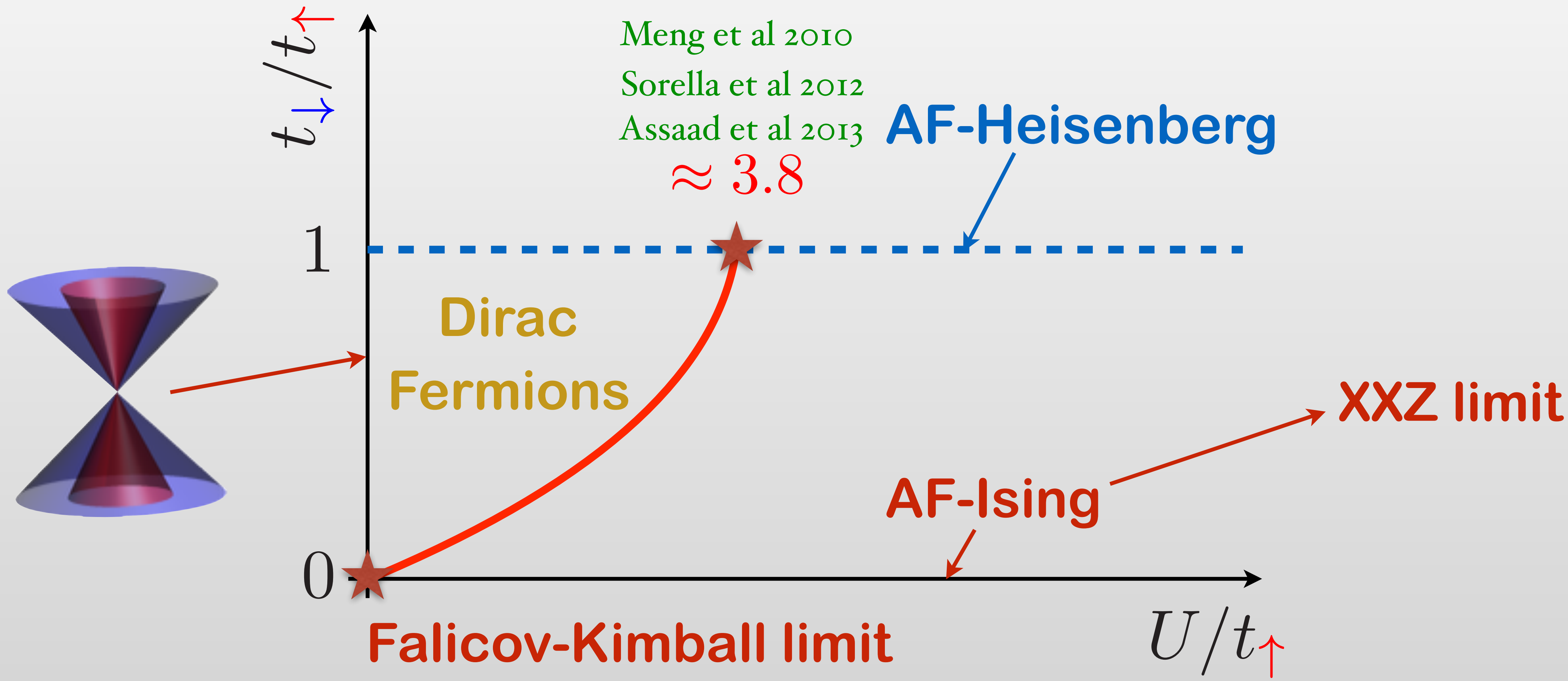

- How to connect the phase boundary ?
- What is the universality class ?

It is an exciting time

better scaling

entanglement & fidelity

sign problem

Thanks to my collaborators!

Mauro Iazzi

Philippe Corboz

Jakub Imriška

Ping Nang Ma

Gergely Harcos

Ye-Hua Liu

Matthias Troyer

ELSEVIER

Discrete Mathematics 200 (1999) 95–99

DISCRETE MATHEMATICS

Popular distances in 3-space

Paul Erdős [a], Gergely Harcos [b], János Pach [a,c,*]

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[b] *Department of Mathematics, University of Illinois at Urbana–Champaign, 1409 West Green Street, Urbana, IL 61801, USA*

[c] *Courant Institute, New York University, 251 Mercer Street, New York, NY 10012, USA*

Received 19 April 1998; accepted 9 June 1998

Mauro Iazzi

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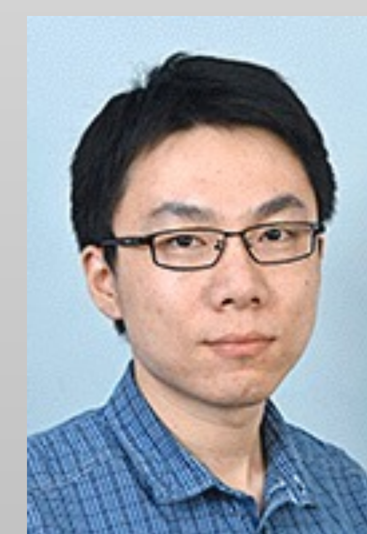

Matthias Troyer


中国科学院物理研究所
Institute of Physics Chinese Academy of Sciences
广告
欢迎博士后，博士生加入！
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Wenan Guo
Li Huang
Zi Yang Meng
Ninghua Tong
Lei Wang
Yilin Wang
Tao Xiang
Zhiyuan Xie
...